A Learning Model of Search and Matching

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Agenda

- Why does trade in some markets (labor, housing, taxis) take time?

- Build microfoundation of search frictions.
  1. Gains from trade depend on idiosyncratic match quality
  2. Gathering information takes time

- Questions:
  - Positive: How do unemployment rates and durations, worker-firm match qualities, etc. respond to shocks?
  - Normative: is the search process efficient? Room for policy?

Related Work

- Geographic models (Lagos, 2000)
  - Microfoundation for search
  - Not based on matching
- Stock-flow matching (Coles & Smith, 1998; Coles & Muthoo, 1998)
  - Focus on match quality
  - Match qualities instantaneous observable
  - Lack of a match only if LLN fails
- Assignment algorithms (Gale & Shapley, 1962; Roth, 1982)
  - Study which worker takes which job
  - No delays in matching
  - Information-gathering typically not modeled
- Competitive search equilibrium (Moen, 1997)

Outline

Production Technology

- Continuous time
- A measure-1 continuum of workers
- Free entry of single-worker firms
- Worker-firm pair (job) produces output flow $y \in \{y_L, y_H\}$
- $\pi = \Pr[y = y_H]$ is iid across all pairs
- Jobs are destroyed at exogenous rate s

Meeting Technology

- No frictions in meeting counterparties
- At any point in time:
  - E workers employed
  - I workers being interviewed for a job
  - Q workers queueing for an interview. Inflow rate u
  - J firms are waiting to find a worker to interview. Inflow rate e
- Workers find firms at Poisson rate μ
  - If $J > 0$, $\mu = \infty$
  - If $J = 0$ and $Q > 0$, $\mu = \frac{e}{Q}$
- Firms find workers at Poisson rate η
  - If $Q > 0$, $\eta = \infty$,
  - If $Q = 0$ and $J > 0$, $\eta = \frac{u}{J}$
  - Starting a search costs κ

Interviewing Technology

- Meeting immediately produces a first impression z_0:

$$z_0 = \Pr\left[y = y_H | z_0\right]$$

  - $\mathbb{E}\left(z_0\right) = \pi$
  - $\Pr\left[z_0 = \pi\right] = 1$ is special case

- If interview continues, both observe stochastic process x_t

$$dx_t = \mu\left(y\right) + \sigma dB_t$$

- Posterior $z_t = \Pr\left[y = y_H | \mathcal{I}_t\right]$ follows

$$dz_t = \frac{\mu^H - \mu^L}{\sigma} z_t\left(1 - z_t\right) dB_t$$

- For now, workers can only interview with one firm at a time

- Interviewing a worker costs c per unit of time

Preferences

- Firms and workers are risk neutral
- Discount the future at rate ρ
- While unemployed (either Q or I), workers obtain utility flow b

Outline

Markets

- A hiring policy is $h = \{w, z_L, z_H\}$
  - w: wage the worker will be paid if hired
  - z_L: cutoff beliefs such that the interview is terminated
  - z_H: cutoff beliefs such that the worker is hired

- Directed search: search for a specific hiring policy h

- Once the interview starts, both are committed to h

Value for Firm

- HJB equation:

$$\rho V(z;h) = -c + \mathbb{E}(dV)$$

- Using Ito's Lemma:

$$\rho V(z;h) = -c + \frac{1}{2}\left(\frac{\mu^H - \mu^L}{\sigma} z(1-z)\right)^2 V''(z;h)$$

with value matching condition

$$V(z_L;h) = 0$$
$$V(z_H;h) = \frac{1}{\rho + s}\left[z_H y_H + (1-z_H) y_L - w\right]$$

- The value of starting to look for a worker with policy h is

$$V_0(h) = \frac{\eta(h)\,\mathbb{E}(V(z_0;h))}{\rho + \eta(h)}$$

Value for Worker

- $\bar{W}$: value of being newly unemployed

- HJB equation:

$$\rho W\left(z;h,\bar{W}\right)=b+\frac{1}{2}\left(\frac{\mu^{H}-\mu^{L}}{\sigma}z\left(1-z\right)\right)^{2}W^{''}\left(z;h\right)$$

  with value matching conditions:

$$W\left(z_{L};h,\bar{W}\right)=\bar{W}$$

$$W\left(z_{H};h,\bar{W}\right)=\frac{w+s\bar{W}}{\rho+s}$$

- The value of starting to look for an interview under policy h is

$$W_{0}\left(h\right)=\frac{b+\mu\left(h\right)\mathbb{E}\left(W\left(z_{0};h\right)\right)}{\rho+\mu\left(h\right)}$$

Equilibrium Definition

A (steady state) competitive equilibrium is:

1. A value for waiting workers $\bar{W}$.
2. A subset $H_0 \subseteq H$ of hiring policies that firms are willing to consider.
3. Meeting rates $\eta(h)$ and $\mu(h)$ for firms and workers for all $h \in H$.

such that

1. Free entry:$V_0(h) \leq \kappa$ for all $h \in H$, with equality if $h \in H_0$.
2. Worker optimization: $W_0(h) \leq \bar{W}$ for all $h \in H$, with equality if $\eta(h) > 0$.
3. Frictionless meeting: function: $\max\{\eta(h), \mu(h)\} = \infty$.

Characterization

- Consider the following problem:

$$\bar{W} = \max_h \mathbb{E}\left[W\left(z_0; h, \bar{W}\right)\right]$$
$$s.t.$$
$$\mathbb{E}\left[V\left(z_0; h\right)\right] \geq \kappa \tag{1}$$

- Any equilibrium has $\bar{W}$ satisfying (1)
- If $\bar{W} > \frac{b}{\rho}$ satisfies (1) and $\mathbb{E}\left[W\left(z_0; h^*, \bar{W}\right)\right] = \bar{W}$, the following is an equilibrium:
  - $\bar{W}$
  - $H_0 = \{h^*\}$
  - Matching rates
    - For h^*: $\eta\left(h^*\right) = \infty$ and $\mu\left(h^*\right) = \infty$
    - For $h \neq h^*$ such that $\mathbb{E}\left[W\left(z_0; h, \bar{W}\right)\right] \geq \bar{W}$: $\eta\left(h\right) = \infty$ and $\mu\left(h\right) = \frac{\rho\bar{W} - b}{\mathbb{E}\left[W\left(z_0; h, \bar{W}\right)\right] - \bar{W}}$
    - For h such that $\mathbb{E}\left[W\left(z_0; h, \bar{W}\right)\right] < \bar{W}$, $\eta\left(h\right) = 0$ and $\mu\left(h\right) = \infty$

Efficiency

- Planner problem: choose z_L and z_H for newly unemployed worker
- Objective: maximize expected NPV of output (call this P_0)
- Value function:

$$\rho P(z;h) = b - c + \frac{1}{2}\left(\frac{\mu^H - \mu^L}{\sigma} z_t (1 - z_t)\right)^2 P''(z_t)$$

with value matching conditions:

$$P(z_L;h) = P_0$$

$$P(z_H;h) = \frac{1}{\rho + s}\left[z_H y_H + (1 - z_H) y_L + s P_0\right]$$

- Optimality imposes smooth pasting:

$$P_z(z_H;h^*) = \frac{y_H - y_L}{\rho + s}$$

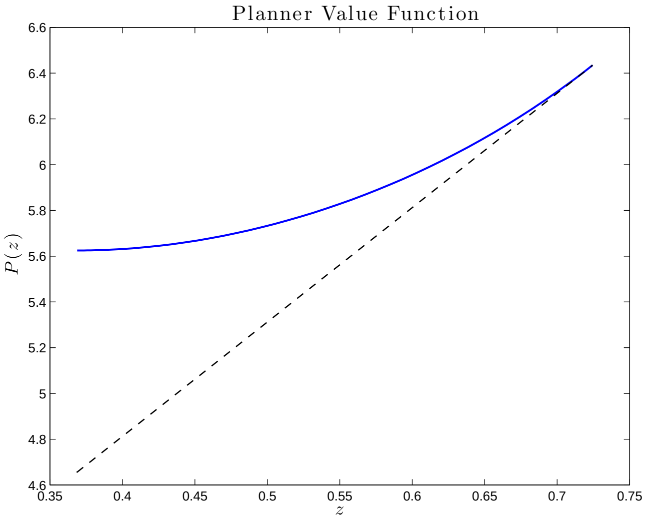
$$P_z(z_L;h^*) = 0$$

- Fixed-point condition

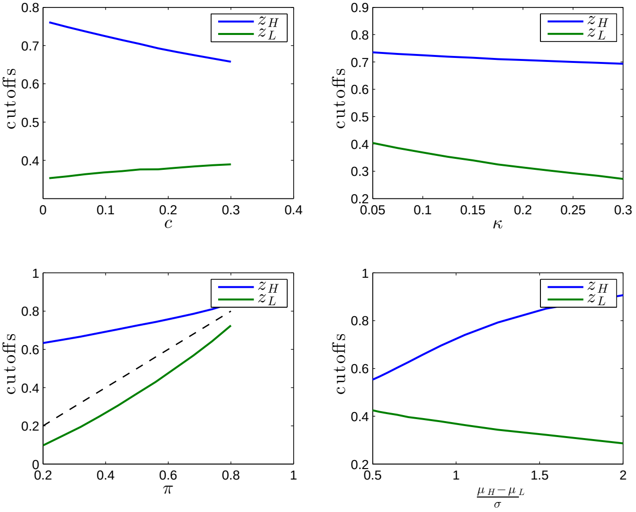
$$P_0 = \mathbb{E}\left(P(z_0;h^*)\right) - \kappa$$

- Same problem that characterized the equilibrium

Value Function



Some Comparative Statics



Outline

The Model

Competitive Equilibrium

Equilibrium Under Nash Bargaining

Non-Exclusive Interviews

The Bargaining Problem

- Worker and firm bargain during the interview
- Each instant, they can:
  - terminate and get $\bar{W}$ and 0 respectively
  - continue the interview
  - start the job
- They bargain over:
  - which option to choose
  - a payment made to the worker (or firm) to continue the interview
  - the wage in case the job starts
- The firm's bargaining power is β

Bargaining Outcome

- Taking $\bar{W}$ as given, same problem as the planner: hiring policy to maximize net surplus:

$$\rho S(z) = b - c + \frac{1}{2}\left(\frac{\mu^H - \mu^L}{\sigma} z_t (1 - z_t)\right)^2 S''(z_t)$$

with value matching conditions:

$$S(z_L) = \bar{W}$$

$$S(z_H) = \frac{1}{\rho + s}\left[z_H y_H + (1 - z_H) y_L + s\bar{W}\right]$$

and smooth pasting conditions.

$$S'(z_H) = \frac{y_H - y_L}{\rho + s}$$

$$S'(z_L) = 0$$

General Equilibrium

- Division of surplus:

$$V(z) = \beta S(z)$$
$$W(z) = \bar{W} + (1-\beta) S(z)$$

- Free entry:

$$\frac{\eta \mathbb{E}[\beta S(z_0)]}{\rho + \eta} = \kappa$$

- Value of unemployment:

$$\bar{W} = \frac{b + \mu \mathbb{E}[(1-\beta) S(z_0)]}{\rho + \mu}$$

- Frictionless meeting technology:

$$\max\{\mu, \eta\} = \infty$$

- Efficiency only if β such that

$$\bar{W} = \mathbb{E}[(1-\beta) S(z_0)]$$

  with $\bar{W}$ taken from planner's problem
  - Analogous to Mortensen-Hosios condition
  - Outside this case, either workers or firms must wait for interviews

Outline

Assumptions

- A worker can interview simultaneously with many firms
- Meeting technology:
  - Workers join unemployment pool (no distinction between I and Q)
  - Firms pay κ to pick a worker at random from pool (who may or may not have other interviews going on)
- Contract:
  - Posted wages
  - Firm chooses cutoffs z_L, z_H (not posted and no bargaining on this)
  - A worker can leave an interview at any point if hired elsewhere
  - (Similar issues arise with Nash Bargaining)

A Shortcut

- Let $\theta(t)$ be the hazard rate of getting a job
- Assume firm and worker treat $\theta(t)$ as a constant
- Unfortunately, $\theta(t)$ turns out not to be constant in equilibrium
  - Constant θ can be justified if parties cannot keep track of time

Value for Firm

$$(\rho + \theta) V(z) = -c + \frac{1}{2} \left(\frac{\mu^H - \mu^L}{\sigma} z_t (1 - z_t) \right)^2 V''(z_t)$$

with value matching conditions:

$$V(z_L) = 0$$
$$V(z_H) = \frac{1}{\rho} [z_H y_H + (1 - z_H) y_L - w]$$

and smooth pasting conditions

$$V'(z_L) = 0$$
$$V'(z_H) = \frac{1}{\rho} [y_H - y_L]$$

- Worker exit hazard θ enters like discount rate ρ
- Value of starting a search is

$$V_0 = \mathbb{E}(V(z_0))$$

Value for Worker

- While unemployed

$$\rho W_0 = b + \theta \left[W(w) - W_0 \right]$$

- While employed at wage w

$$\rho W(w) = w + s \left[W_0 - W(w) \right]$$

- Solving:

$$W_0 = \frac{b + \frac{\theta w}{\rho + s}}{\rho \left(1 + \frac{\theta}{\rho + s} \right)}$$

Equilibrium

A (steady state) competitive equilibrium is:

1. A value for unemployed workers $\bar{W}$.
2. A subset $H_0 \subseteq H$ of wages that firms are willing to consider.
3. Employment hazards $\theta(w)$ for all wages $w \in H$.

such that

1. Free entry: $V_0(w) \leq \kappa$ for all $h \in H$, with equality if $h \in H_0$.
2. Worker optimization: $W_0(w) \leq \bar{W}$ for all $h \in H$, with equality if $\theta(w) < \infty$.