

Stat 209

ANCOVA, CNRL equations

Week 5 math notes

precursor: t-test $Y = \beta_0 + \beta_1 G$

$\hat{\beta}_1 / \text{se}(\hat{\beta}_1)$
pooled t-test (not Welch)

$G=0,1$
group membership

$$Y = \gamma_0 + \gamma_1 G + \gamma_2 X$$

link w/ data (A)

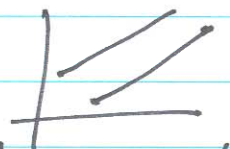
ancova

$$\hat{\gamma}_1 = \bar{Y}_1 - \bar{Y}_0 - \hat{\gamma}_p (\bar{X}_1 - \bar{X}_0)$$

constant treatment effect

$$\hat{\gamma}_2 = \hat{\gamma}_p$$

ave w/in group slopes



cf. overheads derive on board HSB school-level ex

more general model (CNRL)

$$(B) Y = \beta_1 + \beta_2 G + \beta_3 X + \beta_4 XG$$

see Berk's book
ATE intrus

"interaction" term

diff regression lines (cf Berk) w/in group
 $\hat{\beta}_2$, careful, diff of lines at $X=0$

$$D(X) = \beta_2 + \beta_4 X$$

treatment effect as function of X

$$(C) D(X) = \hat{\beta}_2 + \hat{\beta}_4 X$$

sample estimate

sampling variance

$$S_{D(X)}^2 = S_{D(c_a)}^2 + S_{44} (X - c_a)^2$$

center of accuracy

$$(D) c_a = -\frac{S_{24}}{S_{44}}$$

$$(E) D(c_a) = \hat{\gamma}_1; S_{D(c_a)}^2 = S_{22} + S_{24} c_a$$

$S_{ij} = \text{cov}(\hat{\beta}_i, \hat{\beta}_j)$

$D(X)/S_{D(X)}$ pick-a-point

Regions of significance, J-N

(H) $D(X)$ "significantly" different from 0

simultaneous R^1 $2F_{2, n-4}$
cf CNRL paper p.318 R^1 working-hotelling

CNRL Math Notes

model $Y = \beta_1 + \beta_2 G + \beta_3 X + \beta_4 XG + \epsilon$

Grp 1 $E(Y|X, G=1) = \beta_1 + \beta_2 + (\beta_3 + \beta_4)X$

Grp 0 $E(Y|X, G=0) = \beta_1 + \beta_3 X$

Treatment effect (diff of regressions) $A(X) = \beta_2 + \beta_4 X$

func of X iff $\beta_4 \neq 0$

abscissa of point of intersection (w/in group regression) "cut-off"

$X^0 = -\beta_2/\beta_4$

ATI research assignment on "aptitude" to differential instruction.

Inference

From sample obtain estimates $\hat{\beta}_i \quad i=1, \dots, 4$

$D(X) = \hat{\beta}_2 + \hat{\beta}_4 X$

S_{ij} , elements of $\hat{Cov}(\hat{\beta}_i)$
4x4

$\hat{X}^0 = -\hat{\beta}_2/\hat{\beta}_4$ ratio estimator (biased)

inference for $A(X)$ pick-a-point

$D(C_a)$ ancova treatment effect; $D(\bar{X}_G)$ average treatment effect

$D(\bar{X}_{sub})$ average treatment effect for subgroup

average treatment effect

sampling variance $S_{D(X)}^2 = S_{22} + S_{24}C_a + S_{44}(X-C_a)^2$ where $C_a = -S_{24}/S_{44}$

$A(X)$ inference: $D(X)/S_{D(X)}$ via t-distrib $N-4$ df, usual CI

Inference for $A(X)$ J-N region of significance

X-values s.t. reject $D(X) = 0$

R non-simultaneous $D(X) \pm \sqrt{F_{1, N-4}^{\alpha} S_{D(X)}^2}$ concatenate $D(X)/S_{D(X)}$ tests or CI

proper

R' simultaneous Working-Hotelling band about $D(X)$

hyperbolas $D(X) \pm \sqrt{2 F_{2, N-4}^{\alpha} S_{D(X)}^2}$ R' {values on X-axis outside WH band}

($R > R'$)

Potthoff 1966