Math notes: Random Effects Models (lme, HLM, SAS Proc Mixed)

Stat 209 [redacted]

week 4

1. Berk test story (2-level)

		level
$score_{ij}$ = student j score in school i		student
$income_{ij}$ = " " income in school i		student
$ratio_i$ = school student-teacher ratio		school

These "multilevel" models decompose $\beta^{wp}_{4\times 1}$ slopes-as-outcomes

Level 1. within school $score_{ij} = n_{0i} + n_{1i}\, income_{ij} + e_{ij}$

Level 2 between school $n_{0i} = \gamma_{00} + u_{0i}$ (no systematic link w/ ratio)

$n_{1i} = \gamma_{10} + \gamma_{11} ratio_i + u_{1i}$ (within school slope incr. w/ ratio)

Intercept n_{0i} (level, mean it cent w/in school)

slope n_{1i} random param vary over schools

$Ave(n_{1i})$ is $\beta^{wp}_{score\ inc}$

Do params of Level 1 model differ systematically over schools?

Combined model: $score_{ij} = \gamma_{00} + \gamma_{10} income_{ij} + \gamma_{11} ratio_i \times inc_{ij}$ (product term)

$+ [(inc_{ij} \times u_{1i}) + u_{0i} + e_{ij}]$ combin of errors

(fit by lmc, SAS or SPSS approach (lmList)) [reverse, contextual effects]

HSB model and results (Lab 2 B-R book, Singer)

Level 1 $math_{ij} = \alpha_{0i} + \alpha_{1i} cses_{ij} + \varepsilon_{ij}$ (cses $x - \bar{x}$ so α_0 mean)

Level 2 int $\alpha_{0i} = \gamma_{00} + \gamma_{01} meanses_i + \gamma_{02} sector_i + u_{0i}$

slope $\alpha_{1i} = \gamma_{10} + \gamma_{11} meanses_i + \gamma_{12} sector_i + u_{1i}$

```
> bryklme = lme(mathach ~ meanses*cses + sector*cses,
              random = ~ cses|school, data = Bryk)
> summary(bryklme)   Linear mixed-effects model fit by REML
Random effects:
 Formula: ~cses | school
            StdDev    Corr
(Intercept) 1.5426150 (Intr)
cses        0.3182015 0.391
Residual    6.0597955
Fixed effects: mathach ~ meanses * cses + sector * cses
                    Value Std.Error   DF  t-value p-value
(Intercept)  γ̂00  12.127931 0.1992919 7022 60.85510   0e+00
meanses      γ̂01   5.332875 0.3691684  157 14.44564   0e+00
cses         γ̂10   2.945041 0.1556005 7022 18.92694   0e+00
sectorCatholic γ̂02 1.226579 0.3062733  157  4.00485   1e-04
meanses:cses γ̂11   1.039230 0.2988971 7022  3.47688   5e-04
cses:sectorCatholic γ̂12 -1.642674 0.2397800 7022 -6.85076 0e+00
Number of Observations: 7185  Number of Groups: 160
```

Lab 2 lmc fits combined model in lab text (substitute L2 → L1)

variance components error terms

week 5 ancova equivalence

Cath schools higher mean, more egalitarian

[group level ancova non-equiv groups week 5]

Nonlinear, ex Logistic Level 1 model nlme

a. clinical trial, drug, control, curve 0,1 w/in 8 clinics NLMixed

b. Thailand repeat 0,1 outcome Level 1 sex preds presch. Level school 2 mean SES for L1 intercept

week 9 Growth curves

Multiple Regression v. Random Effects Models for (old-fashioned) Contextual Effects

OLS regression, multilevel data

$$Y_{ij} = \beta_0 + \beta_1 \bar{X}_{i\cdot} + \beta_2 X_{ij} + \epsilon_{ij}$$

$$\beta_1 = \beta_{Y\bar{X}\cdot X} = \beta^b_{Y\bar{X}} - \beta^{w\text{-}p}_{YX}$$

NELS data $\hat{\beta}^b = 3.6$ $\hat{\beta}^{w\text{-}p} = 2.1$ $\hat{\beta}^t = 7$

Contextual effect = effect of group on individual

mult regr interpretation "increase in Y for increase $\bar{x}$ with x constant"

As if by experiment? or delusional?

Part II
Refer to the two-level random-effects model story, taken from the Berk text, on class handout 1/31.
In the Level-1 model the outcome is student test score, and within-school predictor is student's family income.
Can you construct a level-2 model such that the resulting combined model (i.e. what would be fit by lme) is the contextual effects regression model for student test score and student income [see class handouts 1/29 and again 1/31 for the form of the contextual effects regression]
Under this multilevel model, what is the interpretation of the contextual effect?

Stat 209 problem

vars

Y_{ij} = score$_{ij}$

X_{ij} = income$_{ij}$

Solution?

random effects model

Level I $\quad Y_{ij} = \eta_{0i} + \eta_{1i} X_{ij} + u_{ij}$

Level II $\quad \eta_{0i} = \gamma_0 + \gamma_1 \bar{X}_{i\cdot}$ (deterministic)

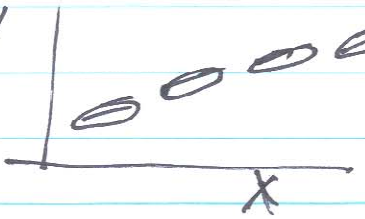
$\eta_{1i} = \beta^{w\text{-}p}_{YX}$ (all schools, same slope)

combined model

$$Y_{ij} = \gamma_0 + \gamma_1 \bar{X}_{i\cdot} + \beta^{w\text{-}p} X_{ij} + u_{ij}$$

delusional also? $\gamma_1 = \beta_{Y\bar{X}\cdot X}$ effect group level on outcome

"ancova" model Kreft-deLeeuw text



within schools same slope.