Multiple Regressions for Multi-level Contextual Estimates

Kreft deLeeuw Burstein

a) regress Y on $\bar{X}$, $X-\bar{X}$ over all cases

$\bar{X}$, $X-\bar{X}$ orthogonal

"Cronbach" regression

NELS data

obtain $\beta_{Y\bar{X}\cdot(X-\bar{X})} = \beta^{b}_{\bar{Y}\bar{X}}$ between

$\beta_{Y(X-\bar{X})\cdot\bar{X}} = \beta_{Y(X-\bar{X})} = \beta^{w-p}_{YX}$ within pooled $Y-\bar{Y}$ on $X-\bar{X}$

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b) Y on $\bar{X}$, X

NELS data

$\beta_{Y\bar{X}\cdot X} = \beta^{b}_{\bar{Y}\bar{X}} - \beta^{w-p}_{YX}$

effect of group membership on individual

$\boxed{\text{"contextual effect"}}$

? how would have student have done it in different group ?

$\beta_{YX\cdot\bar{X}} = \beta^{w-p}_{YX} = \beta_{(Y-\bar{Y})(X-\bar{X})}$ within-pooled

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c) Y on X, $X-\bar{X}$

$\beta_{YX\cdot(X-\bar{X})} = \beta^{b}_{\bar{Y}\bar{X}}$

$\beta_{Y(X-\bar{X})\cdot X} = \beta^{w-p}_{YX} - \beta^{b}_{\bar{Y}\bar{X}}$

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some relations, adjusted variables interpretation

note: $X\cdot\bar{X} = X - \frac{\mathrm{Cov}(X,\bar{X})}{\mathrm{Var}(\bar{X})}\bar{X} = X-\bar{X}$

so in (b) $\beta_{YX\cdot\bar{X}} = \beta_{Y(X\cdot\bar{X})} = \beta_{Y(X-\bar{X})}$ relative standing

in (a) also $(X-\bar{X})\cdot\bar{X} = X-\bar{X}$ $\beta_{Y(X-\bar{X})\cdot\bar{X}} = \beta^{wp}_{YX}$

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more ~~comp~~ of the same,

$\bar{X}\cdot X$ residuals for $\bar{X}$ on X regression

$\bar{X}\cdot X = \bar{X} - \frac{\mathrm{Cov}(X,\bar{X})}{\mathrm{Var}(X)}X = \bar{X} - \eta^2_{X}X$ in (b) $\beta_{Y\bar{X}\cdot X} = \beta_{Y(\bar{X}\cdot X)}$

etc

b) $\beta_{YX\cdot\bar{X}}$ coeff of X in $\bar{X}$, X regression(b)

$= \beta_{Y(X\cdot\bar{X})}$

week 1

what is $X\cdot\bar{X}$? residuals from predicting X from $\bar{X}$

logically that's $X-\bar{X}$ w/in group

brute force in handout

$$X\cdot\bar{X} = \underbrace{X - \frac{Cov(X,\bar{X})}{Var(\bar{X})}\bar{X}}_{\text{residual predicting } X \text{ from } \bar{X}} = X - \bar{X}$$

slope X on $\bar{X}$ = 1

$$\beta_{Y(X-\bar{X})} = \frac{Cov(\bar{Y} + (Y-\bar{Y}),\ X-\bar{X})}{Var(X-\bar{X})}$$

$$= \frac{Cov(Y-\bar{Y},\ X-\bar{X})}{Var(X-\bar{X})}$$

$$= \beta_{Y-\bar{Y},\,X-\bar{X}} = \beta^{w\text{-}p}_{YX}$$