Weeks 6 [redacted]

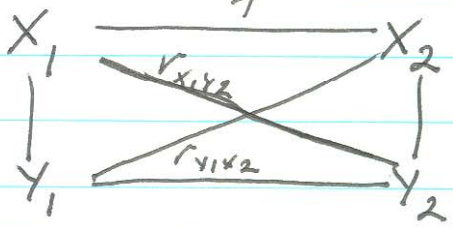
Stat 209 [redacted]

Reciprocal Effects, Simultaneous Equations

CLC (cross-lagged correlation)

compare $r_{X_1 Y_2}$ to $r_{Y_1 X_2}$ (sig)

causal predominance to the larger



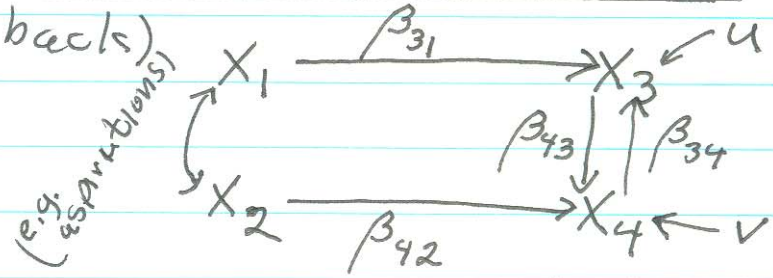
Non-Recursive (feedback)

$$X_3 = \beta_{31} X_1 + \beta_{34} X_4 + u$$

$$X_4 = \beta_{42} X_2 + \beta_{43} X_3 + v$$

$E(X_i) = 0 \quad E(u) = E(v) = 0$

(just ident $\sigma_{uv} = 0$)



X_1, X_2 exogenous $\quad X_3, X_4$ endogenous

u, v uncorr w/ exogenous

OLS estimates

Simultaneity Bias

$$E(\hat{\beta}_{31}^{OLS}) = \beta_{31} - \frac{\sigma_{14}\sigma_{4u}}{\sigma_{11}\sigma_{44} - \sigma_{14}^2}$$

$$E(\hat{\beta}_{34}^{OLS}) = \beta_{34} + \frac{\sigma_{11}\sigma_{4u}}{\sigma_{11}\sigma_{44} - \sigma_{14}^2}$$

$$E(\hat{\beta}_{42}^{OLS}) = \beta_{42} - \frac{\sigma_{23}\sigma_{3v}}{\sigma_{22}\sigma_{33} - \sigma_{23}^2}$$

$$E(\hat{\beta}_{43}^{OLS}) = \beta_{43} + \frac{\sigma_{22}\sigma_{3v}}{\sigma_{22}\sigma_{33} - \sigma_{23}^2}$$

By endogeneity $\sigma_{34} = \beta_{34}\sigma_{44} + \sigma_{4u}$

$\sigma_{44} = \beta_{43}\sigma_{34} + \sigma_{uv}$, $\sigma_{3v} = \beta_{34}\sigma_{4v} + \sigma_{uv}$

$\sigma_{4v} = \beta_{43}\sigma_{3v} + \sigma_{vv}$

IV estimates (consistent)

$$\hat{\beta}_{31}^{IV} = \frac{s_{13}s_{24} - s_{14}s_{23}}{s_{11}s_{24} - s_{12}s_{14}}$$

(IV sub x_2 for x_4)

whereas $\hat{\beta}_{31.4}^{OLS} = \dfrac{s_{13}s_{44} - s_{14}s_{34}}{s_{11}s_{44} - s_{14}^2}$

$$\hat{\beta}_{34}^{IV} = \frac{s_{11}s_{23} - s_{12}s_{13}}{s_{11}s_{24} - s_{12}s_{14}} \qquad \hat{\beta}_{42}^{IV} = \frac{s_{14}s_{23} - s_{13}s_{24}}{s_{12}s_{23} - s_{13}s_{22}} \qquad \hat{\beta}_{43}^{IV} = \frac{s_{12}s_{24} - s_{14}s_{22}}{s_{12}s_{23} - s_{13}s_{22}}$$

Simultaneous Eq's

Lab 3 (Task 3) PSID data

$$\text{hours} = \alpha_1 \log\text{Wage} + \beta_{10} + \beta_{11}\text{educ} + \beta_{12}\text{age} + \beta_{13}\text{kids5} + \beta_{14}\text{nwife} + u$$

$$\log\text{Wage} = \alpha_2 \text{hours} + \beta_{20} + \beta_{21}\text{educ} + \beta_{22}\text{exper} + \beta_{23}\text{exper}^2 + v$$

Duncan - Occupational aspiration;

Rindflus - Education, fertility (Freedman)

B1 p.2 Non-recursive Models Stat 209

```
> # Now to Duncan Haller Portes 1968 (Peer influences on Aspirations) Week 6
> # path diagram and data from Fox Soc Meth 1979
>  dunccor = matrix(nrow = 6, ncol = 6, c(1,.222,.1861,.3355,.4105,.2598,.222,1,.2707,.2302,.3240,.27
+  .1861,.2707,1,.2950,.293,.3607,.3355,.2302,.2950,1..2995,.5007,.4105,.3240,.2930,.2995,1,.4216,
+  .2598,.2786,.3607,.5007,.4216,1))
> dunccor
```

published corr matrix

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	1.0000	0.2220	0.1861	0.3355	0.4105	0.2598
[2,]	0.2220	1.0000	0.2707	0.2302	0.3240	0.2786
[3,]	0.1861	0.2707	1.0000	0.2950	0.2930	0.3607
[4,]	0.3355	0.2302	0.2950	1.0000	0.2995	0.5007
[5,]	0.4105	0.3240	0.2930	0.2995	1.0000	0.4216
[6,]	0.2598	0.2786	0.3607	0.5007	0.4216	1.0000

x_1: Respondent's intelligence
x_2: Respondent's family SES
x_3: Friend's family SES
x_4: Friend's intelligence
y_5: Respondent's occupational aspiration
y_6: Friend's occupational aspiration

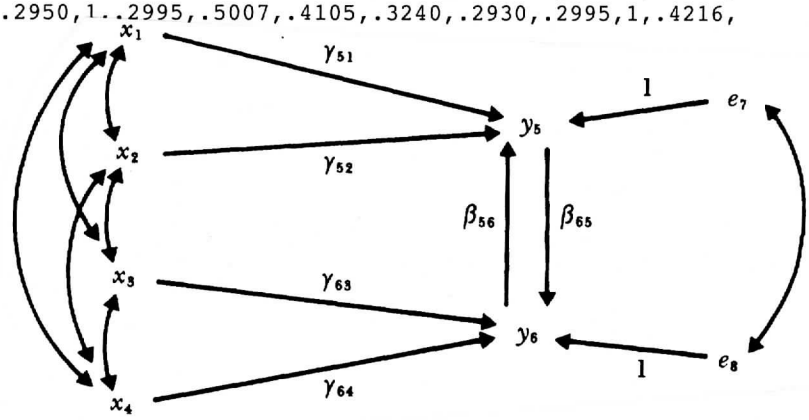


Figure 1. Overidentified nonrecursive model.

create data (0,1)

could do 7SLS by matrix easier to create data use tsls

```
> library(MASS)
> duncdatemp = mvrnorm(329, c(0,0,0,0,0,0),dunccor, empirical = TRUE)
> cor(duncdatemp) # matches above

> focreg = tsls(duncdatemp[,6] ~ duncdatemp[,3] + duncdatemp[,4] + duncdatemp[,5],
+ ~ duncdatemp[,1] +duncdatemp[,2] + duncdatemp[,3] + duncdatemp[,4])
> summary(focreg)
 2SLS Estimates
Model Formula: duncdatemp[, 6] ~ duncdatemp[, 3] + duncdatemp[, 4] + duncdatemp[,5]
Instruments: ~duncdatemp[, 1] + duncdatemp[, 2] + duncdatemp[, 3] + duncdatemp[,4]
                  Estimate Std. Error    t value  Pr(>|t|)
(Intercept)     -1.896e-17    0.04457 -4.254e-16 1.000e+00
duncdatemp[, 3]  1.567e-01    0.05445  2.877e+00 4.278e-03
duncdatemp[, 4]  3.521e-01    0.05505  6.396e+00 5.554e-10
duncdatemp[, 5]  3.419e-01    0.12478  2.740e+00 6.484e-03
Residual standard error: 0.8084 on 325 degrees of freedom
```

friend eq Y_6 predictors X_3 X_4 Y_5

```
> #got to have as many instruments as predictors
> rocreg = tsls(duncdatemp[,5] ~ duncdatemp[,1] + duncdatemp[,2] + duncdatemp[,6],
+ ~ duncdatemp[,1] + duncdatemp[,2] + duncdatemp[,3] + duncdatemp[,4])
> summary(rocreg)
 2SLS Estimates
Model Formula: duncdatemp[, 5] ~ duncdatemp[, 1] + duncdatemp[, 2] + duncdatemp[,6]
Instruments: ~duncdatemp[, 1] + duncdatemp[, 2] + duncdatemp[, 3] + duncdatemp[,4]
                  Estimate Std. Error    t value  Pr(>|t|)
(Intercept)     -1.859e-17    0.04658 -3.991e-16 1.000e+00
duncdatemp[, 1]  2.721e-01    0.05255  5.179e+00 3.923e-07
duncdatemp[, 2]  1.512e-01    0.05364  2.819e+00 5.113e-03
duncdatemp[, 6]  4.034e-01    0.10431  3.867e+00 1.330e-04
Residual standard error: 0.8449 on 325 degrees of freedom
```

respondent eq Y_5 predictors Y_6 X_1 X_2

```
> # both 2SLS results match Fox Soc Meth p.145 results
OLS Comparisons
lm(formula = duncdatemp[, 5] ~ duncdatemp[, 1] + duncdatemp[,2] + duncdatemp[, 6])
Coefficients:     Estimate Std. Error   t value Pr(>|t|)
(Intercept)     -2.032e-17  4.625e-02 -4.39e-16 1.000000
duncdatemp[, 1]  2.945e-01  4.860e-02     6.059 3.78e-09 ***
duncdatemp[, 2]  1.762e-01  4.887e-02     3.605 0.000361 ***
duncdatemp[, 6]  2.960e-01  4.934e-02     5.999 5.28e-09 ***
Residual standard error: 0.8389 on 325 degrees of freedom
lm(formula = duncdatemp[, 6] ~ duncdatemp[, 3] + duncdatemp[,4] + duncdatemp[, 5])
Coefficients:     Estimate Std. Error   t value Pr(>|t|)
(Intercept)     -1.308e-17  4.436e-02 -2.95e-16 1.000000
duncdatemp[, 3]  1.752e-01  4.772e-02     3.672 0.000281 ***
duncdatemp[, 4]  3.714e-01  4.782e-02     7.767 1.06e-13 ***
duncdatemp[, 5]  2.590e-01  4.779e-02     5.420 1.16e-07 ***
Residual standard error: 0.8047 on 325 degrees of freedom
```

OLS

5,6 weights a little different

C1 Freedman, case study Rindfus Ed + Fertility ASR, Stat 209

```
> rindcor = matrix(nrow = 11, ncol = 11,
+ c( 1.000,-0.144,-0.244,-0.323,-0.129,-0.056 , 0.053 ,-0.043 , 0.037 , 0.370 , 0.186,
+ -0.144, 1.000 , 0.156, 0.088, 0.315, 0.150,-0.152, 0.030, 0.035,-0.222,-0.189,
+ -0.244,   0.156,   1.000, 0.274, 0.150,-0.039, 0.014, 0.028, 0.002,-0.328,-0.115,
+ -0.323,   0.088,   0.274,   1.000, 0.218,-0.030,-0.149,-0.060,-0.032,-0.185,-0.118,
+ -0.129, 0.315, 0.150, 0.218, 1.000, 0.071,-0.292,-0.011,-0.027,-0.211,-0.177,
+ -0.056, 0.150,  -0.039,  -0.030, 0.071, 1.000,-0.052, 0.067, 0.018,-0.157, 0.111,
+  0.053,  -0.152,   0.014,  -0.149,  -0.292,  -0.052,   1.000,-0.010,-0.002,-0.012, 0.098,
+ -0.043,   0.030,   0.028,  -0.060,  -0.011,   0.067,  -0.010,   1.000, 0.009,-0.171,-0.122,
+ 0.037,   0.035,   0.002,  -0.032,  -0.027,   0.018,  -0.002,   0.009,   1.000,  0.038, 0.216,
+  0.370,  -0.222,  -0.328,  -0.185,  -0.211,  -0.157,  -0.012,  -0.171,   0.038,   1.000, 0.380,
+ 0.186,  -0.189,  -0.115,  -0.118,  -0.177,   0.111,   0.098,  -0.122,   0.216,   0.380, 1.0))
> rindcor
        [,1]   [,2]   [,3]   [,4]   [,5]   [,6]   [,7]   [,8]   [,9] [,10] [,11]
 [1,]  1.000 -0.144 -0.244 -0.323 -0.129 -0.056  0.053 -0.043  0.037  0.370  0.186
 [2,] -0.144  1.000  0.156  0.088  0.315  0.150 -0.152  0.030  0.035 -0.222 -0.189
 [3,] -0.244  0.156  1.000  0.274  0.150 -0.039  0.014  0.028  0.002 -0.328 -0.115
 [4,] -0.323  0.088  0.274  1.000  0.218 -0.030 -0.149 -0.060 -0.032 -0.185 -0.118
 [5,] -0.129  0.315  0.150  0.218  1.000  0.071 -0.292 -0.011 -0.027 -0.211 -0.177
 [6,] -0.056  0.150 -0.039 -0.030  0.071  1.000 -0.052  0.067  0.018 -0.157  0.111
 [7,]  0.053 -0.152  0.014 -0.149 -0.292 -0.052  1.000 -0.010 -0.002 -0.012  0.098
 [8,] -0.043  0.030  0.028 -0.060 -0.011  0.067 -0.010  1.000  0.009 -0.171 -0.122
 [9,]  0.037  0.035  0.002 -0.032 -0.027  0.018 -0.002  0.009  1.000  0.038  0.216
[10,]  0.370 -0.222 -0.328 -0.185 -0.211 -0.157 -0.012 -0.171  0.038  1.000  0.380
[11,]  0.186 -0.189 -0.115 -0.118 -0.177  0.111  0.098 -0.122  0.216  0.380  1.000
> rinddat = mvrnorm(1766,rep(0,11), rindcor, empirical = TRUE)
> cor(rinddat)
        [,1]   [,2]   [,3]   [,4]   [,5]   [,6]   [,7]   [,8]   [,9] [,10] [,11]
 [1,]  1.000 -0.144 -0.244 -0.323 -0.129 -0.056  0.053 -0.043  0.037  0.370  0.186
 [2,] -0.144  1.000  0.156  0.088  0.315  0.150 -0.152  0.030  0.035 -0.222 -0.189
 [3,] -0.244  0.156  1.000  0.274  0.150 -0.039  0.014  0.028  0.002 -0.328 -0.115
 [4,] -0.323  0.088  0.274  1.000  0.218 -0.030 -0.149 -0.060 -0.032 -0.185 -0.118
 [5,] -0.129  0.315  0.150  0.218  1.000  0.071 -0.292 -0.011 -0.027 -0.211 -0.177
 [6,] -0.056  0.150 -0.039 -0.030  0.071  1.000 -0.052  0.067  0.018 -0.157  0.111
 [7,]  0.053 -0.152  0.014 -0.149 -0.292 -0.052  1.000 -0.010 -0.002 -0.012  0.098
 [8,] -0.043  0.030  0.028 -0.060 -0.011  0.067 -0.010  1.000  0.009 -0.171 -0.122
 [9,]  0.037  0.035  0.002 -0.032 -0.027  0.018 -0.002  0.009  1.000  0.038  0.216
[10,]  0.370 -0.222 -0.328 -0.185 -0.211 -0.157 -0.012 -0.171  0.038  1.000  0.380
[11,]  0.186 -0.189 -0.115 -0.118 -0.177  0.111  0.098 -0.122  0.216  0.380  1.000
```

create data

DADocc
RACE
NOSIB
FARM
REGN
ADOL
REL
CIG
FECUND
ED
AGE

var list

```
> #Rindfus model, Freedman page 356
```

```
> agereg = tsls(rinddat[,11] ~ rinddat[,10] +  rinddat[,2] +  rinddat[,3] +  rinddat[,4] +
+    rinddat[,6] +  rinddat[,7] +  rinddat[,8] +  rinddat[,9],  ~  rinddat[,3] + rinddat[,4]
+    rinddat[,8] +  rinddat[,9] +  rinddat[,1] +  rinddat[,2])
> summary(agereg)
 2SLS Estimates
Model Formula: rinddat[, 11] ~ rinddat[, 10] + rinddat[, 2] + rinddat[, 3] +
    rinddat[, 4] + rinddat[, 5] + rinddat[, 6] + rinddat[, 7] +
    rinddat[, 8] + rinddat[, 9]
Instruments: ~rinddat[, 3] + rinddat[, 4] + rinddat[, 5] + rinddat[, 6] +
    rinddat[, 7] + rinddat[, 8] + rinddat[, 9] + rinddat[, 1] +
    rinddat[, 2]

                Estimate Std. Error    t value  Pr(>|t|)
(Intercept)   -1.298e-17    0.02083 -6.234e-16 1.000e+00
rinddat[, 10]  4.851e-01    0.08429  5.755e+00 1.020e-08
rinddat[, 2]  -1.052e-01    0.02463 -4.270e+00 2.057e-05
rinddat[, 3]   7.692e-02    0.03123  2.463e+00 1.387e-02
rinddat[, 4]  -9.995e-03    0.02380 -4.199e-01 6.746e-01
rinddat[, 5]  -3.628e-02    0.02558 -1.418e+00 1.563e-01
rinddat[, 6]   2.128e-01    0.02415  8.809e+00 0.000e+00
rinddat[, 7]   8.559e-02    0.02324  3.683e+00 2.374e-04
rinddat[, 8]  -5.421e-02    0.02489 -2.178e+00 2.953e-02
rinddat[, 9]   1.966e-01    0.02114  9.300e+00 0.000e+00
```

ED signif

Ed → Age

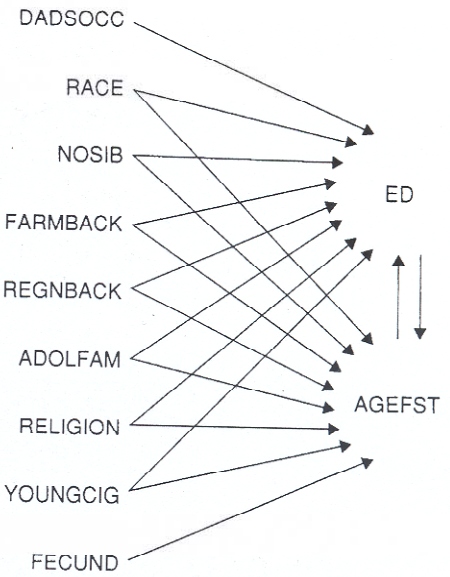
```
> edreg = tsls(rinddat[,10] ~ rinddat[,11] +  rinddat[,2] +  rinddat[,3] +  rinddat[,4] +
+   rinddat[,5] + rinddat[,6] +  rinddat[,7] +  rinddat[,8] +  rinddat[,1],  ~  rinddat[,3]
+   rinddat[,4] +  rinddat[,5] +  rinddat[,6] +  rinddat[,7] +
+   rinddat[,8] +  rinddat[,9] +  rinddat[,1] +  rinddat[,2])
> summary(edreg)
 2SLS Estimates
Model Formula: rinddat[, 10] ~ rinddat[, 11] + rinddat[, 2] + rinddat[, 3] +
    rinddat[, 4] + rinddat[, 5] + rinddat[, 6] + rinddat[, 7] +
    rinddat[, 8] + rinddat[, 1]
Instruments: ~rinddat[, 3] + rinddat[, 4] + rinddat[, 5] + rinddat[, 6] +
    rinddat[, 7] + rinddat[, 8] + rinddat[, 9] + rinddat[, 1] +
    rinddat[, 2]

                Estimate Std. Error    t value  Pr(>|t|)
(Intercept)    4.833e-18    0.01954  2.473e-16 1.000e+00
rinddat[, 11]  1.473e-01    0.09256  1.591e+00 1.118e-01
rinddat[, 2]  -7.652e-02    0.02489 -3.074e+00 2.143e-03
rinddat[, 3]  -2.166e-01    0.02111 -1.026e+01 0.000e+00
rinddat[, 4]  -2.331e-02    0.02182 -1.068e+00 2.856e-01
rinddat[, 5]  -1.093e-01    0.02380 -4.592e+00 4.703e-06
rinddat[, 6]  -1.456e-01    0.02461 -5.917e+00 3.943e-09
rinddat[, 7]  -9.243e-02    0.02103 -4.395e+00 1.173e-05
rinddat[, 8]  -1.278e-01    0.02278 -5.608e+00 2.372e-08
rinddat[, 1]   2.484e-01    0.02469  1.006e+01 0.000e+00

Residual standard error: 0.8213 on 1756 degrees of freedom
```

age not signif

preg ⇏ dropout ?

```
> # ed is significant in age eq, age not signif in ed eq
> #therefore ed --> age  Freedman p.182
```


Figure 1. A Model of the Relationship between Educational Attainment and the Beginning of Motherhood.



$$\widehat{ED} = b_0 + b_1 DADSOCC + b_2 RACE + b_3 NOSIB + b_4 FARMBACK + b_5 REGNBACK + b_6 ADOLFAM + b_7 RELIGION + b_8 YOUNGCIG + b_9 AGEFST + U$$

$$\widehat{AGEFST} = c_0 + c_1 RACE + c_2 NOSIB + c_3 FARMBACK + c_4 REGNBACK + c_5 ADOLFAM + c_6 RELIGION + c_7 YOUNGCIG + c_8 FECUND + c_9 ED + V$$