1970's

week 5 Stat 209 Math notes, part 2

STAT 209

Regression Adjustment in Quasi-experiments

outcome Y

group $G = 1, 0$

Head Start
Bryk, Weisberg
in Oakes

mis-specified model

$$Y = \beta_0 + \beta_1 G + u$$

t-test.
but G, u not indep

adjustment (premeasure) X (pre-test?)

$$\hat{\alpha} = \bar{Y}_1 - \bar{Y}_0 - \hat{\beta}(\bar{X}_1 - \bar{X}_0)$$

estimated effect

IV alternative
X as instrument for G to correct bias in $\hat{\beta}_{YG}$ (t-test)

$\hat{\beta} = 0$ t-test no adj

$\hat{\beta} = 1$ gain scores if X "pretest"

$\hat{\beta} = \hat{\beta}_{YX \cdot G}$ standard ancova (underadjusts, overadjusts)

correcting for underadjusts

$\hat{\beta} = \hat{\beta}_{YX \cdot G} / rel(X)$ adjustment for measurement error in slope

$\hat{\beta} = \hat{\beta}_{YX \cdot G} / r_{YX \cdot G} = \dfrac{s_{Y \cdot G}}{s_{X \cdot G}}$ validity correction Campbell-Erlebacher st. change score [week 9]

$\hat{\beta} = \hat{\beta}_0$ control slope Belson equiv to $D(\bar{X}_1)$ adjustment

Belson, W. A. (1956), "A technique for studying the effects of a television broadcast," Applied Statistics, 5, 195–202.

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numerical illustrations

Anderson et al (1980) Ch.12 Table 12.1 Head Start Data

	Pre	Post	$r_{prepost}$	n
Innovative curriculum	17.1 (6.1)	23.3 (4.6)	.67	157
Standard Head start	14.6 (6.2)	18.9 (5.8)	.78	669

pre diff 2.5 post diff 4.4

Inference?

$\hat\beta$ options

t-test	$\hat\beta = 0$	$\hat\alpha = 4.4$
gain	$\hat\beta = 1$	$\hat\alpha = 1.9$
ancova	$\hat\beta = \beta_{YX\cdot G} = .76$	$\hat\alpha = 2.5$
C-E	$\hat\beta = \frac{5.6}{6.2} = .9$	$\hat\alpha = 2.1$
Belson	$\hat\beta = .73$	$\hat\alpha = 2.57$

Analytic results

Weisberg (1979) Ancova bias positive or negative

Potential outcomes setup: W_i outcome if T $(Q=1)$

$\alpha_i = W_i - Z_i$ treatment effect. Z_i outcome if C $(Q=0)$

observable $Y_i = Z_i + Q\alpha$ (set $\alpha_i = \alpha$) $P = E(Q)$

for non-random assignment ($\rho_{ZQ} \neq 0$)

$$\mu_{Y_1} - \mu_{Y_0} = \alpha + (\mu_{Z_1} - \mu_{Z_0})$$ ← selection bias

Can ancova with covariate X reduce or eliminate bias?

result. bias from ancova $\delta = (\mu_{Z_1} - \mu_{Z_0})\left(\frac{\rho_{ZQ\cdot X}}{\rho_{ZQ}}\right)\left(\frac{\sqrt{1-\rho^2_{ZX}}}{\sqrt{1-\rho^2_{XQ}}}\right)$

H.I.W uses $\pi = \delta / (\mu_{Z_1} - \mu_{Z_0})$ "proportion bias"

Weisberg Ancova Results

STAT 209
week 5

Potential Outcomes
person i

W_i outcome if T $(Q=1)$
Z_i outcome if C $(Q=0)$

Recast week 2
Holland
treatment/control difference
$\alpha_i = W_i - Z_i$
(set $\alpha_i = \alpha$)

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observable $Y_i = Z_i + Q_i \alpha$
$P = E(Q)$ prop in T

Non-random assignment $\rho_{ZQ} \neq 0$

observables
point-biserial ρ_{ZQ}

$\mu_{Y_1} - \mu_{Y_0}$

$$\mu_{Z_1} - \mu_{Z_0} = \frac{\sigma_Z}{\sqrt{P(1-P)}} \rho_{ZQ}$$

$= \alpha + (\mu_{Z_1} - \mu_{Z_0})$ selection bias

recall week 2 FACE results BIAS $= E(Y_c|S=t) - E(Y_c|S=c)$

Ancova with covariate X, saves the day?

(18) $E(Y|X,Q) = \mu + \beta X + (\delta + \alpha) Q$ [note: matching works, eqs 9–12]
observables

can only estimate $\delta + \alpha$, want α

bias from ancova (19)

$$\delta = \rho_{ZQ\cdot X} \frac{\sigma_{Z\cdot X}}{\sigma_{Q\cdot X}} \quad (19)$$

$$= \rho_{ZQ\cdot X} \frac{\sigma_Z}{\sqrt{P(1-P)}} \frac{\sqrt{1-\rho_{XZ}^2}}{\sqrt{1-\rho_{XQ}^2}}$$

proportion of bias

$$\pi = \frac{\delta}{\mu_{Z_1} - \mu_{Z_0}} = \frac{\rho_{ZQ\cdot X}}{\rho_{ZQ}} \frac{\sqrt{1-\rho_{XZ}^2}}{\sqrt{1-\rho_{XQ}^2}}$$

WEISBERG

Table 1
Range of π for Different Combinations of ρ_{ZQ}, ρ_{XZ}, ρ_{ZX}

Basic situation	Case	Sign (ρ_{ZQ})	Sign (ρ_{XQ})	Sign (ρ_{ZX})	π
1	1	+	+	+	$-\infty$ to $+1$
	2	+	−	−	$-\infty$ to $+1$
	3	−	+	−	$-\infty$ to $+1$
	4	−	−	+	$-\infty$ to $+1$
2	5	−	−	−	1 to $+\infty$
	6	−	+	+	1 to $+\infty$
	7	+	−	+	1 to $+\infty$
	8	+	+	−	1 to $+\infty$

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ancova can overadjust or increase bias

Case 1 right direction, but can overadjust
Case 2 adjustment in wrong direction