Math Notes Week 2, part 1

week 7, week 3 Standardized Variables

Stat 209
[redacted]

Standardized Vars (sample)

$y^* = \dfrac{Y - \bar{Y}}{s_y}$

$X_i^* = \dfrac{X_i - \bar{X}_i}{s_{x_i}}$

in sample

single predictor $\hat{\beta}^* = \dfrac{s_x}{s_y} \hat{\beta}_{YX} = r_{xy}$

multiple X_i $\hat{\beta}_i^* = \dfrac{s_{x_i}}{s_y} \hat{\beta}_{YX_i \cdot X_{[i]}}$

for x_1, x_2

week 2 in pop $\beta^*_{YX_1 \cdot X_2} = \dfrac{\rho_{YX_1} - \rho_{YX_2}\rho_{X_1X_2}}{1 - \rho^2_{X_1X_2}}$

Comperability Example

β_1 universal (M, F) set $\beta_1 = 2$

Happiness $= \beta_0 + \beta_1$ Money $+ \varepsilon$

variance	(Happiness)	(money)	(ε)	β^* resulting
M	5	1	1	$2\sqrt{1/5} = .894$
F	6	.75	3	$2\sqrt{.75/6} = .71$

report: >25% gender difference

Freedman Text Hookes Law

ch 5 pp 81-2

response schedule

Length $= a + bX + \varepsilon$
weight

a, b depend on spring
$a =$ length $X = 0$
$b = \Delta L / \Delta X$

warping due to standardizing

$v = Var(\text{weights})$ $\sigma^2 = Var(\varepsilon)$ $s^2 = \hat{\sigma}^2$

$Var(\text{Length}) = b^2 v + \sigma^2$ $\Rightarrow$ standardized coeff

$= b\sqrt{\dfrac{Var(\text{Length})}{Var(\text{weight})}} = b\sqrt{\dfrac{v}{b^2 v + \sigma^2}}$

v, σ^2 determined by procedure, comperability?

Standardized Regression Coefficients

Starting w/ the Correlation Matrix (not the data
[c.f. Lab 1, week 3]

[bold]
$\underset{n \times p}{X}$ transformed to 0, 1

$\underset{(p \times p)}{r_{XX}} = X'X$ correlation of predictors

r_{YX} $(1 \times p)$ corr of response w/ predictors

coefficients

$$\underset{p \times 1}{\hat{b}} = \underset{p \times p}{r_{XX}^{-1}} \; \underset{p \times 1}{r_{XY}}$$

st. regr coeffs

$r_{XX} \hat{b} = r_{XY}$

squared mult correl $\underset{\text{scalar}}{R^2} = \hat{b}^t r_{XY}$

residual standard error $\sqrt{1-R^2}$

standard errors $s.e.(\hat{\beta}_K)$ $\sqrt{(1-R^2)\, r_{XX}^{-1} / n-p}$

sqrt diagonal elements of

$$\left(\frac{1-R^2}{n-p}\right) r_{XX}^{-1}$$