THIRD-VARIABLES WEEK 2 STAT 209

Partial, part correlations (spurious associations)

Consider X_1, X_2, X_3 (may be measured w/ error)

$$r_{13\cdot 2} = \frac{r_{13} - r_{12}\, r_{23}}{\sqrt{(1-r_{12}^2)(1-r_{23}^2)}}$$

adjusted vars

$r_{13\cdot 2} = r_{(1\cdot 2)(3\cdot 2)}$

also $r_{12\cdot 345} = r_{(1\cdot 345)(2\cdot 345)}$ etc

From Stat 60

$H_0: \rho = 0$

t-statistic $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

t_{n-2} critical value

for $H_0: \rho_{12\cdot 3} = 0$

$\rho_{12\cdot 3} = \rho_{(1\cdot 3)(2\cdot 3)}$ $r_{12\cdot 3}\sqrt{n-3}/\sqrt{1-r}$ test statistic

part correlations $r_{(1\cdot 2)3}$ $r_{1(3\cdot 2)}$

$$R^2_{y\cdot x_1 x_2} = r^2_{yx_1} + r^2_{y(x_2\cdot x_1)}$$

Dichotomous Data: Spurious Correlation, Confoundi

Simpson's paradox: conditional, marginal tables

Death penalty ex. odds ratios flip w/ 3rd variable

DP, DR, VR U.C. Berkeley grad admissions

SIMPSON'S PARADOX (marginal vs conditional odds ratios) DEATH PENALTY ex

Stat 141 ex

```
> deathP = matrix(c(19,17, 141,149), nr = 2,
+ dimnames = list("Def" = c("Wh", "Blk"), "DP" = c("Y","N")))
> deathP   # unconditional, marginal table
     DP
Def    Y   N
  Wh  19 141
  Blk 17 149
> prop.table(deathP,1)
     DP
Def           Y         N
  Wh  0.1187500 0.8812500
  Blk 0.1024096 0.8975904
> # so where's the racial bias? Wh seems more likely to fry
> deathPWvic = matrix(c(19,11, 132,52), nr = 2,
+ dimnames = list("Def" = c("Wh", "Blk"), "DP" = c("Y","N")))
> deathPBvic = matrix(c(0,6, 9,97), nr = 2,
+ dimnames = list("Def" = c("Wh", "Blk"), "DP" = c("Y","N")))
> prop.table(deathPWvic,1)
     DP
Def           Y         N
  Wh  0.1258278 0.8741722
  Blk 0.1746032 0.8253968
> prop.table(deathPBvic,1)
     DP
Def           Y         N
  Wh  0.00000000 1.0000000
  Blk 0.05825243 0.9417476
> # for each level  of Victim race, Black Def more likely to receive DP
reversal by conditioning instance of Simpson's Paradox (e.g. marginal vs cond'l or)
```

DP × Def association, dichotomous vars

2 cond'l tables

condition on race of victim

Mediation / Moderation

STAT 209 [redacted]

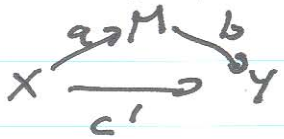
Sources

Kenny web page (Stat209 week 2)

MacKinnon Ann Rev Psych 2007 linked observational

Kraemer Arch Gen Psych linked RCT

Baron-Kenny $X \xrightarrow{c} Y$



OLS (or logistic) for a b c c'

steps $\rho_{XY} \neq 0$ (1) $\rho_{XM} \neq 0$ (2) $\beta_{YM \cdot X} \neq 0$ (3) (b) $\rho_{YX \cdot M} = 0$ (4) complete

Gauss - normal eq's

$$\beta_{12} = \beta_{12 \cdot 3} + \beta_{32}\beta_{13 \cdot 2} \qquad Y=1 \quad X=2 \quad M=3$$

(c) (c') (a) (b)

$$c - c' = \beta_{32}\beta_{13 \cdot 2} = ab$$ amount of mediation

Inference "Sobel" $Var(c-c') = b^2 s_a^2 + a^2 s_b^2$

asymptotic var product 2 rv (Mood/Graybill)

delta-method (first term)

R-implementation Multilevel package (pdf)

Bence measurement error Cochran (Stat 209 week 1)

Holland encouragement designs (Stat 209 week 2,3) next mtg

MacKinnon Fig 1 Eq 1-3

plot pp 599-600

inf Sobel, bootstraps mediated moderation CNRL p 606

Kraemer Fig 1 table

moderator pre-existing mediator, during treatment.