model $Y = \beta_1 + \beta_2 G + \beta_3 X + \beta_4 XG + \varepsilon$

Grp 1 $E(Y|X, G=1) = \beta_1 + \beta_2 + (\beta_3 + \beta_4) X$

Grp 0 $E(Y|X, G=0) = \beta_1 + \beta_3 X$

Treatment effect (diff of regressions) $\Delta(X) = \beta_2 + \beta_4 X$ — func of X iff $\beta_4 \neq 0$

abscissa of point of intersection (w/in group regression) "cut-off" $X^0 = -\beta_2/\beta_4$ — ATI research assignment on "aptitude" to differential instruction.

Start with

$D(\bar{X}_1)$ diff of regressions at treatment group mean on X

notate the within group slopes Y on X as: $\hat{\delta}_1$ treatment slope

$\hat{\delta}_0$ control slope

$$D(\bar{X}_1) = \hat{\beta}_2 + \hat{\beta}_4 \bar{X}_1$$

$$= (\bar{Y}_1 - \hat{\delta}_1 \bar{X}_1) - (\bar{Y}_0 - \hat{\delta}_0 \bar{X}_0) \quad [\hat{\beta}_2 \text{ part}]$$

$$+ (\hat{\delta}_1 - \hat{\delta}_0)\bar{X}_1 \quad [\beta_4 \bar{X}_1 \text{ part}]$$

cancel terms and regroup

$$D(\bar{X}_1) = (\bar{Y}_1 - \bar{Y}_0) - \hat{\delta}_0(\bar{X}_1 - \bar{X}_0)$$

gives form of Belson estimator! use control group slope in ancova-style adjustment.