Struggling to Understand the Stock Market

The 2001 Ely Lecture

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Abstract:

The stock market moves in huge low-frequency waves in relation to other aggregate measures, such as GDP. Individual stocks sell at wildly different values relative to the underlying cash flows currently accruing to shareholders. Many economists attribute these observations to irrationality. Irrational markets breed opportunities for active investment strategies, but experience suggests that passive investors actually earn higher returns. Moreover, an examination of cash flows earned by corporations suggests that their level, and even more their expected rates of growth, are broadly consistent with the principle of rational markets that securities values reflect the present discounted values of cash earnings.

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I. Introduction

Economists are as perplexed as anyone by the behavior of the stock market. Figure 1 shows a broad measure of stock-market value in relation to GDP since 1950. In addition to saw-tooth movements including the contraction in late 2000, the value of the stock market has large, low-frequency swings, moving upward from 1950 to 1965, then downward to 1982, and upward until early 2000.

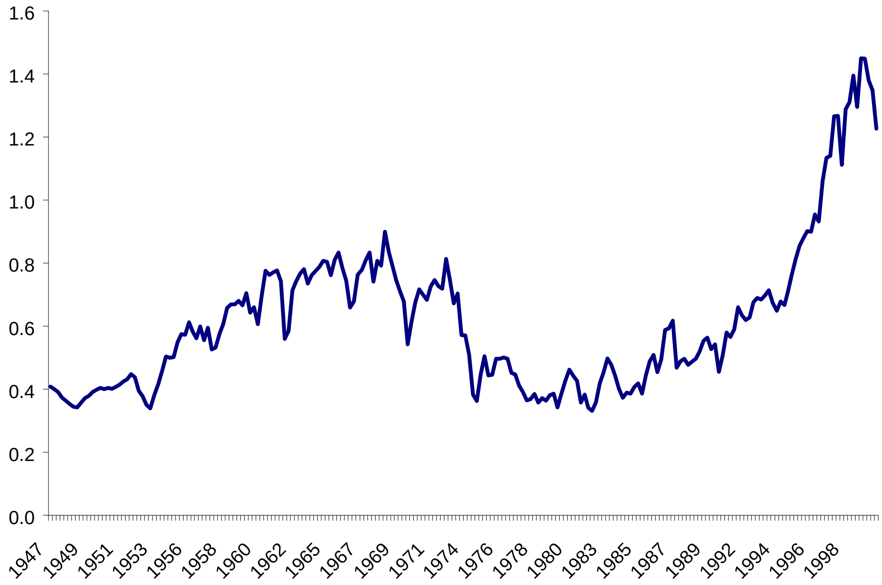

Figure 1. Value of Equity Claims on Non-Farm, Non-Financial Corporations as a Ratio to GDP, 1947 through 2000
Source: Equity value from Federal Reserve Board Flow of Funds Accounts, extended to the end of 2000 in proportion to the S&P 500; GDP from National Income and Product Accounts

I entertain the hypothesis that these large movements are the result of rational (if not accurate) appraisal of the cash likely to be received by shareholders in the future. The hypothesis receives some support from work by financial economists showing that irrational markets create profit opportunities for active traders and that passive traders consistently earn higher returns. Most of my discussion will be complementary to the work of financial economists—I will look at the fundamentals underlying stock-market values.

The lecture considers three potential contributors to the big movements shown in Figure 1:

1. Changes in the value of debt claims
2. Changes in the value of the plant and equipment that corporations own
3. Changes in the value of intangibles owned by corporations and in the value of claims of stakeholders who are not securities holders

I correct Figure 1 by adding data on the market value of debt and find that most of the large swings remain. I find that movements of the stock of plant and equipment are also of little help in understanding the big swings. Changes in the inferred values of intangibles and stakeholder claims account for the great bulk of the large movements of stock-market values. I examine corporate cash flows to seek confirmation that intangibles are either contributing to value or diminishing it. My conclusion is tentatively in favor of the intangibles/stakeholders hypothesis, because cash flows move in a way that is consistent with securities values but depart tremendously from the likely movements of the earnings of hard assets alone.

A rational stock market measures the value of the property owned by corporations. Some types of corporate property—especially the types held by high-tech companies—have values that are exquisitely sensitive to the future growth of the cash they generate. Both the high value of these types of property and the volatility of the value are consistent with the present value model.

I reject market irrationality in favor of the hypothesis that the financial claims on firms command values approximately equal to the discounted expected future returns. The stock market's movements are generally consistent with rational behavior by investors. There is no need to invoke fads, animal spirits, or irrational exuberance to understand the movements shown in Figure 1. Instead, the key concepts are intangibles and their valuation based on the level and especially the growth of their cash flows.

II. Rational Markets

It is convenient to think about financial markets in a simple economy where transactions occur in one period in anticipation of random events that take place in the second period. Suppose there is a finite number of possible states of the world in the second period. People consume a single good. Basic security *s* pays one unit of the consumption good in state *s* and nothing otherwise. The market meets in the first period and determines the price, p_s, of each of the basic securities.

To avoid profitable riskless arbitrage among securities, the price of a more complicated security, paying x_s in each of various states, is $V = \sum p_s x_s$.

Rationality is a concept rooted in the theory of choice. Consider an investor who assembles a portfolio to maximize expected utility: $\max \sum \pi_s u(c_s)$ subject to the wealth constraint $\sum p_s c_s = W$. Here π_s is the probability the investor assigns to state *s*. The first-order condition describing the optimal portfolio is $\pi_s u'(c_s) = \lambda p_s$. Thus securities prices obey $p_s = \frac{1}{\lambda}\pi_s m_s$, where m_s is marginal utility in state *s*. The price depends on two factors: (1) it is proportional to the probability of the state, and (2) it is proportional to marginal utility in that state. Marginal utility is equal across states for an investor with no risk aversion (linear utility). With concave utility, marginal utility distinguish good times (low *m*) from bad times (high *m*). For example, if the states differ by the productivity of the technology for producing the consumption good, then high-productivity states will have low values of *m*. The more valuable securities are those that pay off in states with higher probability and those that pay off in states with higher marginal utility (securities with less risk).

In this setting, one judges rationality by the relation between the observed prices, p_s, and the corresponding values of $\frac{1}{\lambda}\pi_s m_s$. A security is overpriced if the probability of the state and the marginal utility in that state are too low to explain the observed price.

Most suggestions of irrationality appear to deal with mistakes in probability rather than mistakes in marginal utility. For example, the widespread belief that Yahoo's market value of $140 billion in early 2000 was irrationally high derived from a belief that investors overstated the probability that the company would make high profits, not from a belief that investors mistakenly thought that Yahoo would do particularly well in bad times.

In strictly stationary settings, the standard is straightforward for judging whether a person's subjective probability is correct. For example, when the payoff derives from the toss of a fair coin, it is easy to determine the rationality of the resulting security price. In the case of judging Yahoo's high value, the standard is less clear. There is no solid way to show that investors have the probability wrong. Yahoo is something new—there is no long history of similar companies to form a probability distribution from data drawn from a stationary distribution.

As Mordecai Kurz [1994] has stressed, rational beliefs about probabilities are only loosely constrained in a non-stationary world. An individual who believes that new principles govern the economy will not rationally use historical data to form beliefs about today and the future. Rather than deriving probabilities from past experience, the individual will think through what will happen in the future.

In an economy where securities are frequently and seriously mis-priced relative to actual probabilities, an intelligent investor can earn a higher return from buying underpriced securities and selling them if they become overpriced, in comparison to a policy of holding a stable portfolio representative of the entire market. Mark Rubinstein

[2000] observes that the evidence goes strongly against irrational pricing by this criterion. Managers of mutual funds—who deploy vast resources and face strong incentives—consistently demonstrate the superiority of the passive strategy. Actively managed funds generate returns below passive funds by about the amount of the trading and other costs involved in active management. Rubinstein considers this to be the strongest evidence in favor of rational markets.

A substantial body of opinion holds that stock prices move too much—they over-react to news, and they move sharply even when there is no news. Even if we don't know when a stock is over-priced, we can say that its price moves too much. A line of investigation begun by Robert Shiller formulates the idea as a statistical test. Paul Krugman [2000] wrote: "[O]ne thing is for sure: [The stock market] fluctuates more than it should...it's more a series of random leaps that a random walk." Though many economists agree with Shiller and Krugman, it is important to understand that excess volatility implies that active trading strategies yield higher returns, a proposition that gains no systematic support from the evidence.

If the marginal utility-Lagrange multiplier part of the earlier decomposition of a security price is a known constant, $\frac{1}{1+\rho}=\frac{m}{\lambda}$, then the multi-period version of the security valuation analysis implies that the price of a stock paying cash x to shareholders in future periods is the present discounted value, $p=E\left[\sum\left(\frac{1}{1+\rho}\right)^{\tau}x_{\tau}\right]$. If there is excess volatility, we can introduce a variable N (for noise): $p=E\left[\sum\left(\frac{1}{1+\rho}\right)^{\tau}x_{\tau}\right]+N$. Now subtract $(1+\rho)\,p_t$ from p_{t+1} and write the result in the form $\frac{p_{t+1}-p_t+x_t}{p_t}-\rho=\varepsilon_t+\frac{N_{t+1}-(1+\rho)\,N_t}{p_t}$. The left-hand side is the excess return—the difference between the realized return (including capital gain) and the discount rate, ρ. The right-hand side is an unpredictable random variable, ε_t, arising from changes in expected future dividends, plus the quasi-difference of the noise component, as a ratio to the stock price.

Recall that the evidence suggests that experts using active trading strategies make lower returns than passive investors. One implication of this finding is that the excess return, $z_t = \dfrac{p_{t+1} - p_t + x_t}{p_t} - \rho$, is not serially correlated. If it were serially correlated, experts trading on that observation would make higher returns than do passive investors. The expectation update, ε_t, is serially uncorrelated by construction. The noise term will be serially uncorrelated only in the special case where $N_{t+1} - (1+\rho) N_t = \eta_t$, a serially uncorrelated random variable. This special case—an asset bubble—has received substantial attention in the literature on market rationality. A stock price could zoom off toward infinity with a noise term growing at the discount rate, and that particular form of pricing error would not show up as a predictable component of the excess return. It is essentially impossible to build a model where intelligent people believe that the value of a stock will become larger and larger in relation to all other quantities in the economy. A fundamental efficiency condition holds that the discount rate exceeds the rate of growth of output and other quantities.

Peter Garber [2000] takes a close look at the Dutch tulip mania and other supposed historical examples of bubbles. He concludes that they are the urban legends of financial markets.

All other forms of noise create predictable excess returns. In particular, any form of transitory noise creates opportunities for beating the market. In general, when the stock price rises, there is a chance that it rose because of the transitory noise and will therefore fall in the near future. The trading rule is to buy right after negative returns and sell after positive returns. Fund managers have not beaten the market with this strategy.

Robert Shiller [2000] nicely summarizes the evidence against rationality of the stock market. Most of the evidence involves predictability of returns. Subsequent ten-year returns tend to be lower when stock prices are high relative to earnings. Firms with low market values in relation to book value tend to have higher subsequent returns. He also notes that there is direct evidence of noise in stock prices because the present discounted value of actual dividends is less volatile than actual stock prices. This proposition translates directly into the observation I made earlier that noise in stock prices makes returns predictable. The difficulty with this line of attack on rational markets is its neglect of the $\frac{m}{\lambda}$ part of the asset pricing equation. The modest predictable element of returns is easily consistent with modest variations in $\frac{m}{\lambda}$. Modern financial economics speaks of the puzzle of time-varying risk premiums, not a clear finding of irrationality. The same point applies in the discovery of the market-to-book effect—it may reveal something about risk, the factor captured by $\frac{m}{\lambda}$.

Shiller and other believers in irrational markets draw on two other types of evidence. One is obvious pricing errors. Shiller cites a high valuation given to eToys shortly after the company went public. I will come back to this topic at the end of the lecture. Examples of pricing errors noted by others in this camp include the failure of two equivalent claims on Shell Oil to trade at the same price and the valuation of 3Com below the value of its holdings of Palm. These examples demonstrate that arbitrage can be expensive and that disagreements about the values of securities can be substantial when arbitrage does not close the gap. These and related pricing puzzles such as the discounts on closed-end mutual funds raise the question of how much of the value held by a corporation will eventually flow to shareholders and how much will be diverted to managers and other stakeholders.

The second category of evidence supporting irrationality is movements in stock prices without corresponding news. No new information about the fundamentals of the economy became evident on the days in October 1929 and October 1987 when the market fell remarkably. Interpretation of these kinds of events takes us squarely back to the issue of how people estimate probabilities in a non-stationary world. Much of Shiller's discussion of his ideas about how social processes result in price changes would be embraced by believers in the basic principle of asset valuation. It shows that one person values another's opinion in assessing probabilities in a non-stationary environment.

My tentative conclusion is that—despite its substantial movements—the stock market operates on the principle of recording the properly discounted value of the future cash shareholders expect to receive. The rest of the lecture will make this hypothesis and say something about where it leads us.

III. Debt

Debt is a burden on the stock market. This proposition is just a restatement of the Modigliani-Miller theorem. A firm can issue debt, retire equity, and lower its stock-market value accordingly. (To bring about a decline in its stock price rather than in the value of its outstanding shares, the firm would need to pay the proceeds out as dividends). Bulges of debt could be one explanation for periods of low aggregate value of stocks. Figure 2 tests this idea by breaking the total value of financial claims into debt and equity components. The data are from Hall [2000a] and place the value of debt on an approximate market value basis. As in Figure 1, both are stated as ratios to GDP as a convenient normalization.

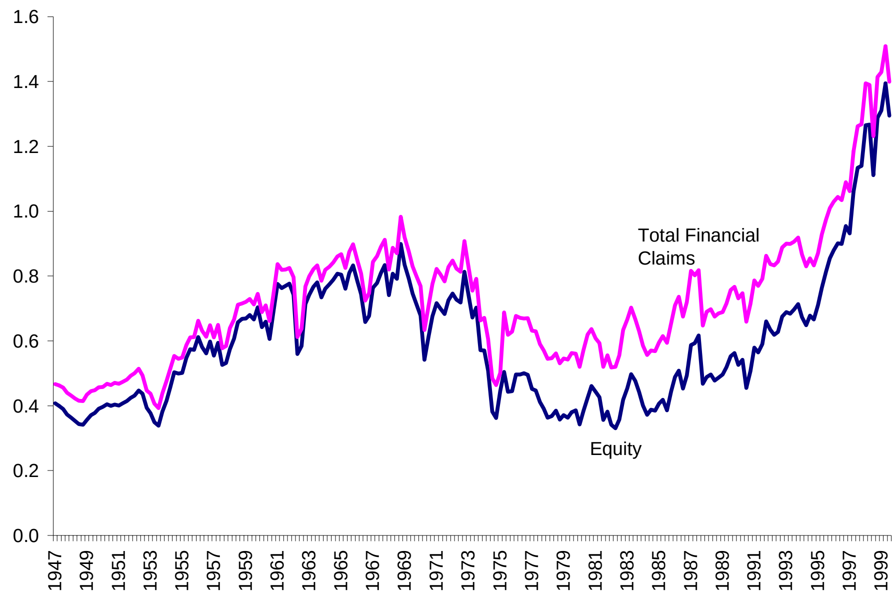

Figure 2. Total Financial Claims on Non-Farm, Non-Financial Corporations, as a Ratio to GDP, 1947-1999.
Source: Hall [2000a]

Figure 2 makes it clear that debt-type claims on firms did grow during the time of low equity values in the 1970s and 1980s, but not enough to alter the basic finding of large swings in total value.

IV. Variations in Hard Asset Values

For much of the rest of the lecture, I will use the following accounting setup:

Financial claims	*Non-financial assets*
Value of equity outstanding	Value of plant and equipment
Value of debt outstanding	Value of inventories
Value of payables and other financial obligations	Value of intangibles: intellectual property, first-to-market advantage, monopoly franchise, and the like
Less value of equity, debt, receivables, and other financial claims on others	*Less* value appropriated by stakeholders other than securities holders
= Net financial claims outstanding	= Net value of non-financial assets

Under the hypothesis of rational securities markets, the two net values are equal. We can read the value of the firm's net non-financial assets from securities markets. Data for net financial claims are shown in Figure 2.

In the simple world of many general-equilibrium macro models, the only non-financial asset is physical capital. If there is only one kind of output and output serves as numeraire, and there are no adjustment costs, then the value of securities reveals the *quantity* of capital. The result is what one might call the *real stuff* theory of the stock market—the market moves only as much as the quantity of real stuff owned by corporations. Returns earned by shareholders are exactly the marginal product of capital. Of course, it remains true that the value of the stock market is the present discounted value of earnings. Each unit of capital earns the marginal product of capital, and the marginal product also equals the discount rate.

Figure 3 shows that real stuff is not an important part of the story of postwar movements of total financial claims. The value of hard assets is a stable fraction of GDP and its small movements are negatively correlated with those of total financial claims.

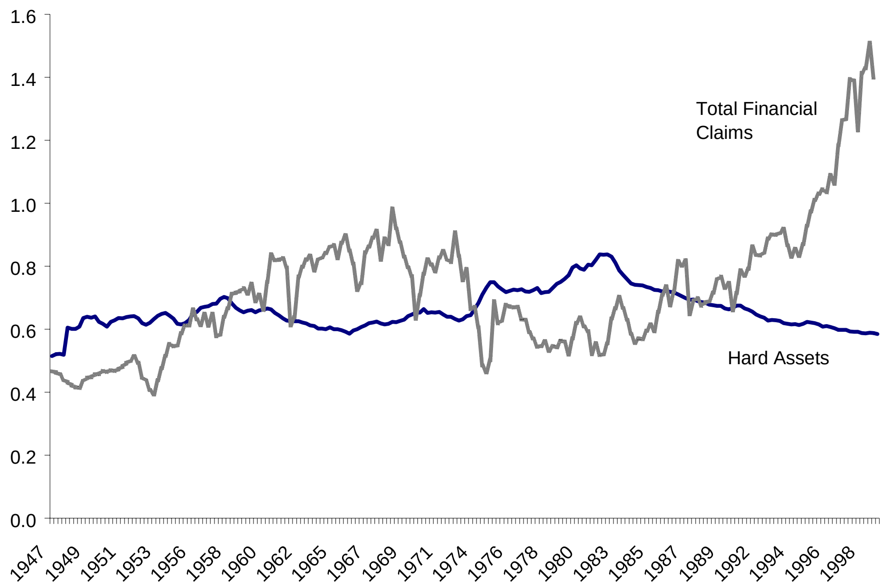

Figure 3. Hard Assets (Plant, Equipment, and Inventory) of and Total Financial Claims on Non-Farm, Non-Financial Corporations, as a Percent of GDP
Source: Hall [2000a]. Plant and equipment are valued at estimated replacement cost.

In the presence of adjustment costs in investment, the value of capital in place will fluctuate in relation to the price of capital, as discussed in Abel [1999]. Corporations will earn transitory rents from scarce installed capital when the demand for capital rises unexpectedly and the value of that capital will reflect the rents. Hall [2000b] shows that these valuation effects for hard assets are probably a small part of the story.

V. Intangibles

Figure 4 shows the net value of intangibles, calculated as a residual by subtracting the value of hard assets from total financial claims. Huge low-frequency movements of the residual occurred over the period. During two episodes—the 1950s and the 1970s-80s—securities values were below—sometime way below—the reproduction cost of capital. The likely contribution of adjustment costs was in the wrong direction for most of these years, as investment was strong. During these years, according to securities markets, the value appropriated by other stakeholders considerably exceeded the value of intangibles. The single hardest episode to understand during those years is the plunge of residual value from almost 30 percent of GDP in 1972 to minus 30 percent in 1974.

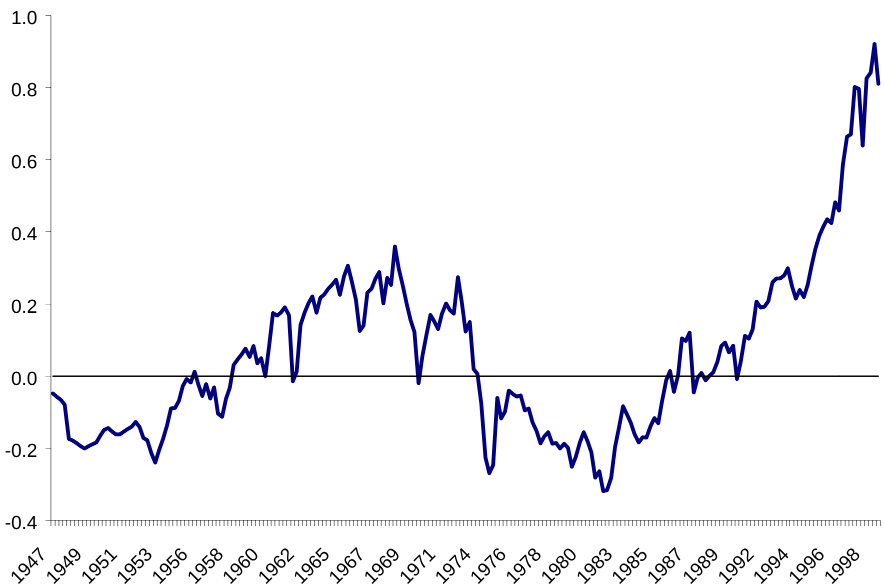

Figure 4. Net Value of Intangible Assets of Non-Farm, Non-Financial Corporations, as a Ratio to GDP
Calculated as the difference between the net value of financial claims and the value of hard assets, as a ratio to GDP, from Hall[2000a].

Intangible values vary across industries even more sharply than over time. Figure 5 shows residual value in 1998 for two-digit industries in 1998 with more than $10 billion in securities value. The industry assignments are only approximate, because many companies operate in more than one industry. The industries with the highest values of the residual—those with high intangible values and low offsets for other stakeholders—are the technology-users: insurance, banks, and business services. Industries with low levels of intangibles and heavy burdens from other stakeholders, such as utilities, oil and gas extraction, primary metals, and air travel, have zero or negative residuals. The figure shows wide variation in the residual.

Both the time-series and cross-section evidence demonstrate large amounts of positive and negative intangible value in corporations. In recent times and in technology-using industries, corporations have accumulated enormous stocks of intangible wealth, according to securities values. In the later 1970s and early 1980s, net corporate stocks of intangibles were negative; in a few industries as of 1998, intangibles remained negative.

Both the timing and the industry distribution of positive intangibles suggest a strong association with computers and software. Corporations appear to have built stocks of business know-how, organizational principles, and electronic business models—types of property I have called “e-capital” (Hall [2000b]). Only technical improvements that remain proprietary become part of a corporation’s stock of valuable intangibles. Despite the importance of the computer in the formation of modern intangibles, computer makers own relatively small stocks of intangibles. The availability of computers is a social good, owned by no company, and thus captured ultimately by workers as higher real wages. Companies like Wal-Mart—whose intangibles account for 80 percent of its total value—harness computers to create proprietary business methods.

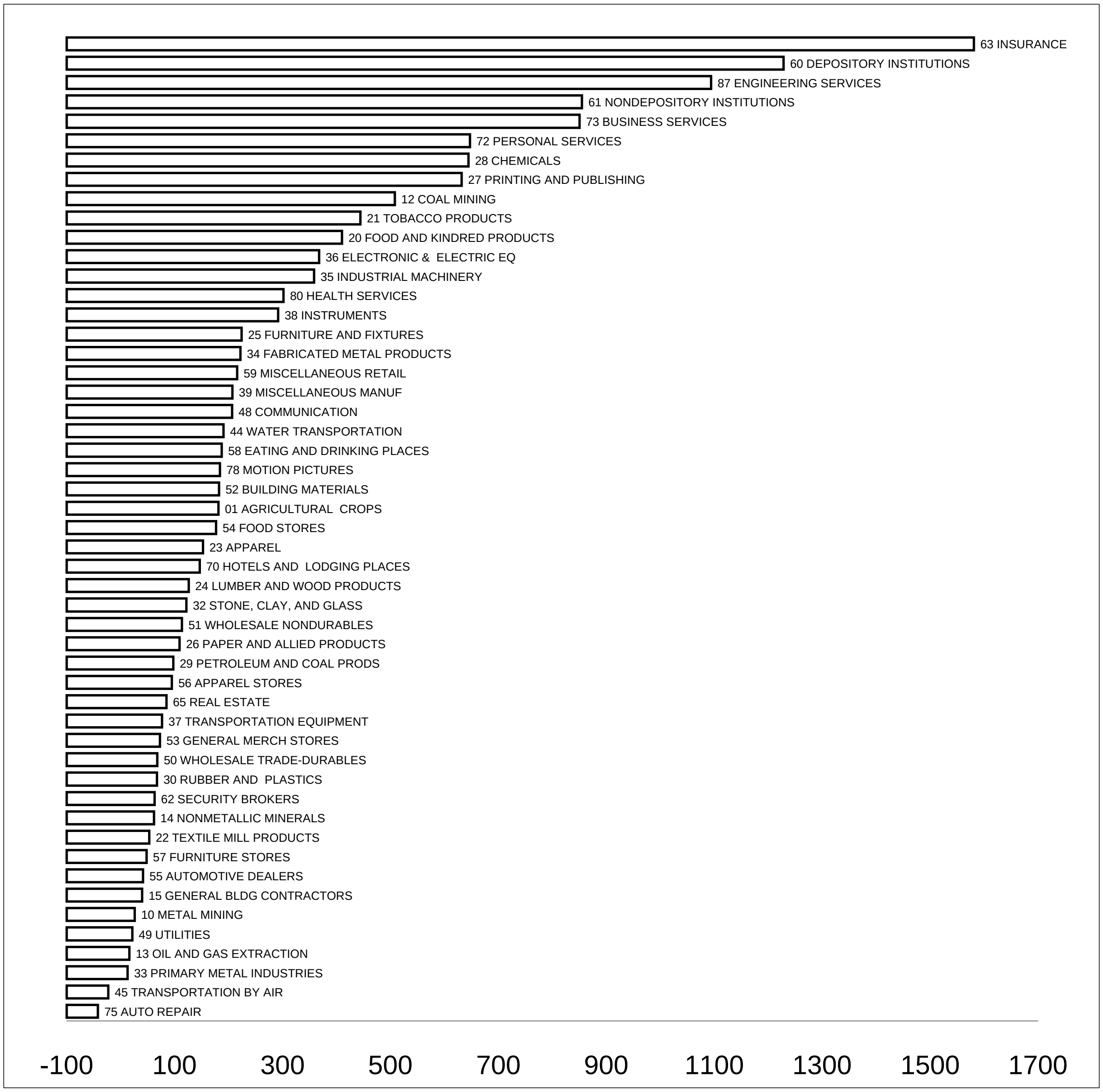

Figure 5. Residual Value as a Percent of Value of Hard Assets for Selected Two-Digit Industries, 1998
Source: Compustat, compiled by Jason Cummins

Negative intangibles are more of a mystery. Shareholders stand at the end of a long line of other claimants on corporate revenue—suppliers, workers, managers, and governments. Though it is common to view the return to capital as about a third of GDP, cash flow actually available to shareholders after taxes is generally around 6 percent of GDP (see Figure 6 below). At times of declines in intangibles into deep negative levels, notably 1972-74, other stakeholders seem to have tightened their grips on corporations.

Intangibles gain their value from the cash they generate. The huge rise in measured intangibles in the 1990s only makes sense if these stocks actually earned cash flow for shareholders. Similarly, the idea that intangible value became negative in the mid-1970s because of losses to other stakeholders only holds up if shareholders did suffer a loss of cash flow. The evidence I will discuss shortly suggests strongly that large swings in measured intangibles do correspond to movements in cash flow. Both the level and rate of growth of cash flow are critical to the story.

VI. Cash Flow and Securities Values

Corporate securities reveal the value of the property corporations own. The property, in turn, derives value from its ability to earn cash in the future. Do variations in corporate cash flow help explain the puzzles of the earlier figures? Are the intangibles accumulated during the 1990s paying their way with growing cash earned by corporations? Figure 6 shows a measure of corporate cash flow in relation to GDP. The numerator is corporate after-tax profit plus interest payments, from the National Income and Product Accounts. This measure accepts the NIPA’s calculation of depreciation as the flow equivalent of investment spending on hard assets but treats intangibles on a strict cash-flow basis.

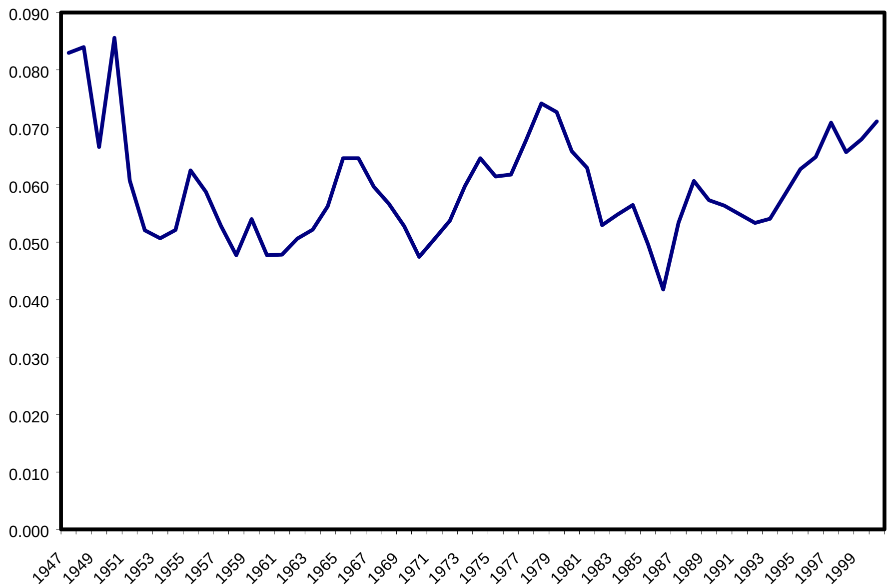

Figure 6. Corporate Cash Flow as a Ratio to GDP, from the National Income and Product Accounts

Corporate cash flow fluctuates around a level of about 6 percent of GDP. Rising cash flow in relation to GDP in the 1990s coincided with large increases in securities values relative to GDP. On the other hand, even higher cash flow in the late 1970s occurred at the same time as extraordinarily low levels of securities values relative to GDP.

The value of corporate assets depends on future as well as current cash flow. In particular, the capitalization factor—the ratio of value to current cash flow—is $\frac{1}{\rho - g}$ for discount rate ρ and constant future growth rate *g*. Figure 7 shows the actual capitalization factor together with movements in the growth rate of real corporate cash flow. The upper line is the ratio of the value of total financial claims on corporations divided by the cash flow variable of Figure 6. The lower line is, in year *t*, the annual growth rate from year *t* to year *t*+5 of cash flow divided by the GDP deflator. In the last four years, the growth rate is from year *t* to 1998, the last year for which the cash flow measure is available.

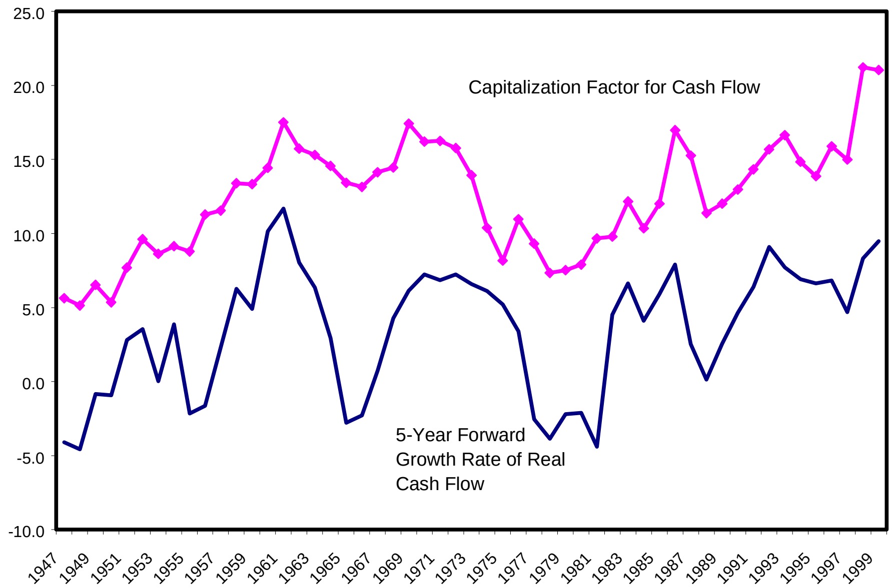

Figure 7. Capitalization Factor for Cash Flow and 5-Year Forward Growth Rate of Real Cash Flow

Figure 7 suggests that growth of cash flow has the expected relation to the capitalization factor. The puzzling drop in the capitalization factor in the mid-1970s coincided with a substantial swing in cash-flow growth, from more than 5 percent at its peak in 1973 to a trough of minus 5 percent in 1982. The recent rise in the capitalization factor corresponds to an increase in real cash-flow growth.

Notice that the capitalization factor jumped to a postwar high in 1999 and fell only a little in 2000. Other valuation ratios, such as the price/dividend ratio for stocks, showed more pronounced elevation over historical levels in the late 1990s. All of these ratios display mean reversion, especially over periods of a century or more—see Figure 1.2 in Shiller [2000]. A high capitalization factor predicts either a decline in the future stock price or an increase in cash flow. Shiller seems to lean more toward the first prediction—the high stock market in the late 1990s resulted from irrational exuberance likely to dissipate in the following years. But Figure 7 suggests that a high capitalization factor also predicts cash-flow growth, in accord with the principles of rational valuation.

VII. Discount Rate

The last influence that might help understand the puzzling movements of securities values is the discount rate. Information about the discount rate is available from the bond market. The Treasury bond market places current prices on future streams of income that are essentially risk-free in nominal terms. If the future price level were certain, a Treasury bond would have the same payoff in every state at a given future date. Its price would reveal the expected value of $\frac{m_s}{\lambda}$ for that date. Corporate returns, on the other hand, are risky, in the sense that payoffs differ across the states of the world. Because corporate returns tend to be higher in states with lower values of $\frac{m_s}{\lambda}$, the price of a claim on a corporation is lower than the price of a Treasury bond whose certain return is the same as the expected return to the corporate claim. In other words, the discount rate to be applied to corporate returns is higher than the discount rate for the corresponding Treasury bond. Still, movements of the observed levels of discounts for Treasury bonds may provide some information about the movements of discounts for claims on corporations.

Figure 8 shows the capitalization factor and the interest rate on 10-year Treasury bonds (available from 1954 onwards). The period of high interest rates—the late 1970s through the late 1980s—coincides roughly with the period of low capitalization factors. At the same time that the bond market put a low value on future interest payments, securities markets put a low value on future cash flow.

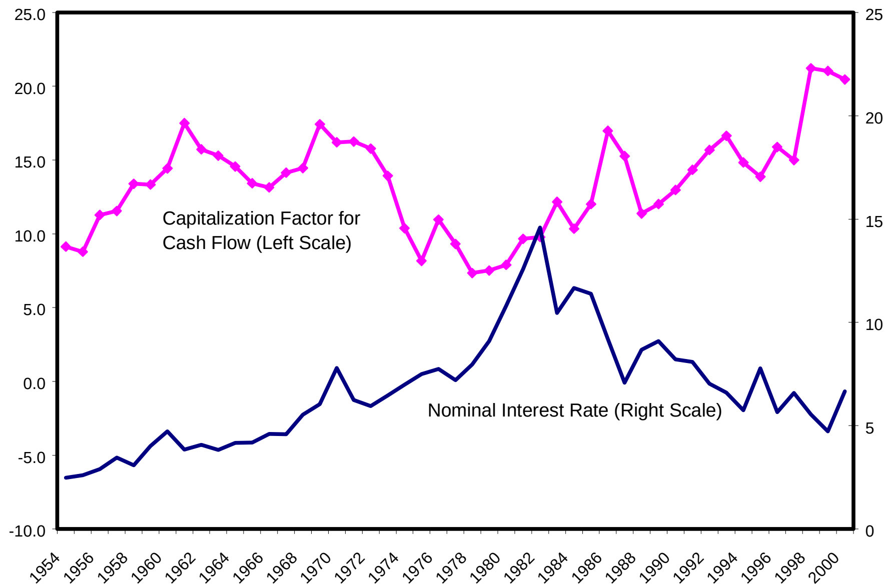

Figure 8. Capitalization Factor and Interest Rate

The interpretation of the bond interest rate is complicated by the fact that bonds, until recently, invariably had payouts defined in dollars rather than in purchasing power. Figure 8 would be more informative if it showed the real interest rate, but that would require solving the hard problem of measuring 10-year inflation expectations. But there is an approach that sidesteps the issue, by comparing the nominal interest rate to the nominal growth of future cash flow. The difference between the nominal interest rate and the nominal growth of cash flow is the same as the difference between the real interest rate and the real growth of cash flow, so inflation adjustment becomes irrelevant.

Suppose that, as of time *t*, cash flow is expected to grow at constant rate g_t from its current level x_t. The present discounted value of the cash-flow stream, v_t, at discount rate $r_t + \phi$ is $v_t = \frac{x_t}{r_t + \phi - g_t}$. Here r_t is the Treasury bond interest rate and ϕ is a risk premium, assumed to be constant. Given observed values of securities prices, cash flow, and the interest rate, I can solve for the cash-flow growth implicit in securities prices:

$g_t = r_t + \phi - \frac{x_t}{v_t}$. Figure 9 shows the growth forecast along with the actual 5-year forward growth rate from Figure 7. I estimate the risk premium as 8.2 percent, the value that equates expected cash-flow growth to actual growth over the whole period. This value is completely in line with other estimates of the premium on corporate assets over government bonds.

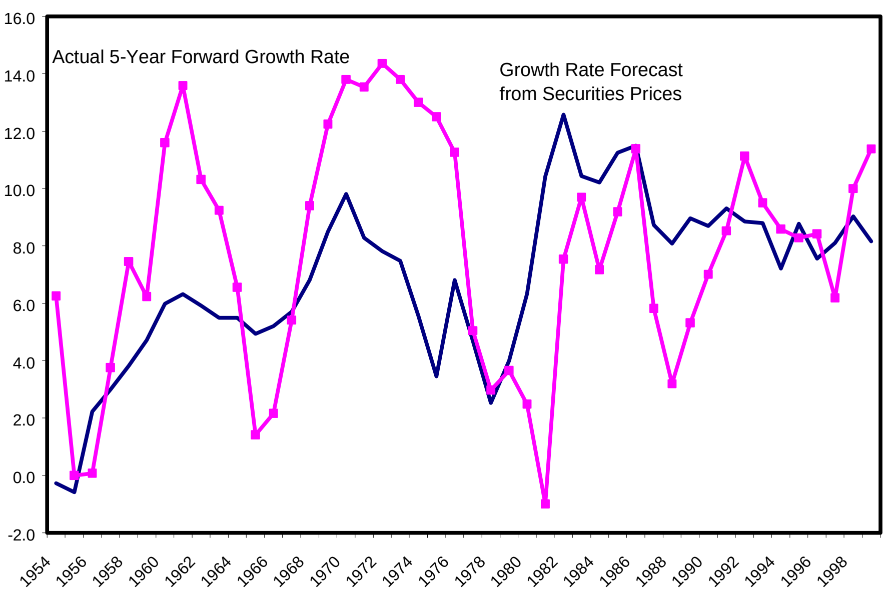

Figure 9. Growth Rate of Cash Flow Implicit in Securities Values and Actual 5-Year Forward Growth Rate of Cash Flow

Notice that the actual growth rate of cash flow has higher volatility than the forecast implicit in securities values. Shiller [1981] pointed out that forecasts should always have this property, which he believed the stock market violated.

Figure 9 tells some important stories about the major movements of the stock market since 1954. The rise in the market through 1960 occurred because of rising expectations of growth in cash flow, which were more than validated by subsequent experience. The decline in expected growth in the mid-1960s was again more than validated by actual experience. Expected cash-flow growth reached its peak in 1970, a year that was also the peak securities values relative to GDP. Then expected growth plunged from nearly 10 percent at its peak to about 4 percent per year during the years from 1974 to 1981. Actual cash-flow growth remained above expected until 1978 and then fell below, reaching a negative value in 1981. Expected growth shot upward to a peak of over 12 percent in 1982, a movement again validated by actual growth. The Tax Reform Act of 1986 was an important factor in the growth of actual cash flow.

In contrast to the excitement of earlier decades, the 1990s were a period of calm from the perspective of Figure 9. Expected cash-flow growth remained in a tight band from 1987 through 2000. Actual growth, though more volatile than expected growth, validated the relatively high level of expected growth. The corresponding story about the extraordinary heights that the stock market reached by 2000 is simple: High expected cash-flow growth coupled with low discount rates call for extreme levels of the capitalization factor. Moreover, as cash-flow growth actually occurs, the high capitalization factor multiplies a high level of the flow, resulting in astronomical levels of the stock market.

VIII. Valuation of Rapid Growth

The discussion to this point has characterized cash-flow growth as a constant. For companies with extraordinary valuations—eBay is the example I will consider—there is no possibility that historical growth can continue into the indefinite future. If it did, the company would account for more than all of GDP in a little over a decade. eBay's earnings are expected to rise by 71 percent in 2001 over 2000 in log terms (Levitan [2000]). The valuation of a fast-growing company requires consideration of the future decline in its growth rate.

Consider a company that generates cash for its shareholders according to $x(t) = x_0 e^{gt - \frac{\delta}{2}t^2}$. The rate of growth of the cash stream is $g - \delta t$. At time zero, growth is at initial rate g. The growth rate declines over time, reaching zero at a time $T = \frac{g}{\delta}$. Figure 10 shows an example of a stream, calibrated roughly to eBay. The present discounted value of the stream is $V = x_0 \int e^{-(\rho - g)t - \frac{\delta}{2}t^2} dt$. Special cases are (i) constant stream: $V = \frac{x_0}{\rho}$, and (ii) constant growth rate: $V = \frac{x_0}{\rho - g}$. The general value is $V = x_0 \frac{1 - \Phi\left(\frac{\rho - g}{\sqrt{\delta}}\right)}{\sqrt{\delta}\phi\left(\frac{\rho - g}{\sqrt{\delta}}\right)}$, where Φ is the cumulative standard normal distribution and ϕ is its density.

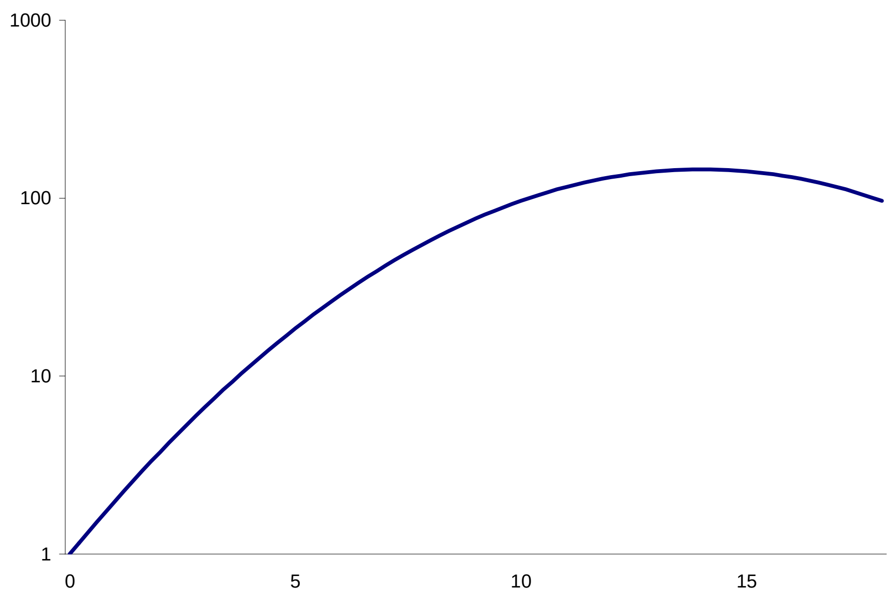

Figure 10. Growth Path of Cash Flow, with *g* = 71 Percent per Year and *T* = 14Years

With a positive, constant growth rate, value becomes indefinitely large as the growth rate g approaches the discount rate r. The growth rate cannot exceed the discount rate. On the other hand, as long as the rate of decline of growth, δ, is positive, the present value will be finite no matter what is the initial rate of growth, g. The value of V for $x_0 = 1$

is the capitalization factor, shown in Figure 12 for the same 71 percent growth rate for eBay.

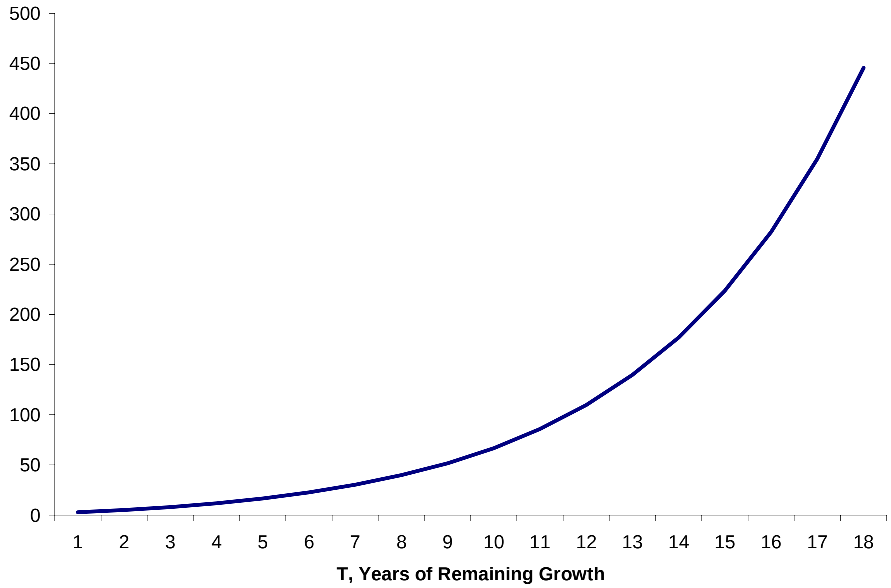

Figure 11. Capitalization Factor as a Function of *T*, the Number of Years of Remaining Growth, with ρ = 18 Percent and *g* = 71 percent per year.

At the beginning of 2000, eBay's market value was $19 billion, about 350 times its prospective earnings of $53 million. Figure 11 shows that a capitalization factor that large makes sense for a company with 71 percent earnings growth if the growth rate will gradually decline to zero over the next 17 years. To sustain the claim that eBay, along with other dot.coms in early 2000, was obviously overvalued, one would need to show that it was implausible that earnings growth would remain high for more than a decade.

In any case, eBay's market value at the beginning of 2001 was just over $8 billion, and its earnings in 2001 were expected to be $108 million, for a capitalization factor of 75. From Figure 11, this value corresponds to a little less than 11 years of future earnings growth. The decline in eBay's market value corresponds to a shortening of about 5 years in the duration of eBay's earnings growth (17 years less 11 years less the year that elapsed).

Changes in relatively distant events have huge effects on current values, for a company with high earnings growth.

Eduardo Schwartz and Mark Moon [2000] carry out a related analysis of Amazon. They model Amazon's earnings growth as a stochastic process that begins at a high rate and gradually approaches a normal rate, with an adjustment speed of 7 percent per quarter. Their model replicates the volatility as well as the high level of Amazon's valuation.

IX. Concluding Remarks

Though the evidence is hardly conclusive, the idea that the stock market values the property owned by corporations seems to stand up reasonably well in data for the United States for the past 50 years. The volatility of the aggregate stock market in relation to other broad measures such as GDP is substantial, but not out of line with the movements of cash flows accruing to corporations after paying all their costs and satisfying all non-shareholder claimants including governments. Even some of the most puzzling episodes in the stock market, such as the collapse of values by 50 percent in 1973 and 1974, seem within the grasp of rational understanding, given the sudden reversal in cash-flow growth that followed soon after. The enormous appreciation of market values in the 1990s is hardly a challenge in view of the consistently high rates of growth of cash flow during the decade. And the reversal of that appreciation in 2000 appears to coincide with diminished cash-flow growth, though it is too early to be confident on that point.

Cash-flow growth is the key factor in understanding movements in the stock market. It is illogical to condemn astronomical price/earnings ratios as plainly irrational without investigating the prospects for growth in future earnings. Streams of future cash growing at high rates are hugely valuable. Growth rates of cash earned by companies exploiting new technologies have been phenomenal. The stock-market values of these companies swing wildly. The pricing of new technology companies tries to avoid the error made in the case of Microsoft—a dollar invested in Microsoft stock in 1990 resulted in a claim on $1.38 in after-tax earnings in 2000 alone. Obviously the market in 1990 guessed absurdly low about Microsoft's cash-flow growth.

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