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Fiscal Stability of High-Debt Nations under Volatile Economic Conditions*

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Abstract. *Using a recursive empirical model of the real interest rate, GDP growth and the primary government deficit in the United States, I solve for the ergodic distribution of the debt/GDP ratio. If such a distribution exists, the government is satisfying its intertemporal budget constraint. One key finding is that historical fiscal policy would bring the current high-debt ratio back to its normal level of 0.35 over the coming decade. Forecasts of continuing increases in the ratio over the decade make the implicit assumption that fiscal policy has shifted dramatically. In the variant of the model that matches the forecast, the government would not satisfy its intertemporal budget constraint if the policy was permanent. The willingness of investors to hold US government debt implies a belief that the high-deficit policy is transitory.*

JEL classification: *E*12, *E*22, *E*32.

Keywords: National debt; deficit; fiscal policy.

1. INTRODUCTION

Today many governments are accumulating large debts. Ratios of debt to GDP near and above 1 are common. Even the United States, historically a low government debt country, is projected to have a debt of 90% of GDP within the next ten years. Currently low worldwide interest rates ameliorate the burden of the debt in many high-debt countries, but interest rates will eventually rise to normal levels. Are high-debt countries likely to collapse under the weight of ever-growing debt when that happens?

I investigate these and related questions in a model that gives full treatment to the uncertainty surrounding the accumulation of debt. Governments gain added tax revenue and often face lower demands for transfer spending in good times, while deficits swell and debt rises alarmingly in bad times, such as the present. Future business cycles are inherently impossible to forecast. Most analyses of government debt make single-value projections, assuming average condi-

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tions for each year in the future. They ignore the tail probability that a sequence of bad outcomes might drive government debt to levels from which there is no escape back to normal. This study computes complete probability distributions and thus quantifies the tail probabilities.

Many of the findings of the study involve the ergodic distribution of the debt/GDP ratio that the model implies. This distribution describes the probabilistic steady state of the model. It is the distribution that the economy converges to from any starting point. It has the property that, if our view of likely outcomes at some future time conforms to the ergodic distribution, that same distribution will describe our beliefs about the distribution across outcomes for all later times. Another way to describe the ergodic distribution is as the distribution of, for example, the debt/GDP ratio among randomly chosen future years.

2. MODEL

The model is a cousin of one in Hall and Reis (2012), where the focus is on the portfolio of the central bank rather than the financial position of the central government. All the variables in the model are in real terms, so there is no possibility that the government can use unexpected inflation to cut the payoffs to bond holders and drive their realized real returns below the promised real returns. The widespread adoption over the past three decades of inflation polices in a tight band around two per cent per year makes this assumption realistic.

I let B be the ratio of the number of bonds to real GDP, y be real GDP and g be its growth factor, the ratio of this year's GDP to last year's. The government's primary deficit as a ratio to GDP is

$$d_t = x_t - \alpha B_{t-1}. \tag{1}$$

Here x is an exogenous component, capturing budget disturbances from wars and other spending events, and α is the endogenous response of the primary deficit toward budget balance when B becomes large. α is all important in the analysis. Governments with an α of 0.1 – meaning that some combination of revenue increases and spending decreases lowers the primary deficit d by one per cent of GDP if the debt/GDP ratio B rises by 10 percentage points – are safe from debt explosions. Governments with no tendency to lean against debt, with α=0, face a likelihood but not a certainty of debt crisis. Current projections of US government debt suggest that not only will US policy fail entirely to lean against the debt but that the primary deficit will be 2.5 per cent of GDP higher than it has been historically. A version of the model that matches the projection has the debt continuing to grow to completely unsustainable levels.

Government debt takes the form of delta bonds – obligations that pay a real coupon that starts at κ and declines by the factor δ each year – see Woodford (2001). A delta bond τ years after issuance sells for $\delta^\tau q$. Accordingly, I count outstanding bonds of that age as δ^τ units of debt when reckoning B. I choose κ so that the typical value of a bond is 1, and thus B is fairly close to the debt/GDP ratio.

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The law of motion of the number of bonds outstanding is

$$B'y' = \frac{(x' - \alpha B)y' + \kappa By}{q'} + \delta By. \tag{2}$$

Here and throughout the study, I use a prime (as in B') to denote a variable one year after the corresponding variable without a prime. The quantity $\frac{(x'-\alpha B)y'}{q'}$ is the number of bonds issued to cover the current primary deficit – it is the amount of the deficit divided by the market price of bonds. The quantity $\frac{\kappa By}{q'}$ covers the coupon payments of κ unit of output per bond, financed by selling new bonds at price q'. The quantity δBy is the number of bonds remaining from the previous year.

Standard principles of modern financial economics are implicit in the model. It could include a stochastic discounter, a function of s and s', in which case it could price any security including delta bonds. Each security's price would be a function of the current state s. But, given that the model includes only one security, the delta bond, it is equivalent to measure its state-dependent price directly from the data, the procedure I follow. Hall and Reis (2012) derives a stochastic discount factor for a delta bond from its state-dependent price; the existence of an SDF guarantees the absence of arbitrage among all asset prices that satisfy the standard asset-pricing condition. See Cochrane (2001) for a complete discussion of these principles.

After dividing both sides by y' and substituting $g' = y'/y$, the law of motion becomes

$$B' = \frac{x' - \alpha B + \kappa B/g'}{q'} + \delta B/g', \tag{3}$$

or

$$B' = \frac{x'}{q'} + \left(\frac{\kappa}{q'g'} - \frac{\alpha}{q'} + \frac{\delta}{g'}\right)B. \tag{4}$$

Some special cases illustrate the evolution of the government debt. First, suppose GDP is constant (g=1), the primary deficit is a positive constant x, bonds are consols (δ=1), with a coupon κ chosen to make q=1, so the interest rate is r=κ. Fiscal policy makes no active attempt to stabilize the debt/GDP ratio (α=0). The law of motion is

$$B' = x + (r+1)B. \tag{5}$$

Then if r>0, B rises without limit, whereas if r<0, the number of bonds outstanding approaches a stationary value $B^* = -x/r$, a positive value.

With a constant positive growth rate of real GDP, $g-1$, the corresponding condition is $r>g-1$ to make it imperative for fiscal policy to lean against debt accumulation with a positive α to prevent a chronic deficit from creating an endless upward spiral in the debt/GDP ratio. There is a debate in the literature on debt policy whether the relevant real interest rate tends to be above or below the rate of growth of real GDP. Currently it appears to be below the growth rate.

I assume that the underlying economy follows a Markov process. The economy has an integer-valued fundamental state, s, with a transition matrix

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$$\omega_{s,s'} = \text{Prob[next state is } s'| \text{ current state is } s]. \tag{6}$$

I let x_s be the exogenous deficit in state s, and similarly for the GDP growth rate g_s and the bond price q_s. The full state of the economy is the pair [s,B]. The number of bonds outstanding is a separate state variable, not a function of the fundamental state s alone. The variables that are functions of the discrete state variable alone are the exogenous component of the primary deficit, x_s, the growth rate of GDP, g_s, and the delta-bond price q_s. This assumption rules out feedback from the level of the debt B to the fundamental conditions in the economy. Within the historical range of variation of B prior to 2008, this assumption made perfect sense. At the end of the study, I consider departures from the assumption.

Each year, the economy transits from s to s' with probability $\omega_{s,s'}$ and from B to B' according to

$$B' = \frac{x_{s'}}{q_{s'}} + \left(\frac{\kappa}{q_{s'}g_{s'}} - \frac{\alpha}{q_{s'}} + \frac{\delta}{g_{s'}}\right)B. \tag{7}$$

Let $\Omega(s'|s)$ be the conditional cumulative distribution function of s' given s; it has mass at integer values of s'. Then let $T(s', B'|s, B)$ be the conditional joint cdf of [s,B] given the prior state:

$$T(s', B'|s, B) = \Omega_{s',s}\mathbb{I}\left(B' - \frac{x_{s'}}{q_{s'}} - \left(\frac{\kappa}{q_{s'}g_{s'}} - \frac{\alpha}{q_{s'}} + \frac{\delta}{g_{s'}}\right)B\right), \tag{8}$$

where $\mathbb{I}(\cdot)$ is the indicator function equal to 0 for a negative argument and 1 for a non-negative one.

The ergodic cdf of [s,B], say $Q(s,B)$, satisfies the invariance condition,

$$Q(s', B') = \int_{s=1}^{k}\int_{B=-\infty}^{\infty} T(s', B'|s, B)dQ(s, B) \text{ for all } s' \text{ and } B'. \tag{9}$$

To approximate the stationary distribution to any desired accuracy, one can choose a set of N regions in the [s,B] space, with central points $\bar{s}_i$ and $\bar{B}_i$, and let

$$q_i = \text{ probability that } Q \text{ assigns to region } i \tag{10}$$

and

$$t_{i,j} = \text{probability that } T \text{ assigns to region } j \text{ conditional on originating from the point } [\bar{s}_i, \bar{B}_i]. \tag{11}$$

Then solve the linear system,

$$q_j = \sum_{i=1}^{N} t_{i,j}q_i \tag{12}$$

and

$$\sum_i q_i = 1. \tag{13}$$

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The solution is

$$q' = \text{bottom row of } [(\text{ all but last column of } \bar{T} - I), \iota]^{-1}, \tag{14}$$

where $\bar{T}$ is the matrix of values of t, I is the identity matrix and ι is a vector of ones.

All the results in the study are near-exact calculations of probabilities, not tabulations of simulations.

Table 1 Data sources

Series	Source
Rate of inflation, GDP price index	NIPA Table 1.1.4
Nominal interest rate on five-year Treasury notes	FRB H.15 data, constant maturity
Unemployment rate	BLS Current Population Survey series LNS14000000
Nominal GDP	NIPA Table 1.1.5
Nominal primary deficit	NIPA Table 3.2, negative of federal government saving less interest payments
Real GDP	NIPA Table 1.1.6
Gross federal debt held by the public	Economic Report of the President, series FYGFDPUB
Forecasts of GDP, inflation and interest rate	CBO Baseline Forecast spreadsheet, August 2012
Forecasts of primary deficit	CBO Baseline Budget Projections spreadsheet, August 2012, adjusted according to Deficits Projected in CBO's Baseline and Under an Alternative Fiscal Scenario spreadsheet

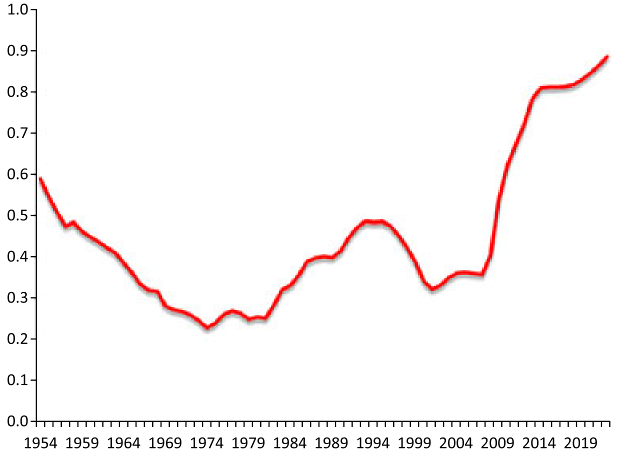

Figure 1 Historical and projected debt/GDP ratio

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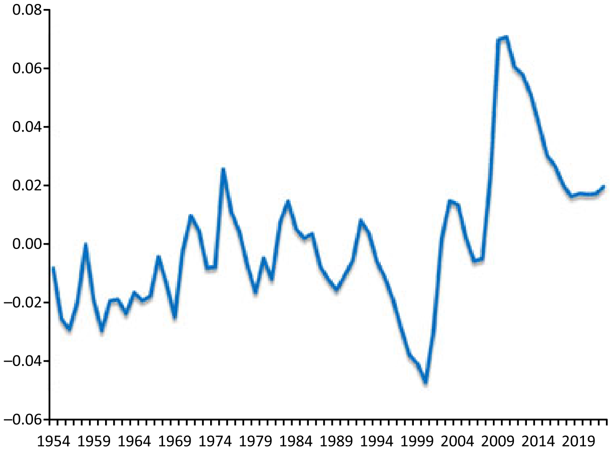

Figure 2 Historical and projected ratio of the primary deficit to GDP

3. THE MODEL APPLIED TO THE UNITED STATES

Table 1 describes the data sources for the United States. Figure 1 shows the debt/GDP ratio calculated from the data, and Figure 2 shows the ratio of the primary deficit to GDP. From the data, I calculate the variables in the model as:

1. Debt/GDP ratio (proxy for B): Debt in the hands of the public in current dollars divided by nominal GDP.
2. Primary deficit as a ratio to GDP (d): The negative of current-dollar government saving, less interest payments, divided by nominal GDP.
3. GDP growth factor (g): This year's real GDP divided by last year's.
4. Bond price (proxy for q): Calculated as $\kappa/(r{+}1{-}\delta)$ where r is the real interest rate, calculated in turn as the nominal five-year bond yield less the expected five-year ahead inflation rate, taken as the fitted value from a regression of the five-year future inflation rate on the current rate and the first through fourth powers of time. The parameter κ is chosen to set the bond price to 1 in state 2.

To define the fundamental states of the economy, I apply k-means clustering analysis (Steinhaus, 1956). This method produces a designated number, k, of clusters from a matrix of data (observations and variables) by finding centroids for each cluster and assigning observations to clusters so as to minimize the sum across all observations of the Euclidean distances of each from a centroid. The variables I use for clustering are the rate of inflation, the real interest rate, the unemployment rate and the GDP growth rate ($g-1$ in the notation of the study). I designate k=6 clusters.

Table 2 describes the resulting set of six fundamental states of the economy. In words, the states are as follows:

1. Boom: Moderate inflation and real interest rate, low unemployment, high GDP growth.
2. Normality: Moderate inflation and real interest rate, low unemployment, above-average GDP Growth.

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Table 2 Fundamental states of the US economy

State	Inflation, π	Real interest rate, r	Unemployment rate, u	GDP growth factor, g	Primary deficit/GDP	Probability	Example years
1	0.024	0.013	0.048	1.058	−0.018	0.16	1955, 1968, 1999
2	0.024	0.025	0.051	1.027	−0.007	0.41	1954, 1969, 2007
3	0.058	0.034	0.059	1.019	−0.006	0.10	1970, 1979, 1990
4	0.033	0.032	0.075	1.045	0.011	0.15	1977, 1986, 1993
5	0.082	0.060	0.082	1.000	0.004	0.06	1975, 1980, 1982
6	0.016	0.002	0.084	1.009	0.041	0.12	1958, 1961, 2009

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3. Monetary stress: High inflation, high real interest rate, high unemployment, low GDP growth.
4. Recovery: Moderate inflation, high real interest rate, high unemployment, high GDP growth.
5. Stagflation: High inflation and real interest rate, high unemployment, low GDP growth.
6. Slump: Low inflation, low real interest rate, high unemployment, low GDP growth.

Table 3 shows the transition probabilities among the six fundamental states of the economy. It shows, for example, that escape from the slump state, number 6, is 38% likely each year the economy is in that state. Escape is to state 1, the boom state, or state 4, the recovery state. Otherwise, it is 63% likely that the economy will remain in the slump. By contrast, it is 71% likely that the economy will remain in its normal state, state 2. Escape from there is about equally likely to states 1 (boom), 3 (monetary stress) and 6 (slump).

To estimate the feedback parameter α, I match the quartiles of the actual distribution of the debt/GDP ratio B, shown in Figure 3, to the distribution implied by the model. The estimate is α=0.11.

Table 3 Transition probabilities among states

From	*To*					
	1	2	3	4	5	6
1	0.45	0.45	0.09	0.00	0.00	0.00
2	0.11	0.71	0.07	0.00	0.00	0.11
3	0.14	0.00	0.43	0.14	0.29	0.00
4	0.00	0.30	0.10	0.60	0.00	0.00
5	0.00	0.00	0.00	0.50	0.50	0.00
6	0.25	0.00	0.00	0.13	0.00	0.63

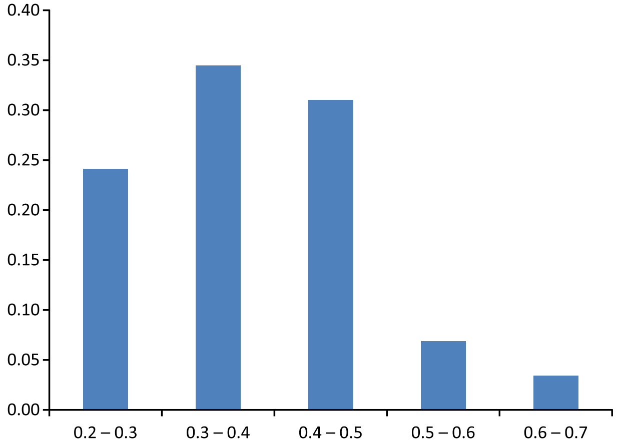

Figure 3 Distribution of the actual debt/GDP ratio

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Manipulation of the model requires setting up the regions in the state space described above. I define a separate set of regions for each of the six values of the discrete fundamental state. The set comprises 500 equally spaced intervals from $\underline{B}$ to $\bar{B}$. Thus, the overall dimension of the state space is 3,000.

Figure 4 shows the ergodic distribution of the debt/GDP ratio (in the sense of the variable B, which is the ratio of the number of bonds to GDP and is closer to reported numbers than is qB, the market value of the debt, because the debt figures never mark the debt to market). This distribution is the marginal over the fundamental states calculated from the full ergodic distribution across all 3,000 compound states.

Table 4 compares the quartiles of the fitted distribution of B from the model to the quartiles of the actual distribution of the debt/GDP ratio.

4. ROLE OF THE DEBT-CORRECTION PARAMETER, α, AND THE POSSIBILITY OF A FISCAL FREE LUNCH

Figure 5 shows the ergodic distribution of the debt/GDP ratio B with α set to zero, so fiscal policy does not lean against debt accumulation. Making this change alone results in a high probability of large *negative* debt, as the government continues to run small surpluses in spite of extinguishing the national debt. To offset this tendency, I introduce a constant in the equation for the

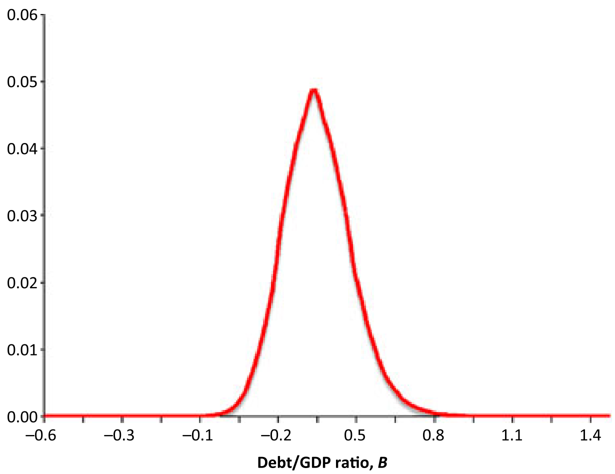

Figure 4 Ergodic distribution of the debt/GDP ratio from the model, base case

Table 4 Quartiles of the actual and fitted distributions of the debt/GDP ratio

Quartile	Fitted	Actual
0.25	0.273	0.318
0.50	0.343	0.364
0.75	0.413	0.453

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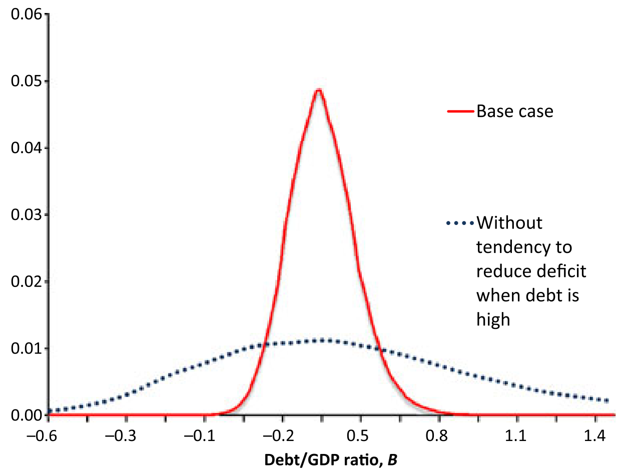

Figure 5 Ergodic distribution of the debt/GDP ratio with zero debt correction and higher primary deficit

growth of debt that corresponds to raising the primary deficit by 0.97 per cent of GDP per year. This raises the primary deficit from its average value in the data of 0.06 per cent of GDP to an average of 1.03 per cent. Figure 5 demonstrates that the debt does not spiral out of control, even without the government leaning against debt accumulation through the effect of the parameter α, and while borrowing to pay for a primary deficit of 1.03 per cent of GDP and pay the interest on a positive amount of debt. There is a non-negligible probability that the debt/GDP ratio will rise to the level of Italy's, which might raise questions about default. There is also a probability that the debt will drop below zero, implying that the government would hold debt claims on the private economy or other governments. Krishnamurthy and Vissing-Jorgensen (2012) show that low values of government debt breed financial instability, as private institutions take on the role of providing liquid debt instruments, a situation that is unstable and leads to financial crisis.

This calculation answers an important question in dynamic public finance: Is the government's borrowing rate sufficiently low that the opportunity to issue debt creates a free lunch? (see, e.g., Bohn, 1995). The answer is a qualified yes. As many earlier studies have discussed, the basic condition for a free lunch is that the real interest on government debt falls short of the growth rate of output. Here, the condition is a bit more subtle because both the borrowing rate (here represented as the valuation of federal debt) and the growth rate are random variables. But the calculations behind Figure 5 show that a small amount of chronic deficit finance of government purchases – 1.03 per cent of GDP – results in the accumulation of a modest amount of debt which remains constant relative to GDP. The model would permit any amount of chronic deficit spending with a corresponding ergodic value of the debt/GDP ratio, but the ergodic level of debt/GDP would grow in proportion to the deficit. The debt/GDP ratio would be unrealistically high for the current level of the US primary deficit.

The basic message of these calculations is that government needs to keep the primary deficit, averaged over the states of the economy, quite close to zero.

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Note that the principle works in reverse – a chronic primary surplus creates, in the long-run ergodic equilibrium, a large negative debt/GDP ratio, interpreted as a huge holding by the government of debt claims on the private economy. Krishnamurthy and Vissing-Jorgensen (2012) have argued, persuasively in my view, that there is an optimal level, around 40% of GDP, for the national debt. Maintaining that ratio in the longer run requires a primary deficit that averages, across states of the economy, very close to zero. With a chronic deficit, debt reaches levels that drive down its price and thus lead to an explosion of debt. With a chronic surplus, the government denies the private economy the benefits of a highly liquid market in safe debt. Private substitutes for that safe debt have proven unstable.

5. THE EVOLUTION OF THE DEBT/GDP RATIO

The model tracks the distribution of its variables from any starting point. With z denoting the column vector of probabilities across the model's 3,000 states, and z_0 the starting point, a vector of zeros and a single 1 in the position of the initial condition, the distribution evolves as

$$z_{t+1} = T' z_t. \tag{15}$$

The marginal cumulative distribution of the debt/GDP ratio in year t is

$$m_t = (\Gamma \otimes \iota) z_t, \tag{16}$$

where Γ is a square matrix of dimension 500 with ones on and below its diagonal and zeros above, $\otimes$ is the Kronecker product and ι is a row vector of six ones.

Figure 6 describes the counterfactual evolution of the distribution of the debt/GDP ratio in the absence of the crisis. It starts with the moderate debt/GDP ratio of 2007, B=0.36, and with the economy in its normal state, s=2. The heavy line shows the mean of the distribution of the debt/GDP ratio from 2007 through 2031. The thin lines show the 5th, 25th, 75th and 95th percentiles of the

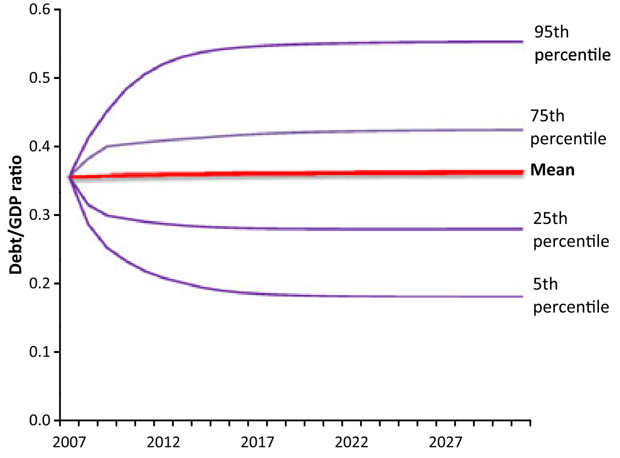

Figure 6 Distribution of debt/GDP ratio starting from normal conditions in 2007

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distribution in each year. Within a decade, the distribution fans out to the ergodic distribution shown in Figure 4. The actual debt/GDP ratio in 2012 is 0.72. The probability, as of 2007, of that value or higher in 2012, according to the model, is 0.00042. The deep and lingering effects of the crisis, and the huge increase in the federal debt resulting from it, were deeply surprising from the perspective of 2007.

Figure 7 shows the model's distribution of the debt/GDP ratio under the hypothesis that the economy had been in the slump state (number 6) in 2007. In that case, the unemployment rate would have been 8.4 per cent instead of the 5.1 per cent in state 2. The deficit would have widened and the mean level of the debt/GDP ratio would have risen from 0.36 to 0.40 in 2010. In later years, the mean level would have fallen gradually back to its ergodic level. The probability that the debt/GDP ratio would have been 0.72 is 0.00208, higher than in Figure 6 but still quite low. Not only was the onset of high unemployment and high deficits a surprise, but an even bigger surprise was the continuation of bad times through 2012. The experience embodied in the model suggests that the debt/GDP ratio should have risen only modestly and begun to fall by 2012, when in fact the ratio has continued to rise to a high level.

Figure 8 starts the model in 2012, with a debt/GDP ratio of 0.72, in the slump state, number 6. The mean of the distribution declines fairly rapidly back to its ergodic level. The figure includes the forecast for the debt/GDP ratio from the Congressional Budget Office (CBO). This forecast embodies the CBO's 'alternative fiscal scenario' that makes reasonable assumptions about likely changes in current tax and spending law, unlike the CBO's main forecasts that assume the retention of current law. For the first five years, the CBO forecast tracks the 95th percentile from the model – from the perspective of the historical experience embodied in the model, it is quite unlikely, but not impossible, that the debt/GDP ratio will continue to rise in coming years even though the mean of the distribution of future values will decline. But starting in 2017, the CBO forecasts faster growth of the debt/GDP ratio, which the model finds quite implausible.

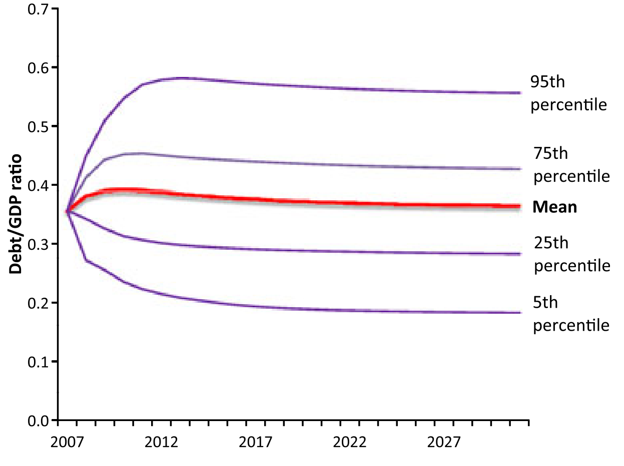

Figure 7 Distribution of debt/GDP ratio starting from slump conditions in 2007

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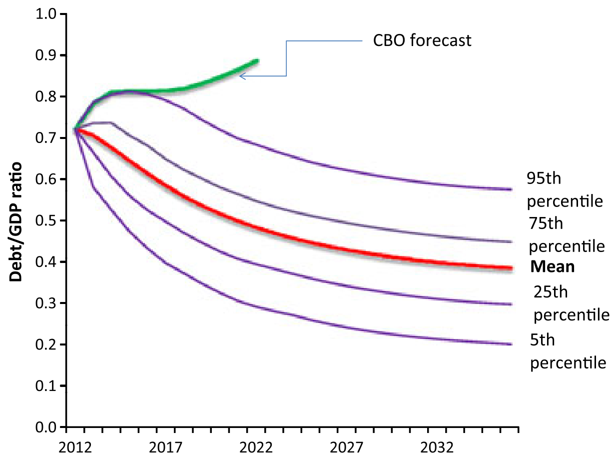

Figure 8 Distribution of debt/GDP ratio starting from slump conditions in 2012

Recent experience seems to indicate that the historical tendency to lower the primary deficit – through revenue increases or spending cuts – is no longer present in US fiscal policy. Obviously the CBO believes that such a change has occurred, or it would not project continuing growth in the debt/GDP ratio. Figure 9 shows how the model's distributions of the future debt/GDP ratio changes if the parameter α, measuring the extent to which fiscal policy leans against high values of the ratio, is set to zero. The mean of the distribution declines, but not as fast as in the base case. The CBO forecast stays within the 95th percentile of the distribution, so the disagreement between the model and the CBO is nowhere as large as in the previous figure with α=0.11.

Figure 10 shows the answer to the question, 'What fiscal policy assumption in the model would align the mean of its distribution of the debt/GDP ratio

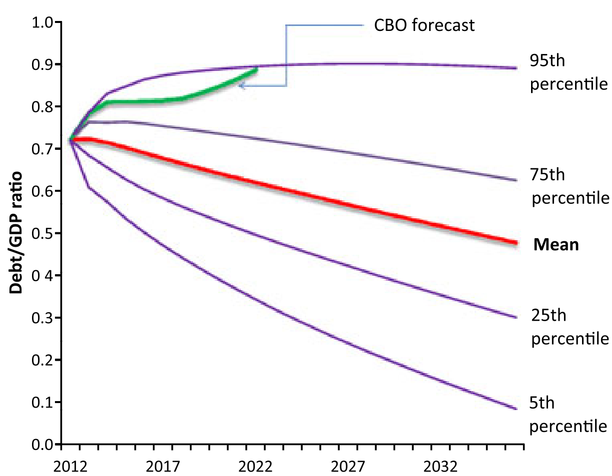

Figure 9 Distribution of debt/GDP ratio starting from slump conditions in 2012: No feedback from debt to deficit

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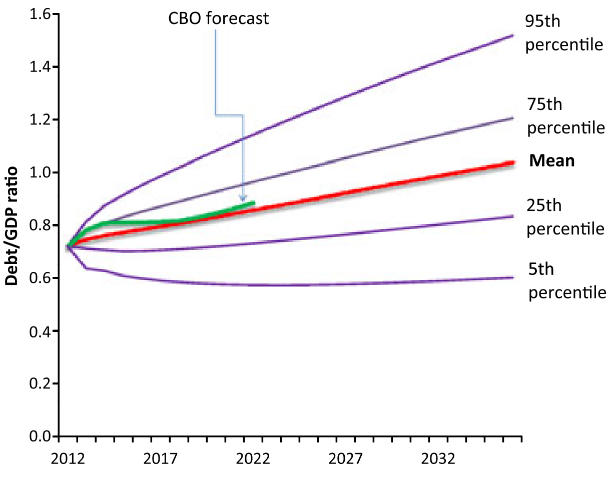

Figure 10 Distribution of the debt/GDP ratio starting from slump conditions in 2012: No feedback from debt to deficit and larger exogenous primary deficit

with the CBO's forecast?' It retains $\alpha=0$, so there is no response of the primary deficit to the growing debt. It shifts the primary deficit upward by 2.5 per cent of GDP. Thus the CBO forecast posits a dramatic departure from earlier fiscal policy, dropping the earlier tendency for policy to lean against the debt and adding a large permanent tendency toward high primary deficits. The standard explanation for this shift is that federal health and retirement spending will rise faster as a ratio to GDP than in earlier years (as the ratio itself becomes so much higher and the growth rate of this category of spending remains roughly constant), and that revenue-augmenting changes in fiscal policy will not keep pace.

6. FACTORS DETERMINING THE VOLATILITY OF THE DEBT/GDP RATIO

The ergodic distribution of the debt/GDP ratio describes the volatility of that variable, in the sense that it is the probability distribution of the ratio in a randomly chosen year. In this section, I describe alterations of the model relative to the base case by comparing the ergodic distribution of a model perturbed along one dimension to the ergodic distribution of the model in the base case, shown earlier in Figure 4.

The first comparison investigates the importance of the volatility of the bond price. When the economy is strong, interest rates tend to be high. An issue in many economies today is that large increases have occurred in government debt during a time of low rates for the economies that have retained the confidence of investors, including France, Britain, Japan and the United States. When worldwide interest rates return to normal, will these economies suffer substantial added stress from payments on their large debts once they are rolled into new bonds paying the higher rates?

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Table 5 shows how bond prices vary by conditions in the model, as indexed by the fundamental state. For example, in a slump, bonds sell for 1.115 times as much as they do in normal times. To put it differently, the burden in terms of future interest from a given level of borrowing is 1.115 times higher in normal times. To get at the issue of the volatility in the debt/GDP ratio arising from variations in bond prices, I solve for the ergodic distribution in a model that differs from the base case only in that the bond price is 1.000 in all six fundamental states. Figure 11 shows the distribution, marked with dots, along with the base-case distribution, the same as in Figure 4. It is apparent that the net effect of bond price volatility is to lower the volatility of the debt/GDP ratio, but by only a small amount. The interest savings in slumps themselves more than offset the post-slump increase in bond interest. The likelihood of ratios in the range from 0.6 to 0.9 is slightly higher when the bond price is constant.

A second source of volatility in the debt/GDP ratio is the volatility of GDP growth. Recall that the ratio evolves according to

$$B' = \frac{x'}{q'} + \left(\frac{\kappa}{q'g'} - \frac{\alpha}{q'} + \frac{\delta}{g'}\right)B. \tag{17}$$

Table 5 Interest rate and bond price by fundamental state

State	Description	Real interest rate	Bond price
1	Boom	0.013	1.061
2	Normality	0.025	1.000
3	Monetary stress	0.034	0.965
4	Recovery	0.032	0.973
5	Stagflation	0.060	0.867
6	Slump	0.002	1.115

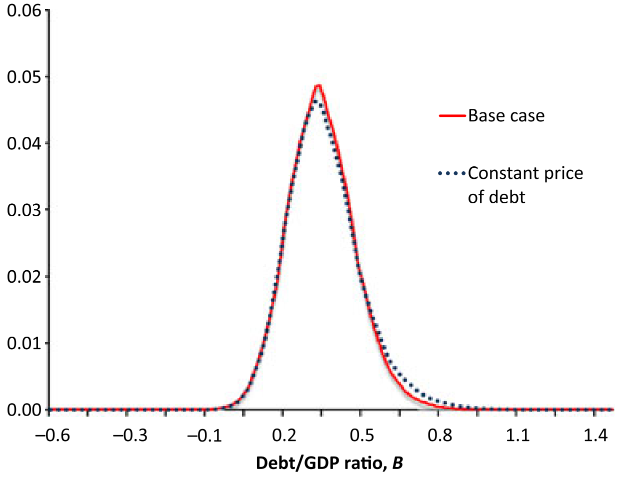

Figure 11 Ergodic distribution of debt/GDP ratio with constant debt price and in base case

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Higher GDP growth, g', lowers the second and fourth terms, where g' appears in the denominator. With more GDP, the burden of servicing the outstanding debt, measured by the second term, and the amount of inherited debt per unit of GDP, measured by the fourth term, are smaller. These effects contribute to variation in B, but, as Figure 12 shows, replacing the variable GDP growth effect by a constant 1 has almost no effect on the ergodic distribution.

Variations in the exogenous part of the primary deficit, x, are obviously an important determinant of the dispersion of the ergodic distribution and thus the volatility of the debt/GDP ratio. Table 6 shows how the exogenous component varies by the fundamental state of the economy. In normal times, the component is 4.6 per cent of GDP (recall that the actual primary deficit is usually around zero because of the offset effect captured by αB in the formula $d=x-\alpha B$). In slumps, the component is much higher, at more than 11% of GDP. Figure 13 shows the ergodic distribution of the debt/GDP ratio in a counterfactual model where the exogenous component is constant across states. The dispersion is somewhat lower than in the base case.

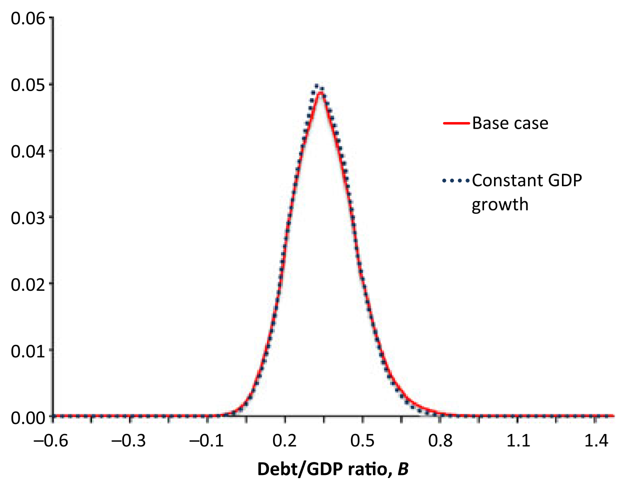

Figure 12 Ergodic distribution of debt/GDP ratio with constant GDP growth and in base case

Table 6 Exogenous component of the primary deficit, by fundamental state

State	Description	Exogenous part of primary deficit
1	Boom	0.023
2	Normality	0.046
3	Monetary stress	0.028
4	Recovery	0.061
5	Stagflation	0.032
6	Slump	0.111

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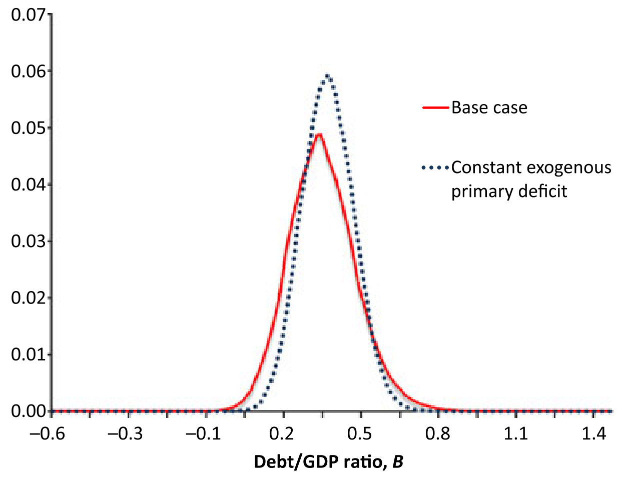

Figure 13 Ergodic distribution of debt/GDP ratio with constant exogenous part of primary deficit and in base case

Figure 12 shows the volatility of the debt/GDP ratio without any variation in the exogenous component of the primary deficit. The considerable volatility shown in the distribution marked with dots arises from the two other fundamental sources, bond prices and GDP growth volatility. The figure demonstrates an important point about volatility, namely, that it is definitely not additive – the overall amount of volatility in the debt/GDP ratio cannot be broken down into components that add up to a total amount. This point is familiar from the calculus of variances. Consider two random variables, each with a standard deviation of 1. The standard deviation of the sum is 1.41. Each variable appears to contribute 1/1.41 = 71% of the standard deviation of the sum.

7. DEPENDENCE OF THE BOND PRICE ON THE DEBT/GDP RATIO

A. Krishnamurthy & A. Vissing-Jorgensen, 2013 observe that US Treasury debt has a higher price, compared with the prices of other future cash payoffs, when the debt/GDP ratio is low. They attribute the higher valuation to a money-like convenience benefit that earns a higher return when Treasurys are scarce. They estimate that a 10% decrease in the debt/GDP ratio lowers the interest rate on Treasurys by 30 basis points (0.3 percentage points). This effect disappears if the ratio exceeds 0.55. In principle, an economy operating in this way should have higher dispersion in its debt/GDP ratio than does the base-case economy: When debt is high, the higher interest rate raises debt more, whereas when debt is low, the lower rate results in less debt accumulation. But Figure 14 shows that this effect is almost undetectable.

A second source of dependence of the distribution of the debt/GDP ratio arises from the observations that governments with shaky finances pay higher interest rates that presumably incorporate default premiums. The distribution of government interest rates across countries arranged by their debt/GDP ratios is not easy to interpret, however. The most indebted advanced economy is Japan, a country

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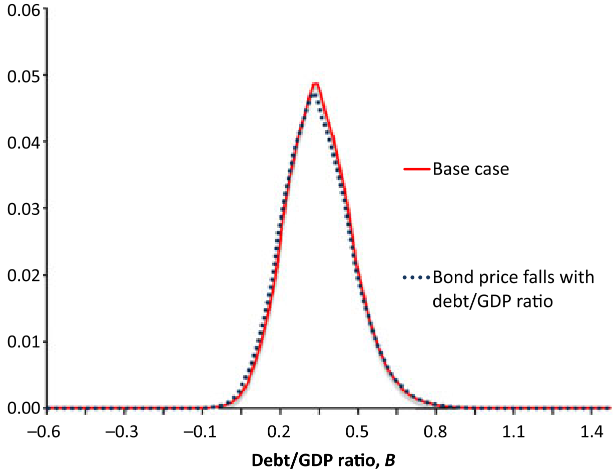

Figure 14 Ergodic distribution of debt/GDP ratio with high bond price when debt is low, reflecting convenience value, and in base case

that pays extremely low interest rates. To illustrate the effect of rising rates for heavy debts, I solved for the ergodic distribution in the case where each 10 percentage point increase in the debt/GDP, when it is above 0.4 ratio, raises the borrowing rate by 50 basis points (0.5 percentage points). Figure 15 shows that the upper tail of the distribution of the debt/GDP ratio lies considerably to the right of the base-case distribution.

8. CONCLUDING REMARKS

The base case of the model in this study describes the US economy over the period starting in 1954. The model embodies a strong tendency to return to normal

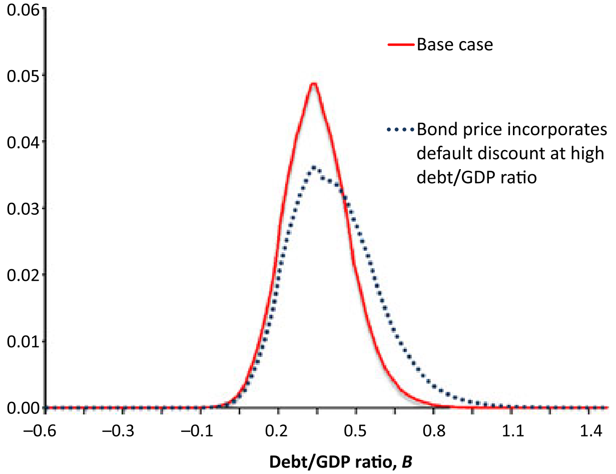

Figure 15 Ergodic distribution of debt/GDP ratio with low bond price when debt is high, reflecting default discount, and in base case

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in all dimensions, including its debt/GDP ratio. The ergodic distribution of the ratio clusters fairly tightly around a ratio of 0.35. Most alterations of the model leave the distribution more or less unchanged.

One important exception is that, in the base case, the model infers that US fiscal policy has quite a strong tendency to lower the primary deficit when government debt is high relative to GDP. This tendency underlies all the conclusions I have reached about the base case. An alternative model with no tendency to adjust fiscal policy to keep debt on target has dangerously high volatility and a substantial likelihood of entering a zone of potential default. The single most worrisome finding of this study is that the CBO – a non-partisan agency with a reputation for professional honesty – projects a path for the debt/GDP ratio that is completely inconsistent with earlier US fiscal policy, in that the debt/GDP ratio continues to rise when economic conditions return to normal.

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