Carnegie-Rochester Conference Series on Public Policy 52 (2000) 1-22
North-Holland www.elsevier.nl/locate/econbase

Reorganization*

Robert E. Hall
Hoover Institution and Department of Economics
Stanford University, Stanford, CA 94305
National Bureau of Economic Research

Abstract

One of the productive activities engaging the work force is reorganizing. When factors of production are better matched, productivity is higher. The probabilistic matching model of Diamond, Mortensen, and others provides a way to make the idea of reorganization precise. Because the flow of organizational effort generates benefits lasting well into the future, it is appropriate to think of organizational capital. Unemployment—job seeking—is one of the inputs to organization. The flow of organizational effort represented by unemployment is analogous to the flow of physical investment. When an adverse technology shock causes job destruction, the economy substitutes the formation of new organizational capital for the flow of output. An increase in the interest rate can cause intertemporal substitution toward lower job destruction and less reorganization, but this effect may not come into play for a brief unexpected increase, and may be overwhelmed by intertemporal substitution in physical capital.

1 Introduction

When an economy is well-organized, it is more productive. Organization results from the matching of factors of production. But the formation of matches and the replacement of degraded matches require resources. The most visible use of resources for reorganization is unemployment. During the time that workers are trying to find positions that exploit their comparative

* *Correspondence to:* Robert E. Hall, Hoover Institution, Stanford University, Stanford, CA 94305. Hall@Hoover.Stanford.edu This research was supported by the National Science Foundation and is part of the research program on Economic Fluctuations and Growth of the NBER. I am grateful to Eva Nagypal for helpful comments and assistance.

PII: S0167-2231(00)00014-2

advantages, they are not producing output. Consequently, the economy faces a trade-off between production and reorganization. The trade-off is similar to the trade-off between producing consumption goods and producing investment goods.

A probabilistic matching model provides a good starting point for studying reorganization. In the model I use, matches gradually deteriorate over time. Eventually, the time comes when it is in the interest of the matched factors (here, two workers) to part company. That decision launches a period when they invest in reorganization. Each worker makes a new, high-productivity match and resumes producing output. In the steady state, these decisions occur randomly over time and the aggregate flow of reorganizational investment is constant over time.

In the comparison of steady states, there is a single decision—the cutoff age where matches are broken and reorganization begins. In an economy with a stringent cutoff that ends matches before they have deteriorated very much, the flow of reorganizational investment is high. Output available for consumption is correspondingly lower. On the other hand, the average match has a higher productivity, so output is higher on that account. If there are no externalities in matching, the decentralized economy will find the optimal balance of reorganization and production. As I will show, there is an exact analogy in steady states between a model with a flow of reorganizational investment and one with a flow of investment in physical capital.

The opportunity to substitute between reorganization and production results in interesting dynamics. First, a period when productivity is temporarily low in the production of goods causes a sharp move toward reorganization, whose value becomes relatively higher during the period. That is, a temporary decline in productivity causes a spike in unemployment. The remaining dynamics are controlled by the speed of matching the burst of unemployed workers.

Second, in a model where the interest rate is an exogenous driving force, a period of a temporarily increased interest rate can also cause a burst of reorganization. This may appear paradoxical. An increased interest rate ordinarily discourages investment. A higher interest rate should keep workers in jobs longer than normal, because the output they are currently producing has a higher value in relation to the improved productivity they would enjoy later if they searched for new jobs. Two realistic factors in the model offset this general tendency, however. Once is that the model takes physical capital to be complementary to labor input. Consequently, a temporarily high interest rate induces disinvestment in that capital, and, at the same time, the release of the corresponding workers to find new jobs. This mechanism is described in detail in Hall [1999a] with accompanying empirical evidence. The other factor is to make the period of a high interest rate occur so quickly that there is no change in the relative price of current and future output. This assumption removes the intertemporal substitution effect toward lower reorganization and leaves only the substitution effect away from physical capital.

2 Matching

Matching reorganizes the economy. I will start by examining a particular matching process, rather than adopt the concept of a general matching function that has become standard since Diamond [1982] and Mortensen [1982]. The primary reason to be more specific about matching is to understand the conflicting forces of agglomeration and congestion in search. Agglomeration—first studied by Diamond—means that search is more efficient when the number of other searchers is higher. Congestion means the opposite—searchers interfere with each other's job-seeking. Common sense suggests that congestion dominates, so it is harder to find a job when unemployment is high than when it is low. Data from the U.S. economy do not support that view, however. Flows of new hires track unemployment in rough proportion, which suggests that the job-finding rate is about the same whether unemployment is high or low. Hall [1991] discusses evidence supporting the hypothesis that the matching rate is constant, but the topic is ripe for further investigation; the existing evidence is far from definitive. Bowlus [1995] presents evidence that match quality is poorer when unemployment is high. In that case, even if jobs are as easy to find in recessions as in normal times, there is a congestion effect with respect to the quality of the matches. This paper does not have a rich enough structure to consider match quality.

One reason that the observed job-finding rate may not be a reasonable guide to the matching probability in the model is that the composition of unemployment changes over the business cycle. In normal times, the unemployed are more likely to be new entrants or re-entrants rather than experienced workers. Their search efficiency is likely to be lower than for experienced workers. When a burst of job destruction raises unemployment, the composition switches toward workers with a better knowledge of the labor market. Thus, the composition effect may hide a decline in the matching probability that affects each type of worker.

Diamond's paper in 1982 studied a pure bilateral matching model, where workers search for partners. The model has no distinct concept of an employer—the firm is just the partnership of two matched workers. The huge literature associated with Mortensen and Pissarides—including Diamond's subsequent work with Blanchard (1990)—has made the employer a separate economic actor (see Mortensen and Pissarides (1999)). Employers draw employment opportunities from a distribution. If the prospective profit warrants, the employer declares a job vacancy and begins to search for a worker. The flow of matches resulting from stocks of searching workers and searching employers is described by a matching function. Generally, the function is taken to have constant returns in the two stocks. The model is closed by the equilibrium condition that the prospective profit of an incremental employer is zero.

I will return to Diamond's simpler model in order to develop ideas about reorganization. In my model, the extent of the economy's organization depends on the quality of the matching of workers in productive partnerships. The firm is a collection of workers. I view the Mortensen-Pissarides model as containing two kinds of labor: production workers and managers. An unemployed manager is called a vacancy. My model has only one kind of labor and the firm is a partnership of two similar workers. There are two essential aspects of the technology—the matching process and the evolution of the productivity of the resulting matches.

For the matching process, I consider the following setup. There are N searchers hoping to form partnerships. They are identical, so they pair off with the first people they find. The economy has S matching stations. Each period, each searcher visits a station at random. The probability that a given searcher will visit a given station is $\frac{1}{S}$. The distribution of the number of searchers at a given station is binomial $(N, \frac{1}{S})$. The probability that a given station will have an odd number of visitors is, for large N and S, $\frac{1-e^{-2N/S}}{2}$ (see the Appendix for derivation). All searchers are matched except one at each station with an odd number of searchers, so the matching function showing the fraction matched is

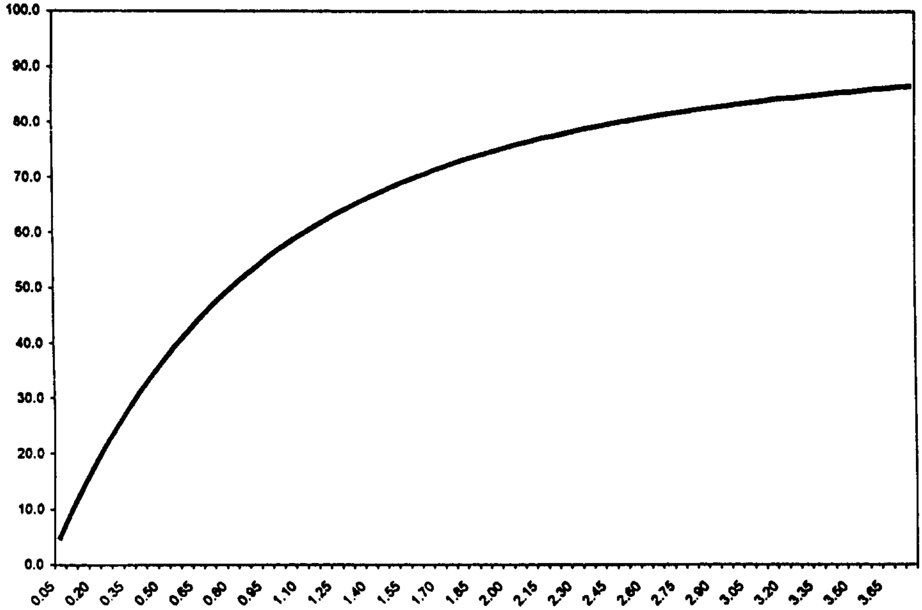
$$m_\infty(N/S) = 1 - \frac{S}{N}\frac{1-e^{-2N/S}}{2}. \tag{1}$$

Figure 1 shows the matching function. It is strictly increasing—the likelihood of one searcher finding a partner rises with the number of searchers. The matching technology has agglomeration effects but no congestion effects. Agglomeration arises in the following way: when there are few searchers, most of them visit stations by themselves, so they are odd searchers who do not form matches. The more searchers there are per station, the lower the fraction of odd outcomes and the higher the fraction matched. There is no limit to the number of matches that can occur at a given station.

A simple modification introduces congestion effects that compete with the natural agglomeration of search. Suppose that the maximum number of partnerships that can be created at a station is one. Each station generates a pair of matched workers if two or more searchers show up, and no match otherwise. The probability that no searcher will visit a given station is, for

Figure 1

Matching Function, No Congestion



The vertical axis shows the percentage of searchers matched in one period. The horizontal axis is the ratio of the number of searchers to the number of matching stations.

large N and $S, e^{-N/S}$. The probability that a single searcher will visit is $\frac{N}{S}e^{-N/S}$. In all other cases, two workers will be matched. The resulting matching function is

$$m_1(N/S) = 2\frac{S}{N}[1 - (1 + \frac{N}{S})e^{-N/S}]. \quad (2)$$

Figure 2 shows the matching function with congestion. When the number of searchers is less than about 1.8 times the number of matching stations, agglomeration effects dominate.[1] Increasing the number of searchers raises the matching rate. As before, agglomeration results from a decline in the fraction of stations visited by a single searcher. Above the critical ratio, congestion dominates. With a higher number of searchers, more of them visit stations that already have two searchers and therefore cannot make another match. Other matching functions, $m_2(N/S), m_3(N/S)$, and so on, could be defined by placing limits of 2, 3, or more on the number of pairs that could be created at each station. For these matching functions, agglomeration effects would continue at higher values of X/N.

My discussion presumes that the number of stations is held constant. If, on the other hand, the number of stations is adjusted to remain in a prescribed proportion, say r, to the number of searchers, then the matching rate would always be the constant, $m_i(r)$.

If the number of matching stations is chosen optimally given the number of searchers, then the ratio N/S will exceed the critical level of 1.8. Otherwise, a reduction in the number of stations would both avoid the costs of those stations and raise the matching rate. The higher the cost of a station, the more the optimal ratio exceeds 1.8. Similarly, if the number of matching stations is chosen before the number of searchers is known, then, roughly speaking, the optimal choice will aim to place the typical value of the ratio N/S somewhat above 1.8.

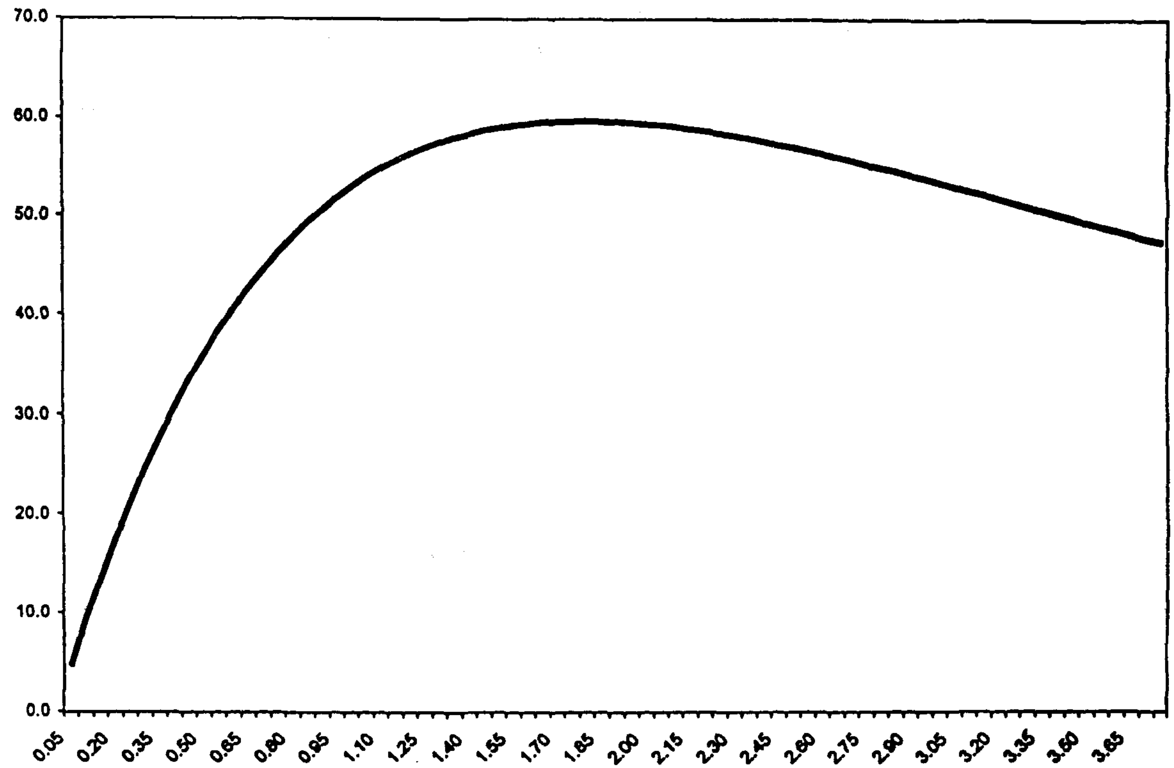
Thus, there are two reasons to expect that the matching rate should be approximately constant: first, the number of matching stations may adjust in proportion to the number of searchers. Second, the number of matching stations may be chosen to be a constant so that the ratio of searchers to stations is close to the maximum of the matching function. Variations in the ratio will take place across the flat part of the matching function near its maximum. In addition, as I mentioned earlier, the data suggest that the job-finding rate is roughly constant across wide variations in the unemployment rate.

The rest of my discussion will assume that there is a fixed matching rate. As I noted earlier, there is evidence supporting this view, but it is far from conclusive.

[1]The ratio of searchers is stations that maximizes the matching rate is the positive root of $e^r = 1 + r + r^2$, which is 1.793.

Figure 2

Matching Function with Congestion



3 Reorganization in the steady state

In the model developed here, the productivity of a match declines with its age. The economy needs to reorganize itself continually, else aggregate productivity would drift downward as all matches degrade. At a certain point in the life of each match, it becomes desirable for the partnership to dissolve and for the former partners to seek new, more productive matches. Absent such deterioration of match quality, the economy would organize itself into permanent partnerships once and for all, and thereafter would never need to reorganize.

In order to focus on the issue of reorganization, I exclude other determinants of the evolution of the joint value of a match over time. In particular, I do not consider the accumulation of match-specific capital, nor do I consider that the partners in a match learn about the value of the match (see Pries [1998]). These factors help to explain a key feature of the U.S. labor market: a pronounced decline in the hazard of separation with job duration (see Hall [1982]). In the simple model of this section, all matches end at the same age, quite contrary to fact. When I take up the dynamics of the model, I alter this assumption, but I do not attempt to be realistic with respect to the separation hazard. What matters for the model is that there are always some matches whose joint value is nearing zero, so that it is likely that separation will occur soon. In my model, this occurs simply as a result of the age of the match. In a more realistic model, it is the result of various random processes—learning about match quality, increases in the values of opportunities outside the match, and idiosyncratic factors in match quality.

Let D be the duration of a match. Each period, a fraction $\frac{1}{D}$ of employed (matched) workers become unemployed. It will be convenient at this point to switch to continuous time. Let u be the fraction of workers unemployed. Then the flow of workers into unemployment is $\frac{1-u}{D}$ and the flow out is mu, where m is the matching or job-finding rate derived in the previous section. In the steady state, the two flows are equal, so $u = \frac{1}{1+mD}$.

The unemployment rate u is the flow of investment in reorganization. A higher investment rate corresponds to a lower job duration:

$$D(u) = \frac{1-u}{mu}. \tag{3}$$

In the steady state, matches will be distributed uniformly in the interval $[0, D]$ in age. I assume that the productivity of a match declines from its initial value of 1 unit of output per worker to $e^{-\omega\tau}$ at age τ. The average output of employed workers is $\frac{1-e^{-\omega D}}{\omega D}$. Total output is

$$(1-u)\frac{1-e^{-\omega D(u)}}{\omega D(u)}. \tag{4}$$

This formula embodies the fundamental trade-off between employment and reorganization. A higher flow of reorganization—a higher value of u—lowers output by reducing employment, but it raises output by increasing the productivity of those who are employed.

3.1 *Comparison of physical and organizational investment*

There is an exact analogy in steady states between the reorganization model and a two-sector model with physical capital. In the capital-goods sector of the analog economy, one unit of labor produces one unit of capital. In the consumption-goods sector, labor and capital combine to produce output according to the constant-returns production function, $Lf(K/L)$. Here L is employment in the consumption sector and K is the capital stock. Let u be the fraction of the labor force in the capital sector and let δ be the rate of deterioration of capital. Normalize the labor force at one. Note that $L = 1 - u$. In the steady state, $K = \frac{u}{\delta}$. Output of consumption goods is $(1-u)f(\frac{u}{(1-u)\delta})$. Let $k = \frac{u}{(1-u)\delta}$ be the capital/labor ratio in the consumption goods sector. Then the output of consumption goods is $(1-u)f(k)$.

In the reorganization model, where u is the flow of labor services into reorganization, output is

$$(1-u)\frac{mu}{1-u}\frac{1-e^{-\omega\frac{1-u}{mu}}}{\omega}. \tag{5}$$

Let $k = \frac{mu}{1-u}$, the steady-state stock of organizational capital. Then the flow of output is $(1-u)f(k)$, where

$$f(k) = m\delta k\frac{1-e^{-\omega/k}}{\omega}. \tag{6}$$

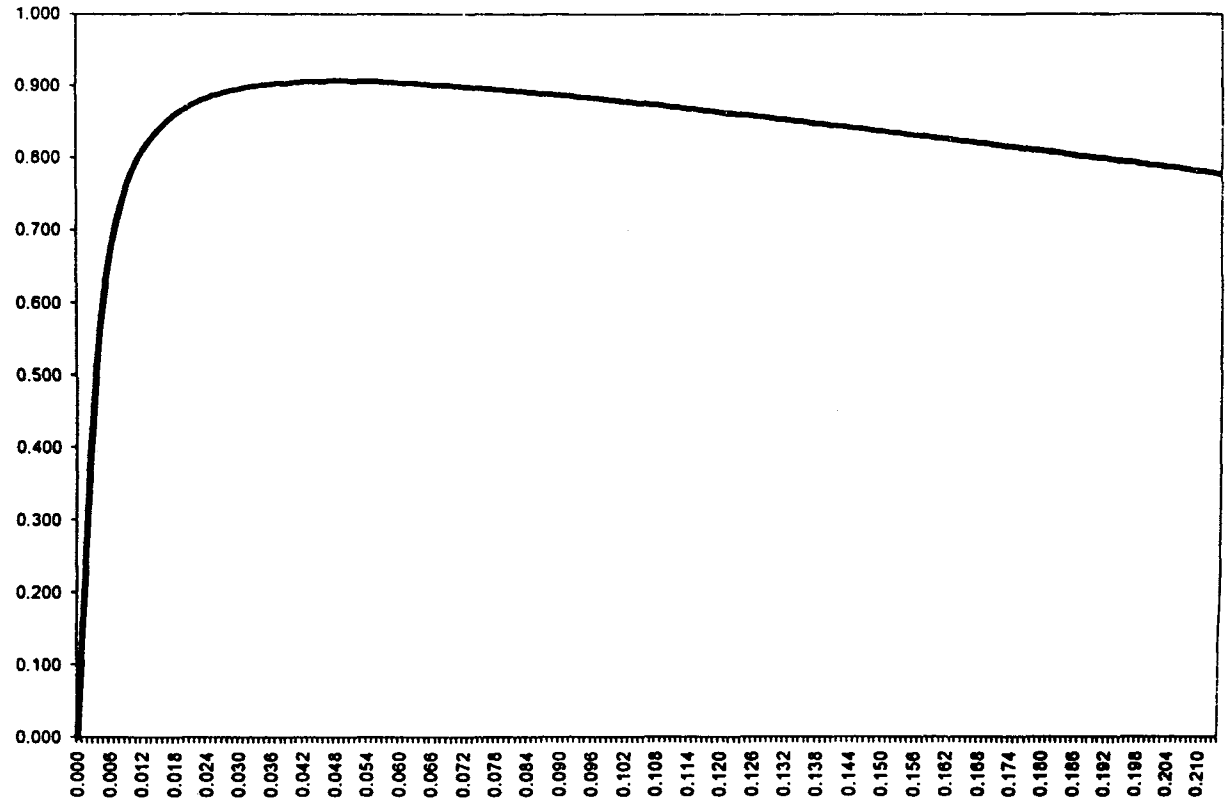
This is a well-behaved neoclassical technology, with $f'(k) \geq 0$ and $f''(k) \leq 0$. Thus the reorganization model is exactly analogous in the steady state to a member of the class of two-sector models.

Figure 3 shows steady-state output as a function of the flow of organizational investment, u. At zero investment, output is zero. Small increments to investment result in sharp increases in output as they dramatically improve the organization of the economy by eliminating matches whose productivities have fallen to low levels thanks to advancing age. Output reaches its maximum at about 5 percent unemployment. Above this level, improvements in productivity from improved organization are more than offset by the decline in employment.

The maximum shown in Figure 3 has the same interpretation as the Golden Rule of Saving. No rational economy would ever invest to a point to the right of the maximum. With a positive interest rate, the economy will

Figure 3

Steady-state Output as a Function of Unemployment



choose a point to the left of the maximum, because the opportunity cost of the organizational capital needs to be considered in choosing the optimum.

3.2 *Reorganization and the vintage capital model*

Organizational investment does not satisfy the conditions for capital aggregation, so there is no strict concept of an organizational capital stock outside the steady state. Rather, the model developed here is analogous to a two-sector vintage capital model, where the surviving stock of each previous period's investment has a distinct role in production. In the analogous vintage capital model, it takes $\frac{1}{m}$ units of labor to produce one unit of capital. Production of final goods requires one unit of labor and one unit of capital. Capital of age τ produces $e^{-\omega\tau}$ units of output. In the steady state, capital remains in use until it reaches a cutoff age D.

3.3 *Optimal investment in reorganization*

By optimality, I mean the maximization of the present value of output, given an exogenous interest rate, taken for now as a constant. Consider for the moment the case of a finite time horizon, T. Let s be the amount of time until $T, s = T - t$. Further, let $U(s)$ be the value associated with searching and let $W(s,\tau)$ be the value associated with a match formed τ years before the horizon, measured s years before the horizon ($\tau \geq s$). These values obey

$$\dot{U}(s) = mW(s,s) - (m+r)U(s). \tag{7}$$

In comparison to the immediate future, the value of searching is higher by the flow of value from the likelihood of finding a job and exiting search, $mW(s,s)$, and lower by the "interest" earned from searching, $(m+r)U(s)$.

The optimal decision between remaining in an aging match and leaving to search for a new one is governed by the comparison of the values of the two states. The candidate value of the aging match, say $\tilde{W}(s,\tau)$, obeys

$$\frac{\partial \tilde{W}(s,\tau)}{\partial s} = e^{-\omega(\tau-s)} - r\tilde{W}(s,\tau). \tag{8}$$

The option to end the match implies

$$W(s,\tau) = max[\tilde{W}(s,\tau), U(s)]. \tag{9}$$

With specified terminal conditions, say $U(0) = 0$ and $W(0,\tau) = 0, \forall\tau$, finding the optimal path is a matter of integrating these equations. Because the system has all negative roots, it has the turnpike property that the terminal conditions become unimportant as the horizon becomes distant. It is straightforward to find the unique match duration D that satisfies the steady-state condition for the unemployment value,

$$0 = mW_0 - (m + r)U \tag{10}$$

where $W_0 = W(s, s)$ is the stationary value of new matches and U is the stationary value of search. In addition, the integral of equation 3.6 is

$$W_0 = \frac{1 - e^{-(r+\omega)D}}{r + \omega} + e^{-rD}U. \tag{11}$$

Finally, the optimality condition for match duration requires

$$e^{-\omega D} = rU. \tag{12}$$

A match should be abandoned when its flow of output is the same as the opportunity cost of the time, as determined by the value of searching.

These equations do not have a closed-form solution, but it is straightforward to solve them numerically by searching over D. As noted earlier, the optimum involves a lower level of investment in reorganization (a lower unemployment rate) than the Golden Rule. In the example developed earlier, where the Golden Rule unemployment rate was 4.8 percent, the optimal unemployment rate at 6 percent annual interest is 4.3 percent.

4 Reorganization and search capital

Whenever a relationship requires search, the successfully matched parties have search capital. Unless something changes in the environment or in the benefits of the match, they will remain matched in order to preserve the capital value of the match. What is the relation between reorganization and search capital?

First, consider an economy that never needs to reorganize. For example, set $\omega = 0$ in the model just developed. In the steady state, the economy will not devote any resources to reorganization. Nonetheless, it will have search capital. Each permanently matched partnership will have a positive joint match value associated with its original search effort. The total amount of search capital will be the resources that would be necessary to place all workers back into partnerships if all of the existing partnerships were destroyed.

In an economy with depreciable search capital, a flow of reorganization effort will be needed to maintain the optimal allocation. What is fundamental is the flow of reorganization, not the idea of a capital stock. The stock could just as well be labeled the stock of search capital. As the previous section noted, there is no useful concept of organizational capital except in comparing steady states.

5 Dynamics

It is more instructive to consider the dynamics in a discrete-time version of the model. I will also introduce two additional features of the model at this stage. First, I will make the process of degradation be random rather than deterministic—this eliminates what would otherwise be extreme and unrealistic echo effects from bursts of job destruction. The modification does not result in a realistic separation hazard over job duration, however. With probability π each period, the partnership drops from producing $(\frac{1}{1+\omega})^{\tau}$ units of output to $(\frac{1}{1+\omega})^{\tau+1}$; otherwise, it continues to produce the same amount. Second, I will bring physical capital into the model by assuming that the partnership is required to hold γ units of output as capital during the period in order to produce. The model is the same as in Hall [1999a] except that I do not consider uncertainty.

The value transition equations for the model are, in notation analogous to that in the previous section,

$$U_t = \frac{1}{1+r_t}[(1-m)U_{t+1} + m(W_{0,t+1} - \gamma)] \tag{13}$$

$$W_{\tau,t} = max\{\frac{1}{1+r_t}[zt(\frac{1}{1+\omega})^{\tau} + (1-\pi)W_{\tau,t+1} + \pi W_{\tau+1,t+1}], \gamma + U_t\}. \tag{14}$$

In equation 13, the new partnership has to invest in γ units of capital in order to set up shop. In equation 14, the productivity of all partnerships is perturbed by a time effect, a_t. The partnership has the option of dissolution, in which case it recovers its capital, γ. The fact that dissolution (job destruction) provides immediate output is important in the dynamic response to increases in interest rates, as discussed more extensively in Hall [1999a].

The solution for the dynamic path of employment and unemployment involves two steps. First, starting from appropriate terminal values (such as the steady state), iterate equations 13 and 14 backwards in time to find the values associated with the productivity states and unemployment. In particular, in each period, note which states will have positive employment because the left side of the max in equation 14 holds and which states will have zero employment because the right side holds. Then iterate the allocation transition equations forward in time. They are:

$$n_{0,t} = mu_{t-1} + (1-\pi)n_{0,t-1} \tag{15}$$

$$\begin{aligned} n_{\tau,t} &= (1-\pi)n_{\tau,t-1} + \pi n_{\tau-1,t-1}, \text{if state } \tau \text{ isviable} \\ &= 0, \text{otherwise} \end{aligned} \tag{16}$$

$$u_t = (1-m)u_{t-1} + d_t \tag{17}$$

Here d_t is job destruction, the sum over all nonviable productivity states of the right-hand side of equation 16.

In the steady state, jobs last until they transit to productivity state D, at which time they are destroyed and the workers enter the reorganization process by becoming unemployed. As discussed in Section 2, the value of D is chosen to maximize the present value of output. The expected duration of a job is $\frac{D}{\pi}$ periods. The steady-state unemployment rate is $u = \frac{1}{1+m\frac{D}{\pi}}$. In the steady state, employed workers are distributed equally among the D productivity states.

I use the steady state as the initial condition at $t = 0$ for the dynamic system 15 through 17. Then I use the value transition equations 13 and 14 with a perturbed value of either the productivity time effect or the interest rate to introduce a shock at $t = 1$. The result is a departure from normal job destruction in period 1. Then I use equations 15 through 17 to calculate the dynamic response to the temporary change in job destruction.

The parameters and steady-state values of the driving forces in my calculations are:

Parameter or variable	*Interpretation*	*Value*
m	Matching rate	0.3 per quarter
ω	Productivity degradation step	1.3 percent
γ	Capital/output ratio	6
π	Probability of productivity degradation	13 percent per quarter
r	Stationary interest rate	2 percent per quarter
D	Threshold productivity step	9
u	Unemployment rate	4.6 percent

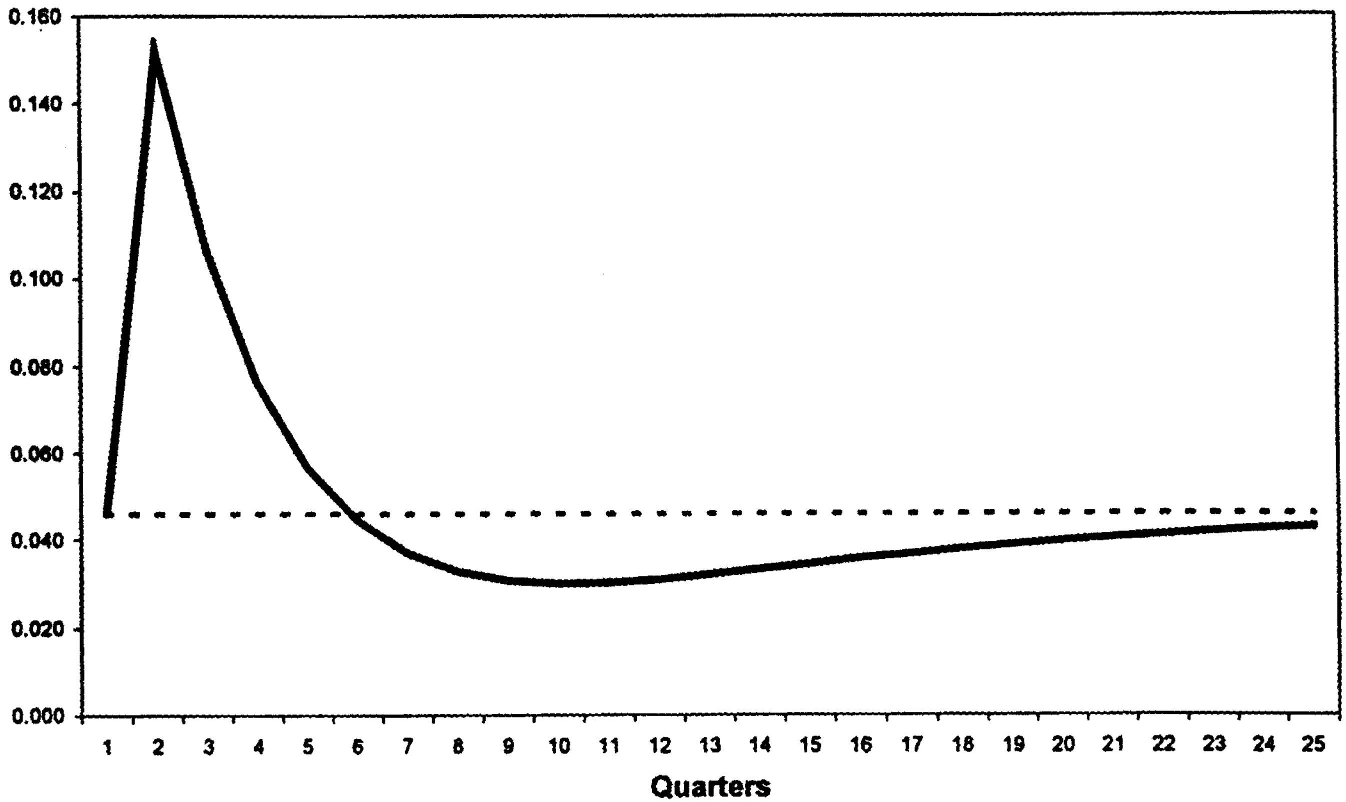
Notice that the allocation transition equations 15 through 17 are a first-order Markoff process during any period when the value transition equations 13 and 14 are at their stationary point. Thus, for any impulse that affects the values only in the first period, the impulse response functions will have the same shape and will be governed by the Markoff process. The key assumption underlying this property is the constancy of the matching rate. Decisions about ending partnerships are not influenced by the number of people currently searching for partners.

Figure 4 shows the impulse response function for unemployment for any impulse that triggers job destruction in one extra duration category in period 1 but returns to normal job duration thereafter.

In the steady state, partners remained paired through the eighth productivity degradation. The impulse underlying Figure 4 causes the cutoff to switch to the seventh degradation just in period one. Unemployment jumps upward by the 11 percent of the labor force who would have been at the eighth step. As the bulge of unemployed workers make new matches,

Figure 4

Impulse Response Function for Unemployment



unemployment declines. After 6 quarters, unemployment drops below its steady-state value. The impulse response function has the property of *concentration*, described in Hall [1999b]. A burst of job destruction is followed by a period of lower than normal likelihood of further job destruction—the impulse response function drops below normal for a period before leveling out to its steady-state average.

5.1 *Technology shocks*

Consider a shock that lowers the time effect in productivity, z_t, only in period 1: $z_t < 1$ and $z_t = 1, t > 1$. The interest rate is at the same level, r, in all periods. The value transitions linking period 1 to the future are:

$$U_1 = \frac{1}{1+r}[(1-m)U_2 + m(W_{0,2} - \gamma)] \tag{18}$$

$$W_{\tau,1} = max\{\frac{1}{1+r}[z_1(\frac{1}{1+\omega})^{\tau} + (1-\pi)W_{\tau,2} + \pi W_{\tau+1,2}], \gamma + U_1\}. \tag{19}$$

Equation 13 shows that the value of being unemployed in period 1 is at its steady-state value in period 1—a decline in the value of working in period 1 does not affect the value of search, because the earliest a new match could be formed would be in period 2, when productivity is back to normal. Two features of the model result in this key simplification: the constant match rate and the exogenous interest rate. Equation 14 shows that the lower value of z_1 implies that fewer productivity states will be viable in period 1. In addition to normal job destruction resulting from the random arrival of matches at the steady-state cutoff productivity state, there is extra job destruction from moving the cutoff to a lower-numbered productivity state. The actual level of cutoff productivity does not change, but the lower value of z_1 means that the cutoff is achieved at a lower-numbered state.

The effect of a temporary decrease in the productivity of goods production is to shift resources into reorganization. If there is a period when it is not as desirable as usual to use labor to produce goods, that becomes a good time to schedule reorganization. This is the idea in Hall [1991] made formal in this model. My earlier paper stressed that there is a perfectly elastic supply of labor for the production of goods, because of the alternative use of labor in reorganizing the economy. Equation 14 embodies that horizontal labor-supply schedule. The opportunity cost of labor used in production, indexed by $U_1+\gamma$, is not affected by the temporary shift in productivity. Any theory of employment volatility has, at its heart, an explanation of highly elastic labor supply. I find this explanation more plausible than, for example, theories where the alternative activities are leisure or time spent working at home (see Hall [1998]).

Because the response to the one-time technology shock causes job destruction only in period 1, Figure 4 applies. The figure shows the response of the unemployment rate to a reduction in aggregate technology of 2 percent. This reduction is sufficient to trigger the destruction of jobs after the seventh step of degradation as well as the usual destruction after the eighth step.

5.2 *Interest-rate shock*

The analysis of a one-shot increase in the interest rate is a bit more complicated, though the story comes out the same. With the technology index held at its steady-state value of 1, the relevant versions of the value transition equations are:

$$U_1 = \frac{1}{1+r_1}[(1-m)U_2 + m(W_{0,2} - \gamma)] \tag{20}$$

$$W_{\tau,1} = max\{\frac{1}{1+r_1}[(\frac{1}{1+\omega}^{\tau} + (1-\pi)W_{\tau,2} + piW_{\tau+1,2}], \gamma + U_1\}. \tag{21}$$

A level of r_1 above its normal level depresses both $W_{\tau,1}$ and U_1 below their normal levels. I will show that the reduction in $W_{\tau,1}$ exceeds the reduction in U_1. As a result, a sufficiently large increase in the interest rate will trigger job destruction beyond what occurs in the steady state.

Consider any productivity state τ that is viable in the steady state. From equations 18 and 21,

$$\frac{d}{dr_1}(W_{\tau,1} - U_t) = -\frac{1}{1+r_1}(W_{\tau,1} - U_1). \tag{22}$$

The hypothesis of steady-state viability implies that

$$W_{\tau,1} \geq \gamma + U_1. \tag{23}$$

Thus the derivative of the viability criterion with respect to the interest rate is strictly negative as long as the capital/labor ratio γ is strictly positive. An increase in the interest rate is likely to result in the destruction of jobs that are near the margin of viability in the steady state.

In general, an increase in the interest rate will discourage all forms of investment, both in physical capital and in reorganization. The experiment considered here, however, relates only to the intertemporal effect on physical capital, which is transmitted to employment because of the complementarity of capital and labor in production. The technology permits the release of output at the beginning of period 1 by reducing the level of capital carried through the period. A high value of the interest rate r_1 signifies a high price of output delivered at the beginning of period 1 in relation to output delivered at the beginning of period 2. This intertemporal substitution mechanism is the subject of Hall [1999a].

On the other hand, the reorganization channel does not allow any alteration in the amount of output delivered at the beginning of period 1. Keeping workers on the job rather than destroying their jobs could increase output in period 2 at the expense of reorganization investment, but would not change the amount of output available at the beginning of period 1 (if γ were zero) and would actually reduce the output available then if γ were positive. A higher interest rate in period 2 would induce job-preservation in period 1 with a sufficiently low value of γ.

If r_1 is sufficiently above its steady-state level to trigger the destruction of jobs in one additional productivity state, then the impulse response function will be the one shown in Figure 4.

6 Concluding remarks

The economy faces an interesting trade-off across steady states and over time between producing goods and reorganizing. A higher permanent flow of reorganization—modeled here as more search effort and higher unemployment—results in higher productivity. But it reduces the fraction of the labor force employed making goods. There is an optimal steady state that balances these factors. The considerations governing the optimum are exactly the same as for the trade-off in the steady state between producing consumption and investment goods.

The intertemporal trade-off seems a promising way to explain the volatility of employment and unemployment. One way or another, any successful theory of volatility has to invoke high elasticity of labor supply. The theory of wage rigidity is one approach, though its foundations in contract theory are yet to be poured. The idea that workers substitute freely between work and leisure or other activities at home—the basis of employment volatility theories in the real business-cycle tradition—receives only modest support from the evidence on individual behavior, despite heroic recent attempts. The idea promoted in this paper takes unemployment seriously and not just as the residual from employment. Unemployment is a productive economic activity. Although unemployment increases arise from adverse developments—declines in productivity or jumps in the interest rate—they can be understood in the framework of value-maximizing behavior. Workers turn to reorganizing—finding better job matches—when the relative reward to work declines temporarily or when the interest rate is high.

The impulses I have considered are strictly temporary. Technology and the interest rate return to normal after a single period. Persistent movements of employment and unemployment are entirely the result of the dynamics of matching. It is well-known that simple matching is completely inadequate to explain the observed persistence of unemployment (Cole and Rogerson

[1996]). But it appears that a more sophisticated view of matching dynamics, incorporating the extreme hazard of separation early in matches, can explain persistence (Hall [1995] and Pries [1998]).

The standard of modern macroeconomics is the dynamic stochastic general equilibrium model. I have not made much progress developing the ideas in this and related papers in general equilibrium. Of course, the exogenous real interest rate could be rationalized in general equilibrium with linear intertemporal preferences, but that is not a plausible specification. In an otherwise standard DSGE model, I believe that it would be fairly easy to replicate my findings in this paper for technology shocks, if the model contains a job destruction mechanism along the lines of the one developed here. The reason is that almost nothing happens to the interest rate in a standard DSGE model in response to technology shocks. The interest rate is locked to the marginal product of physical capital. But it is correspondingly impossible to generate enough movement in the interest rate to trigger job destruction from that source.

Appendix: Derivations of Matching Functions

I am grateful to Elizabeth O'Neil and Patrick O'Neil for the first of these derivations.

A. Unlimited matching at each station

$$\begin{aligned} \text{Let } p &= \frac{1}{S} \text{ and} \\ \pi &= \sum_{i \text{ odd}} \binom{N}{i} p^i (1-p)^{N-i} = (1-p)^N \sum_{i \text{ odd}} \binom{N}{i} \left(\frac{p}{1-p}\right)^i, \end{aligned}$$

the probability of an odd number of searchers visiting a station. To evaluate the summation over odd terms, note that

$$(1+x)^N = \sum_{i=1}^{\infty} \binom{N}{i} x^i,$$

so

$$\sum_{i\text{odd}} \binom{N}{i} x^i = \frac{1}{2}[(1+x)^N - (1-x)^N].$$

Thus

$$\pi = \frac{(1-p)^N}{2}[(1+\frac{p}{1-p})^N - (1-\frac{p}{1-p})^N].$$

Now let $r = N/S$ and consider the limit as N become large and r is held constant. Note that $\lim_{x\to 0}(1-x)^{\frac{1}{x}} = e$. Thus

$$\lim_{N\to\infty} \pi = \frac{1-e^{-r}}{2}.$$

At the S stations, $S\pi$ searchers fail to be matched. This is a fraction $\frac{S\pi}{N}$ of the searchers. The fraction matched is

$$m_\infty(r) = 1 - \frac{S\pi}{N} = 1 - \frac{1-e^{-2r}}{2r}$$

B. Limit of one match per station

The (binomial) probability that no searcher will visit a given station is $(1-\frac{1}{S})^N$, which is $e^{-N/S}$ for large N and S. The probability that one searcher will visit is $N\frac{1}{S}(1-\frac{1}{S})^{N-1}$, which is $\frac{N}{S}e^{-N/S}$ for large N and S. The rest of the derivation appears in the body of the paper.

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