Search-and-Matching Analysis of High Unemployment Caused by the Zero Lower Bound

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A landing on the non-Walrasian continent has been made. Whatever further exploration may reveal, it has been a mind-expanding trip: We need never go back to

\[ \dot{p} = \alpha(D - S) \]

and

\[ q = \min(D, S) \]
Four kinds of agents

(1) *Endowed households* of measure one, with utility $\sum_t c_t$

(2) Workers of measure $\lambda \geq 1$, with the capacity in each period to turn one unit of the primary input into consumption, for which they receive a wage of $w$ units of consumption goods. Their reservation wage is $z$.

(3) Firms, intermediaries who receive the input from endowed households, hire workers at the wage $w$, and return $1 - p$ units of consumption to endowed households for each unit of the input.

(4) A central bank that accepts deposits (reserves) from endowed households that pay interest, in the form of the primary input, at a per-period rate of $r$, the reserve rate.
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Frictionless equilibrium

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The wage is \( w_t = p_t \) and the supply of consumption by firms, integrated with the market for workers, is

\[
\begin{align*}
    c_t &= 0 \text{ if } p_t < z \\
    &\in [0, (1 - z)\lambda] \text{ if } p_t = z \\
    &= (1 - p_t)\lambda \text{ if } p_t > z
\end{align*}
\]

(1)
Equilibrium without frictions

\[ c = 1 - p \]

\[ c = (1 - p)\lambda \]

Classical unemployment

Equilibrium
The central bank

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The period-1 Arrow-Debreu price for period-\( t \) consumption is

\[ a_t = \frac{p_t}{(1 + r)^{t-1}} \]
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$$a_t = \frac{p_t}{(1 + r)^{t-1}}$$

Let $a = \min_t a_t$. The household will choose $c_t = 0$ for all $t$ with $a_t > a$. 
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The quick and dirty explanation is that adding a central bank that sets an interest rate different from the equilibrium rate of a model, without removing an equation, results in an over-determined system of equations that has no solution.
Demand Gap Resulting from a Price and Wage above the Equilibrium Level

\[ c = 1 - p \]

\[ c = (1 - p)\lambda \]

Demand-gap unemployment
DEMAND-GAP UNEMPLOYMENT

A feasible path of the economy exists with prices satisfying the intertemporal equality condition (the consumption Euler equation) of the endowed households and with demand-gap unemployment in every period. The price trajectory is

\[ p_t = \frac{p_T}{(1 + r)^{T-t}} \]

with \( p_T \) less than one but close enough that \( p_1 \geq z \)
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Demand-gap unemployment is

\[ u_t = \lambda - 1, \]

the excess of the labor force over maximum feasible employment.
Clashing theories of unemployment

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The demand-gap model and the DMP model clash.
Search and Matching

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The number of postings is $V = \frac{\phi(q)}{q} U$, where $U$ is the number of searchers. A reasonable specification for $\phi(q)$, based on the matching function $\alpha \sqrt{UV}$, is

$$\phi(q) = \frac{\alpha^2}{q}$$

.
Nash-bargained wage

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The surplus from a match is $p - z$; the worker receives a fraction $\beta$ of the surplus and the firm retains the rest.
Firms expand their efforts to find workers to the point of zero profit:

\[ q(1 - \beta)(p - z) = k \]
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The unemployment rate is

\[ u = 1 - \frac{\alpha^2}{q} \]

.
The wage is

\[ w = z + \beta(p - z) = \beta p + (1 - \beta)z \]
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The labor market imposes a functional relation between unemployment and the price:

\[ u(p) = 1 - \frac{(1 - \beta)\alpha^2(p - z)}{k} \]
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Product market

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A matched household and firm make a Nash bargain for the price of consumption goods, $p$
Nash Bargain in the Product Market

The firm’s outside option is to sell to another household at the prevailing price, $\bar{p}$, but the firm faces a cost $\gamma$ of breaking off bargaining with one household and starting up with another, so the outside option is worth $\bar{p} - \gamma$
Nash bargain in the product market

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In period \( T \), the household has no outside option because there are more households offering to trade their endowments for consumption goods than there are firms able to convert endowment goods to consumption goods, and no opportunity to invest the endowment at the central bank.

The surplus from the potential trade is \( 1 - (\bar{p}_T - \gamma) \).
The bargaining weight for the household is $b$
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The bargained price solves

$$1 - p_T = b[1 - (\bar{p}_T - \gamma)]$$
Nash bargain in period $T$, continued

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In the symmetric equilibrium, where $\bar{p} = p$, the price is

$$p_T = 1 - \frac{b}{1 - b\gamma}.$$
Earlier periods

The endowed household has the option to invest its endowment at the central bank at rate \( r \) for \( \tau \) periods, and pay

\[
\frac{p_{t+\tau}}{1 - u_{t+\tau}}
\]

for conversion in period \( t + \tau \). The effective price is boosted by division by \( 1 - u_{t+\tau} \) to account for the possibility that the household will not be matched to a firm.
EARLIER PERIODS, CONTINUED

The present value in period $t - 1$ of output purchased by saving in period $t - 1$ and purchasing in period $t + \tau$ is

$$X_{t,\tau} = \frac{p_{t+\tau}}{(1 + r)^\tau (1 - u(p_{t+\tau}))}.$$
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The most advantageous outside option is

$$x_t = \min_{\tau} X_{t,\tau}.$$
Earlier periods, continued

This outside option for the household in period $t$ is worth $1 - x_t$. If $x_t > 1$, it has no influence and the bargain becomes the same as in period $T$, in which case I redefine $x_t = 1$. The firm has the same option as in period $T$. The surplus is

$$S_{t-1} = 1 - (1 - x_t) - (\bar{p}_{t-1} - \gamma)$$
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The household’s payoff is

$$1 - p_{t-1} = bS + 1 - x_t$$

$$= b[1 - (1 - x_t) - (\bar{p}_{t-1} - \gamma)] + 1 - x_t. \quad (2)$$
Symmetric equilibrium

\[ p_{t-1} = x_t - \frac{b}{1 - b}\gamma, \]

provided that \( p_t \geq \lambda \) for all \( t \)
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Given \( p_T \), one can compute the equilibrium price path by backward recursion.
ILLUSTRATIVE PARAMETER VALUES

Efficiency of matching: $\alpha = 0.28$
Bargaining weight of jobseekers: $\beta = 0.5$
Bargaining weight of endowment households: $b = 0.5$
Firm’s cost of maintaining a posting of a vacancy: $k = 0.02$
Flow value of not working: $z = 0.5$
Number of years: $T = 10$
Central bank’s real interest rate: $r = 0.01$
**Properties**

Unemployment rate in all years is $u = 0.055$, a normal level for the U.S.
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Elasticity of the unemployment rate with respect to the product price is around 25, a value known to equip the model to turn small observed fluctuations in productivity into meaningful fluctuations in unemployment. The model’s reliance on Nash bargaining with equal bargaining weights—shown in Shimer (2005) to generate pathetically small fluctuations in unemployment—is offset by the model’s different specification of the matching process.
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Paths of Unemployment and Consumption Price Induced by a Central-Bank Interest Rate of 0.01