MS&E 246: Lecture 7
Stackelberg games

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Stackelberg games

In a Stackelberg game, one player (the “leader”) moves first,

and all other players (the “followers”) move after him.

Stackelberg competition

- Two firms (N = 2)
- Each firm chooses a quantity $s_n \geq 0$
- Cost of producing s_n : $c_n\ s_n$
- *Demand curve*:

  Price = $P(s_1 + s_2) = a - b\ (s_1 + s_2)$
- Payoffs:

  Profit = $\Pi_n(s_1, s_2) = P(s_1 + s_2)\ s_n - c_n\ s_n$

Stackelberg competition

In *Stackelberg competition*, firm 1 moves before firm 2.

Firm 2 observes firm 1's quantity choice s_1, then chooses s_2.

Stackelberg competition

We solve the game using *backward induction*.

Start with second stage:

Given s_1, firm 2 chooses s_2 as

$$s_2 = \arg\max_{s_2 \in S_2} \Pi_2(s_1, s_2)$$

But this is just the *best response* $R_2(s_1)$!

Best response for firm 2

Recall the best response given s_1:

$$\max_{s_2 \geq 0} [(a - bs_2 - bs_1)s_2 - c_2 s_2] \implies$$

Differentiate and solve:

$$a - c_2 - bs_1 - 2bs_2 = 0$$

So:

$$R_2(s_1) = \left[\frac{a - c_2}{2b} - \frac{s_1}{2}\right]^+$$

Firm 1's decision

Backward induction:

Maximize firm 1's decision, *accounting for firm 2's response at stage 2.*

Thus firm 1 chooses s_1 as

$s_1 = \arg\max_{s_1 \in S_1} \Pi_1(s_1, R_2(s_1))$

Firm 1's decision

Define t_n = $(a$ - $c_n)/b$.

If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = \left(a - bs_1 - b\left(\frac{t_2}{2} - \frac{s_1}{2}\right)\right) s_1 - c_1 s_1$$

If $s_1 > t_2$, then payoff to firm 1 is:

$$\Pi_1 = (a - bs_1)\, s_1 - c_1 s_1$$

Firm 1's decision

For simplicity, we assume that

$$2c_2 \leq a + c_1$$

This assumption ensures that

$$(a - bs_1)\, s_1 - c_1 s_1$$

is *strictly decreasing* for $s_1 > t_2$.

Thus firm 1's optimal s_1 must lie in $[0, t_2]$.

Firm 1's decision

If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = \left(a - bs_1 - b\left(\frac{t_2}{2} - \frac{s_1}{2}\right)\right)s_1 - c_1 s_1$$

Firm 1's decision

If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = \left(\frac{a}{2} - \frac{b}{2}s_1 + \frac{c_2}{2}\right)s_1 - c_1 s_1$$

Firm 1's decision

If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = -\frac{b}{2}s_1^2 + \left(\frac{a}{2} + \frac{c_2}{2} - c_1\right) s_1$$

Thus optimal s_1 is:

$$s_1 = \frac{a - 2c_1 + c_2}{2b}$$

Stackelberg equilibrium

So what is the Stackelberg equilibrium?

Must give complete strategies:

$s_1^* = (a - 2c_1 + c_2)/2b$

$s_2^*(s_1) = (t_2/2 - s_1/2)^+$

The *equilibrium outcome* is that
firm 1 plays s_1^*, and firm 2 plays $s_2^*(s_1^*)$.

Comparison to Cournot

Assume $c_1 = c_2 = c$.

In Cournot equilibrium:

(1) $s_1 = s_2 = t/3$.

(2) $\Pi_1 = \Pi_2 = (a - c)^2/(9b)$.

In Stackelberg equilibrium:

(1) $s_1 = t/2$, $s_2 = t/4$.

(2) $\Pi_1 = (a - c)^2/(8b)$, $\Pi_2 = (a - c)^2/(16b)$

Comparison to Cournot

So in Stackelberg competition:

-the *leader* has *higher* profits

-the *follower* has *lower* profits

This is called a *first mover advantage.*

Stackelberg competition: moral

Moral:

Additional information available can lower a player’s payoff, if it is common knowledge that the player will have the additional information.

(*Here:* firm 1 takes advantage of knowing firm 2 knows s_1.)