MS&E 246: Lecture 8
Dynamic games of complete and imperfect information

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Outline

- Imperfect information
- Information sets
- Perfect recall
- Moves by nature
- Strategies
- Subgames and subgame perfection

Imperfect information

Informally:

In *perfect information* games, the history is common knowledge.

In *imperfect information* games, players move without necessarily knowing the past.

Game trees

We adhere to the same model of a *game tree* as in Lecture 6.

1) Each non-leaf node v is identified with a unique player $I(v)$.
2) All edges out from a node v correspond to *actions* available to $I(v)$.
3) All leaves are labeled with the *payoffs* for all players.

Information sets

We represent imperfect information by *combining* nodes into *information sets.*

An information set h is:

-a subset of nodes of the game tree

-all identified with the same player $I(h)$

-with the same actions available to $I(h)$ at each node in h

Information sets

Idea:

When player $I(h)$ is in information set h, *she cannot distinguish between the nodes of* h.

Information sets

Let H denote all information sets.

The union of all sets $h \in H$ gives all nodes in the tree.

(We use $h(v)$ to denote the information set corresponding to a node v.)

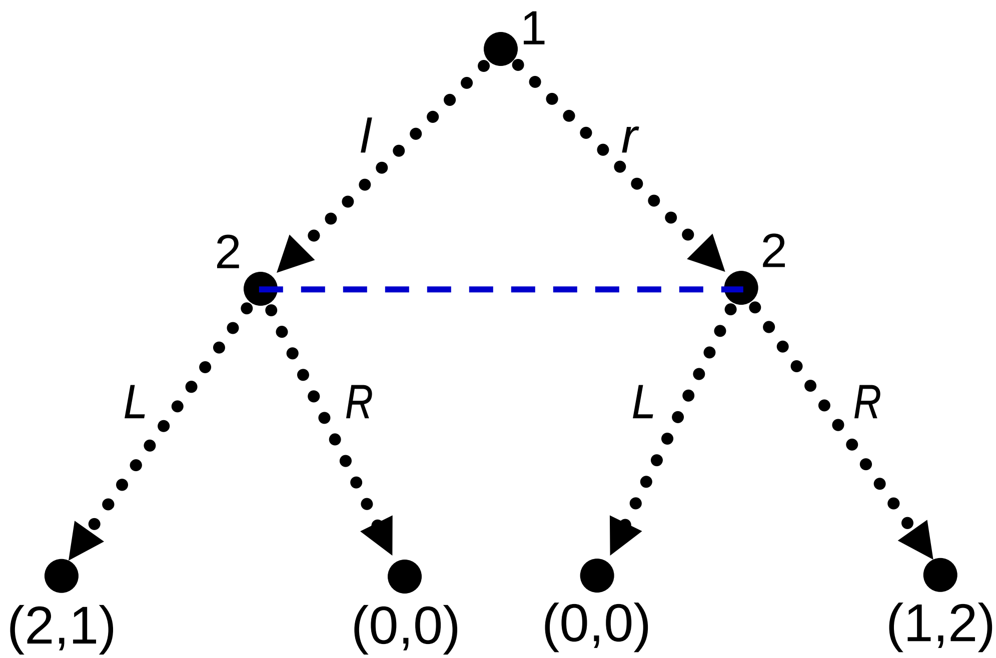
Perfect information

Formal definition of perfect information:

All information sets are singletons.

Example

Coordination game without observation:



- - - -: *Player 2 cannot distinguish between these nodes*

Perfect recall

We restrict attention to games of *perfect recall*:

These are games where the information sets ensure a player never forgets what she once knew, or what she played.

[Formally:

If v, v' are in the same information set h, neither is a predecessor of the other in the game tree.

Also, if v', v'' are in the same information set, and v is a predecessor of v', then there must exist a node $w \in h(v)$ that is a predecessor of v'', such that the action taken on the path from v to v' is the same as the action taken on the path from w to v''.]

Moves by nature

We allow one additional possibility:

At some nodes, *Nature* moves.

At such a node v, the edges are labeled with *probabilities* of being selected.

Any such node v models an *exogenous* event.

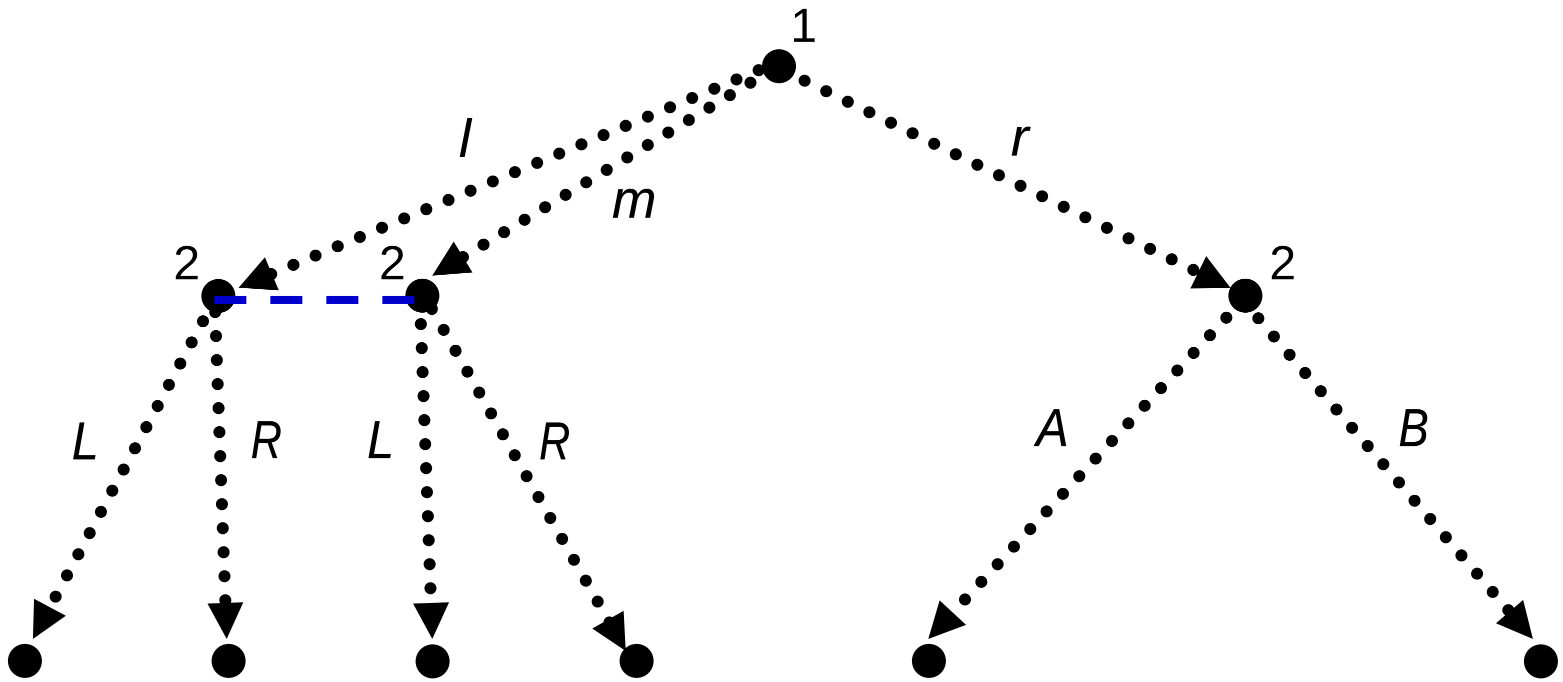
[*Note:* Players use *expected payoffs.*]

Strategies

The *strategy* of a player is a function from *information sets* of that player to an *action* in each information set.

(In perfect information games, strategies are mappings from nodes to actions.)

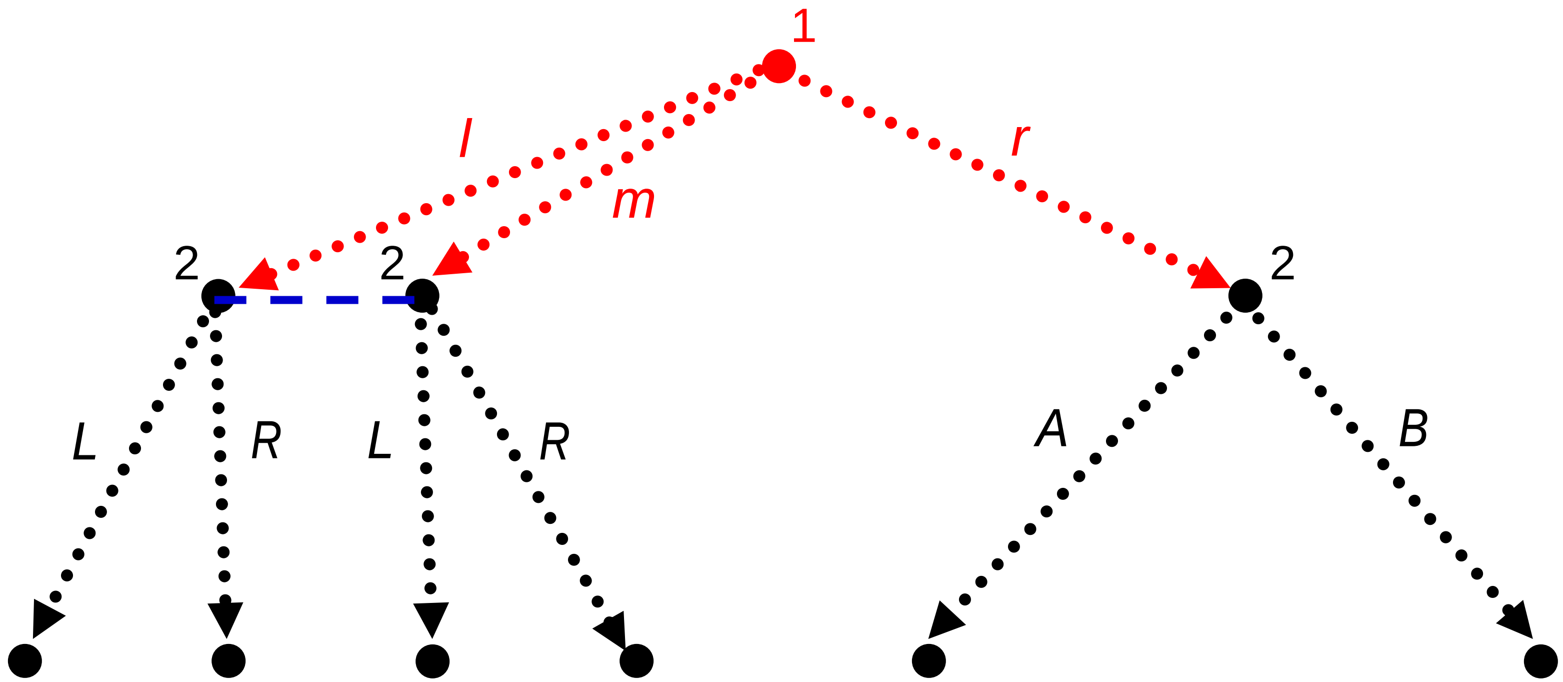
Example



Example

Player 1: 1 information set

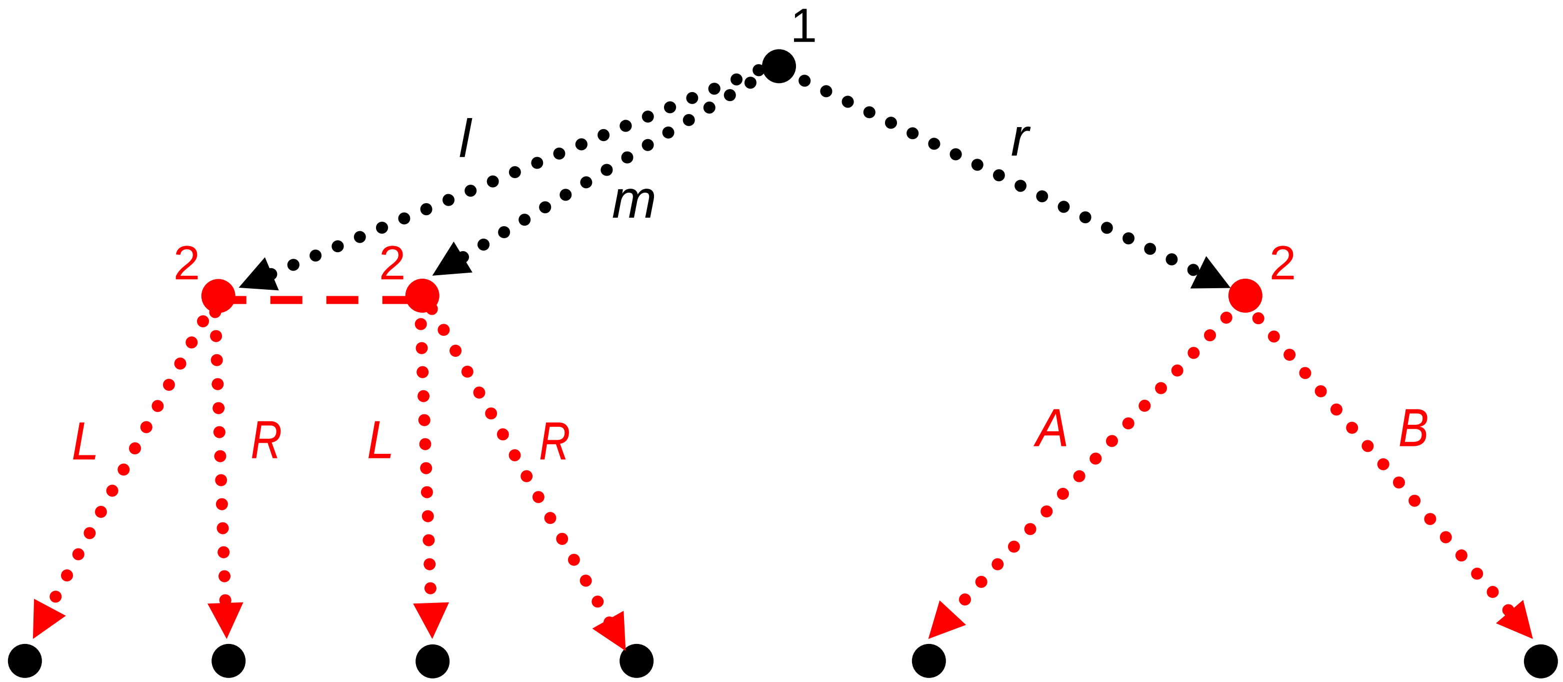
3 strategies – *l*, *m*, *r*



Example

Player 2: 2 information sets

4 strategies – *LA*, *LB*, *RA*, *RB*



Subgames

A *(proper) subgame* is a subtree that:

-begins at a singleton information set;

-includes all subsequent nodes;

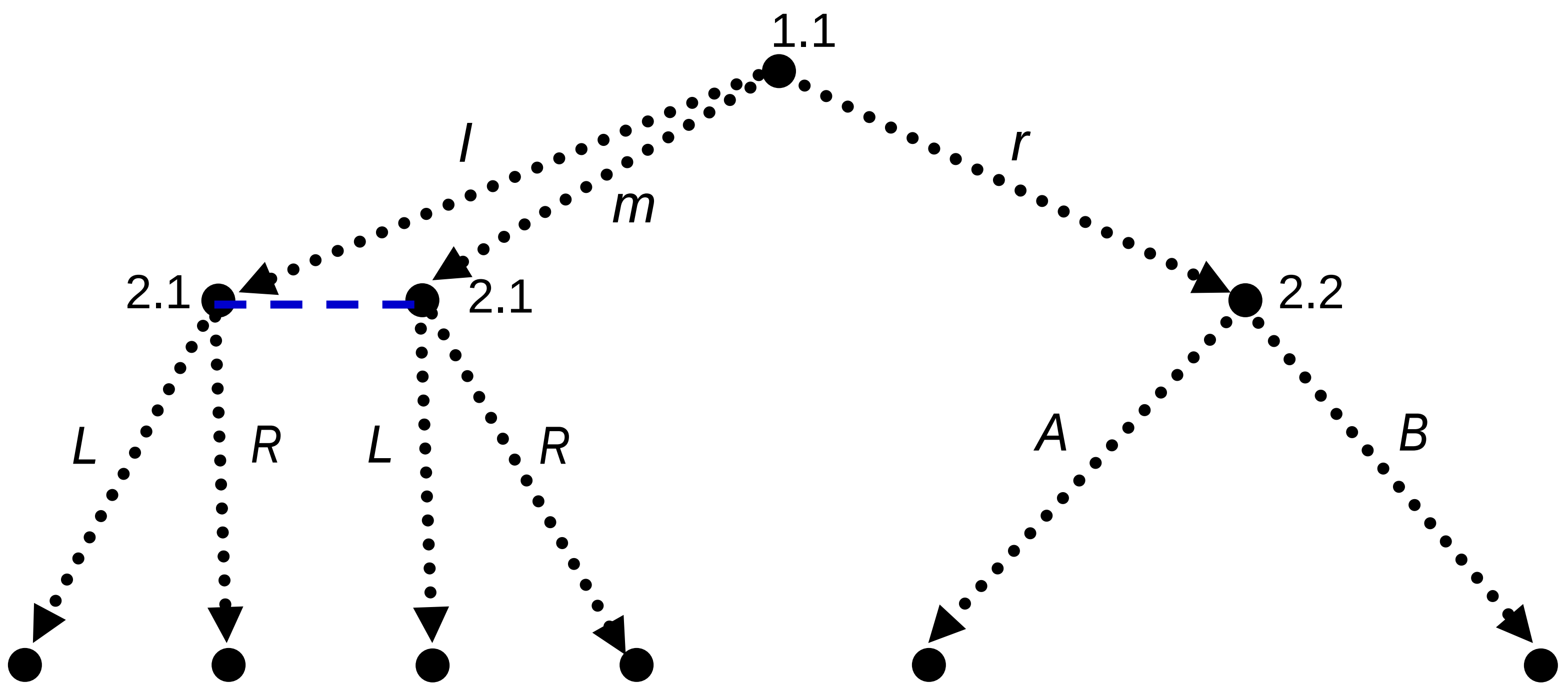
-and *does not cut any information sets*.

Idea:

Once a subgame begins, subsequent structure is *common knowledge.*

Example

This game has two subgames, rooted at 1.1 and 2.2.



Extending backward induction

In games of perfect information, any subtree is a subgame.

What is the analog of backward induction?

Try to find “equilibrium” behavior from the “bottom” of the tree upwards.

Subgame perfection

A strategy vector (s_1, ..., s_N) is a *subgame perfect Nash equilibrium* (SPNE) if it induces a Nash equilibrium in (the strategic form of) every subgame.

(In games of perfect information, SPNE reduces to backward induction.)

Subgame perfection

"Every subgame" includes the game itself.

Idea:

-Find NE for "lowest" subgame

-Replace subgame subtree with equilibrium payoffs

-Repeat until we reach the root node of the original game

Subgame perfection

- As before, a SPNE specificies a *complete contingent plan* for each player.
- The *equilibrium path* is the actual play of the game under the SPNE strategies.
- As long as the game has finitely many stages, and finitely many actions at each information set, an SPNE always exists.