II. Stabilization of moduli in string theory I

❑ Recent developments in fixing moduli near black hole horizon and black hole attractors

❑ 1) A striking role of stringy corrections converting a classical singularity into a regular black hole with the singularity covered by the horizon.

❑ 2) An emergent relation between black hole attractors and cosmology with regard to moduli stabilization. *Explicitly attractive K3.*

A Simple Example of Moduli Fixing

Aspinwall, R.K.

We analyze M-theory compactified on K3xK3 with fluxes and its F-theory limit, which is dual to an orientifold of the type IIB string on $K3 \times T^2/Z_2$

We argue that instanton effects will generically fix all of the moduli.

Before branes are introduced

Moduli space is no more

Cosmology, Supersymmetry and Special Geometry

In familiar case of **Near Extremal Black Holes**

DUALITY SYMMETRY protects exact entropy
  formula from large quantum corrections

DUALITY SYMMETRY (shift symmetry)

 protects the **flatness of the potential**

in D3/D7 inflation model from large quantum
  corrections

Shift Symmetry of $\mathcal{G}$

- Flatness of the effective supergravity inflaton potential follows from the shift symmetry of

$$\mathcal{G} \equiv K + \ln|W|^2$$

$$V = e^{\mathcal{G}}[|\mathcal{G}_{,z}|^2 - 3]$$

We need models where the position of the D3 brane after stabilization of the volume is still a modulus

SHIFT SYMMETRY
and volume stabilization

Distance between branes
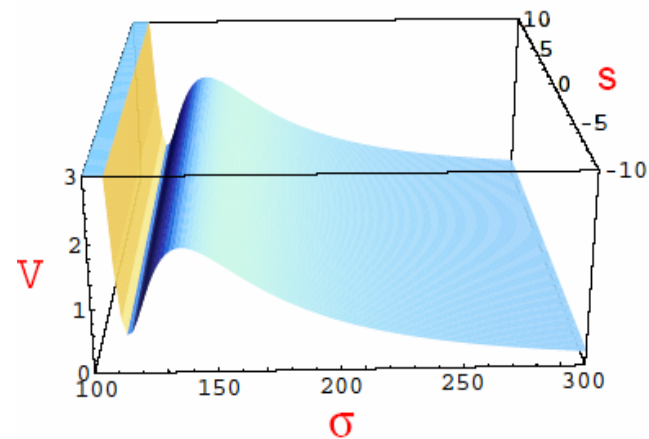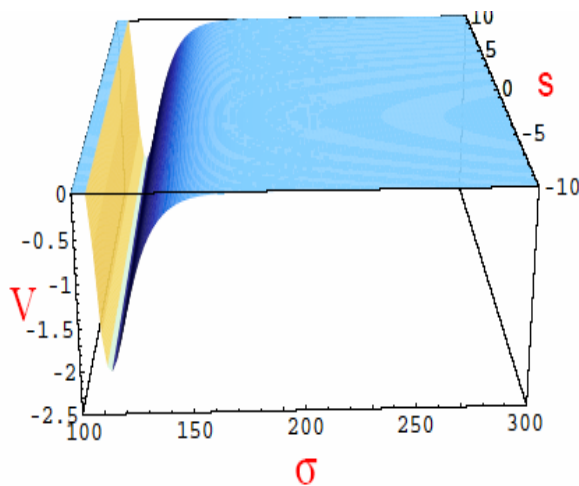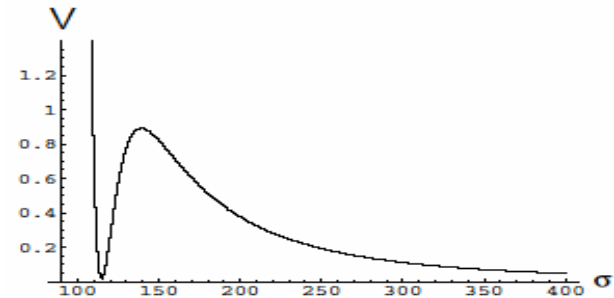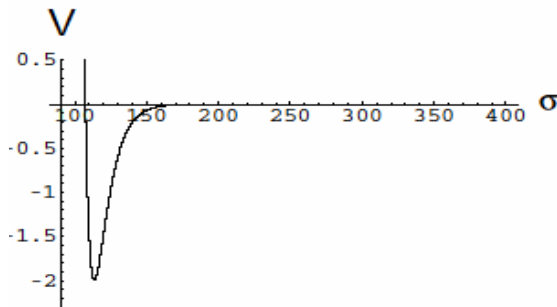
$$\phi = x^4 + ix^5$$

Volume-axion field

$$\rho = \alpha + i\sigma$$

$$\phi \to \phi + \mathrm{Re} f \qquad \phi - \bar{\phi} \to \phi - \bar{\phi}$$

$$\mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi}) \to \mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi})$$

Inflaton Trench

Hsu, R.K., Prokushkin

- **Supersymmetric Ground State of Branes in Stabilized Volume**

SHIFT SYMMETRY

The motion of branes does not destabilize the volume

String Theory and
N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

■ Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

Isometry of the compactified space provides shift symmetry slightly broken by quantum corrections

■ orientifold with fluxes, mobile D3 branes and heavy D7 branes

Coset Space $\quad \dfrac{SU(1,1)}{U(1)} \times \dfrac{SO(2,2+n_3)}{SO(2)\times SO(2+n_3)}$

Duality

Special Kähler geometry

N=2 supergravity with vector multiplets

■ SymplecticVectors

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}' = \begin{pmatrix} A & -B \\ C & D \end{pmatrix} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \qquad Sp(2(n+1), R)$$

$$K = -\log\left[i(\overline{X}^\Lambda F_\Lambda - \overline{F}_\Lambda X^\Lambda) \right]$$

■ Kähler potential is a symplectic invariant

■ **Supersymmetric Black Hole Entropy**

Symplectic Invariant

Duality and symplectic transformations

$$\mathcal{L}_1 = \tfrac{1}{4}(\text{Im } \mathcal{N}_{\Lambda\Sigma})\mathcal{F}_{\mu\nu}^\Lambda \mathcal{F}^{\mu\nu\Sigma} - \tfrac{i}{8}(\text{Re } \mathcal{N}_{\Lambda\Sigma})\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\mu\nu}^\Lambda \mathcal{F}_{\rho\sigma}^\Sigma$$

coupling constants or functions of scalars

$$\partial^\mu \text{Im } \mathcal{F}_{\mu\nu}^{+\Lambda} = 0 \qquad \text{Bianchi identities}$$

$$\partial_\mu \text{Im } G_{+\Lambda}^{\mu\nu} = 0 \qquad \text{Equations of motion}$$

$$\mathcal{F}_{\mu\nu}^\pm = \tfrac{1}{2}\left(\mathcal{F}_{\mu\nu} \pm \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\rho\sigma}\right)$$

$$G_{+\Lambda}^{\mu\nu} \equiv 2i\frac{\partial\mathcal{L}}{\partial\mathcal{F}_{\mu\nu}^{+\Lambda}} = \mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{+\Sigma\,\mu\nu}$$

DUALITY

$$\begin{pmatrix}\tilde{\mathcal{F}}^+ \\ \tilde{G}_+\end{pmatrix} = \mathcal{S}\begin{pmatrix}\mathcal{F}^+ \\ G_+\end{pmatrix} = \begin{pmatrix}A & B \\ C & D\end{pmatrix}\begin{pmatrix}\mathcal{F}^+ \\ G_+\end{pmatrix}$$

Inflaton Shift is a Duality

$$(y^r)' = y^r + \beta^r \qquad s' = s \qquad t' = t \qquad u' = u$$

$$\mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$B = 0$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \beta & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 0 & 0 & -\beta^2/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta^2/2 & 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & \beta & 0 \end{pmatrix}.$$

$$A^T C - C^T A = 0 \ , \ \ B^T D - D^T B = 0 \ , \ \ A^T D - C^T B = I$$

Conclusion: $\mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi})$

■ No fine-tuning required for slow-roll inflation

A Stringy Cloak for a Null Singularity

Dabholkar, R. K., Maloney

hep-th/0410076

■ A first explicitely computable class of string theory/supergravity models when $c_2(R\ldots)^2$ terms modify a classically singular solution with vanishing horizon into a regular black hole with singularity clothed by a finite area of the horizon

$$S_{cl} = \frac{1}{4}A_{cl} = 0 \qquad S = \frac{1}{2}A = 4\pi\sqrt{\frac{c_2\, p\, q}{24}}$$

C_2 depends on topology of Calabi-Yau (second Chern-class coefficient)

■ Simple example, c_2 on K3 is 24 $\qquad S = \frac{1}{2}A = 4\pi\sqrt{p\, q}$

N=2 BPS mass formula, M=|Z|

- The BPS mass is equal to the central charge, which depends on moduli and charges: symplectic invariant

$$M^2_{BPS} = |Z|^2 = |\langle Q, V \rangle|^2 = e^K |q_I X^I(z) - p^I F_I(z)|^2$$

- The ADM mass of the black hole is equal to the value of the central charge when moduli are at infinity

$$M^2_{ADM} = |Z(p, q, z_\infty, \bar{z}_\infty|^2$$

Attractor equations

- Introduce a symplectic vector

$$\Pi = \begin{pmatrix} Y^I \\ F_J(Y) \end{pmatrix} \quad \text{where} \quad Y^I \equiv \bar{Z}\, X^I.$$

- At the attractor point there is an algebraic relation between the fixed values of moduli and charges

$$Y^I - \bar{Y}^I = ip^I, \qquad F_I(Y) - \bar{F}_I(\bar{Y}) = iq_I$$

Calabi-Yau black holes

$$S_{cl} = \frac{A_{cl}}{4} = 2\pi \sqrt{\hat{q}_0 D_{ABC} p^A p^B p^C}$$

■ Classical area=0 if $\qquad D_{ABC} p^A p^B p^C = 0$
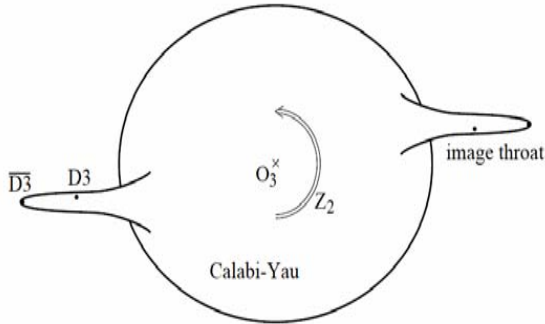
■ Quantum corrected entropy and area

$$A = 4\pi |Z|^2_{r=0} = 8\pi \sqrt{\frac{c_{2A} p^A |\hat{q}_0|}{24}}$$

$$S = 4\pi \sqrt{\frac{c_{2A} p^A |\hat{q}_0|}{24}}$$

Stabilization of moduli via instantons: breaking the isometries of the manifold

- When is this possible?

- **Can we use fluxes and instanton corrections to fix all moduli but the inflaton?**

New class of inflationary models in string theory
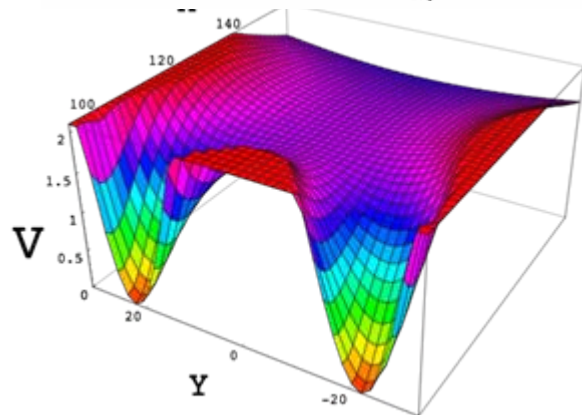

KKLMMT brane-anti-brane inflation

D3/D7 brane inflation

Dasgupta, Herdeiro, Hirano, R.K.

D-term inflation
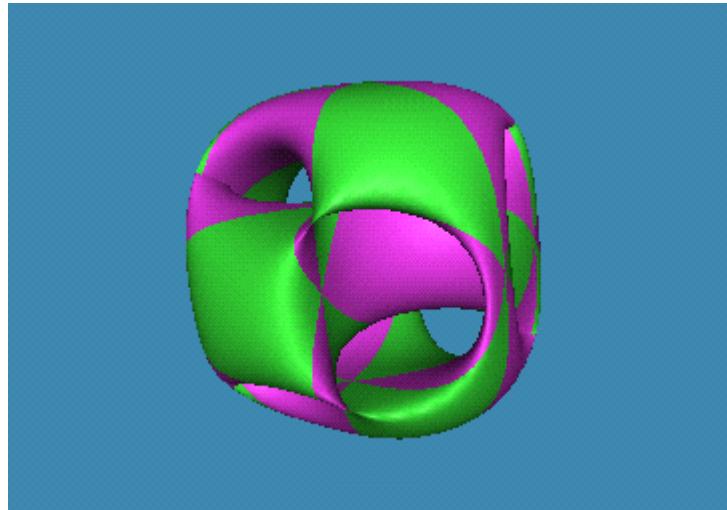

Racetrack modular inflation

Blanco-Pilado, Burgess, Cline, Escoda, Gomes-Reino, Kallosh, Linde, Quevedo

DBI inflation, **Silverstein et al**

Major problem

The mechanism of volume stabilization in this (and many other models of string theory) does not seem to work.

Witten 1996: in type IIB compactifications under certain conditions there can be corrections to the superpotential coming from Euclidean D3 branes. His argument was based on the M-theory counting of the fermion zero modes in the Dirac operator on the M5 brane wrapped on a 6-cycle of a Calabi-Yau four-fold. He found that

such corrections are possible

only in case that the four-fold admits divisors of arithmetic genus one,

$$\chi_D \equiv \sum (-1)^n h^{(0,n)} = 1$$

In the presence of such instantons, there is a correction to the superpotential which at large volume yields a new term

$$W_{\text{inst}} \sim \exp(-a\rho)$$
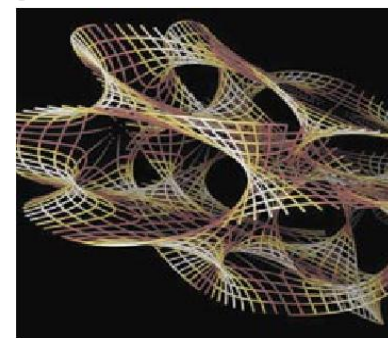
In type IIB string theory the leading exponential dependence comes from the action of an Euclidean D3 brane wrapping a 4-cycle.

Counting fermionic zero modes M5 with fluxes

■ New computation of the normal bundle U(1) anomaly

$$\chi_D(F) = \chi_D - (h^{(0,2)} - n)$$

■ Here n is the dimension of solutions of the constraint equation which depends on fluxes.

■ To have instantons we need $\chi_D(F) = 1$

$$n = h^{(0,1)} + h^{(0,3)} \qquad \chi_D \neq 1$$

Witten's condition is generalized

- New vacua with $\chi_D(F) = 1$ $\chi_D \neq 1$
  $\chi_D = 1$

- it seems the landscape just got another factor 10^{500} bigger

- Examples include "friendly" parts of the landscape

Example of $K3 \times K3$ compactification

An M5 brane wraps a 6-cycle $D = K3 \times \mathbb{P}^1$ of the 4-fold $X = K3 \times K3$. Since both $K3$ and $\mathbb{P}^1$ only have even cohomology, the same is true for the cohomology of the 6-cycle by the Künneth formula. The Dirac equation in the fluxless case hence only has positive chirality solutions

$$\epsilon_+ = \phi|\Omega\rangle + \phi_{\bar{a}\bar{b}}\Gamma^{\bar{a}\bar{b}}|\Omega\rangle \, .$$

Consider a flux which is a $(2,0) + (0,2)$ form on $K3$

$$\frac{F}{\pi} = \Omega_1 \wedge \bar{\Omega}_2 + \bar{\Omega}_1 \wedge \Omega_2$$

We need to contract the flux with $\bar{\Omega}_1$ and project onto the non-harmonic piece. This amounts to contracting Ω_1 with $\bar{\Omega}_1$, which is a number by covariant constancy of the complex structure. We are left with $\bar{\Omega}_2$, the harmonic projection of which is itself. $\bar{\Omega}_1$ hence does not solve the constraint. Thus we lose a zero mode of the Dirac operator, namely $\phi_{\bar{a}\bar{b}}\Gamma^{\bar{a}\bar{b}}|\Omega\rangle$, upon turning on flux. In particular, $\chi_D = 2$, while $\chi_D(F) = 1$.

Stabilization of Kahler moduli is possible!

Choose (0,4) and (4,0) FLUX

- This flux breaks susy in Minkowski and in AdS

- Count the fermionic zero modes on Dirac operator

- (0,0) mode is cut out but (0,2) survives

- Instanton corrections are possible!

- With account of exp. Terms in the superpotential, susy is restored in AdS

- **Moore, Les Houches**

FLUX VACUA AND SUPERSYMMETRIC ATTRACTORS

- We conjecture a universal formulation of supersymmetric attractor equations. It is valid for the flux vacua or for BPS black holes, depending on the choice of the components of flux either in the compact space or in 4d space.

- As an example, we define flux vacua with a rigid *explicitly attractive K3 surface* where a class of moduli are fixed by fluxes. The explicit values of complex structures are extracted from the previously known solution of the attractor equation for the black holes with the same symmetry.

FLUX VACUA AND ATTRACTORS

Explicitly attractive K3 surfaces

- Attractive K3 surfaces are always rigid - any infinitesimal deformation of complex structure would always decrease the rank of the Picard lattice on K3, which is equal to 20.

- Torelli's theorem defines the complex structure of the attractive K3 surface

$$\Omega^{(2.0)} = q - \tau p$$

- However, we may use the solutions of the complete set of attractor equations and give the explicit answer for all moduli in terms of fluxes.

Part of these attractor equations were already used in the definition of the attractive K3 surface by Moore. Moreover, he has proposed the interpretation of the attractor value of the

$$|Z|^2_{fix} = (p^2 q^2 - (p \cdot q)^2)^{1/2} = \frac{A}{4\pi}$$

as an area of the unit cell in the transcendental lattice of the K3 surface.

We found an **explicit definition of the attractive K3 surface**, where the axion-dilaton and all 20 complex structure moduli are given at the fixed points as functions of all fluxes.

$$t^1 = \tau = \frac{p \cdot q}{p^2} - i \frac{(p^2 q^2 - (p \cdot q)^2)^{1/2}}{p^2} \, ,$$

$$t^i = \frac{\overline{\tau} p^{i-1} - q^{i-1}}{\overline{\tau}(p^{n+1} - p^0) - (q^{n+1} - q^0)} \cdot \qquad i = 2, \ldots, n+1$$

We used a known solution of the attractor equation for some particular black holes!