High-resolution spectroscopy of whispering gallery modes in large dielectric spheres

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The mode spectrum of a 3.8-cm-diameter fused-silica sphere has been studied in the vicinity of 1.06 μm. A single-frequency Nd:YAG laser was used to excite whispering gallery modes by means of evanescent wave coupling. The spectrum is in excellent agreement with predictions from Mie theory.

The interaction of optical waves with spherical objects has been a subject of interest since the early treatments by Lorenz, Mie, and Debye, which provide the framework for the description of various natural light-scattering phenomena. More recently, investigations have been extended to nonlinear-optical interactions in small spheres, which are favored by the strong spatial confinement of resonance modes. The observation that a spherical object can sustain high-Q resonator modes dates back to Richtmyer. Spherical resonators for optical waves have been used as laser resonators. Mie theory predicts that the modes' radial spatial extension from the surface toward the center of the sphere lies in the range from a few wavelengths to \((1 - n_s/n_a)R\), where \(R\) is the sphere's radius and \(n_s\) and \(n_a\) are the refractive indices of the sphere and the medium surrounding it. The former modes, called whispering gallery modes, have been observed in stimulated-emission, optical-levitation, and fluorescence experiments. Direct excitation by means of a coupling prism has recently been demonstrated by Braginsky et al. in fused-silica mini-spheres using a He-Ne laser.

Here we report detailed results on the resonance frequencies of whispering gallery modes of large dielectric spheres at the Nd:YAG wavelength \(\lambda_0 = 1.064 \mu\text{m}\).

Two types of mode, TM and TE, can exist in a spherical resonator; in the limit of resonators large compared with the wavelength of the modes, their electric fields are approximately radially and azimuthally polarized, respectively. The electric fields of the eigenmodes are given by

\[
E_r(r, \theta, \phi) = \frac{\epsilon}{k r} j_n(kr) Y_n^0(\theta, \phi) \quad \text{TM modes},
\]

\[
E_r(r, \theta, \phi) = -\frac{q}{\sin \theta} j_n(kr) Y_n^1(\theta, \phi) \quad \text{TE modes},
\]

where \(k = 2\pi n_s c/\lambda\) and \(j_n, Y_n^m (q = -n, \ldots, n)\) are the spherical Bessel and harmonic functions. Resonance occurs when the scattering coefficient associated with a particular mode \(n\) is unity. This is satisfied for an infinite number of reduced eigenfrequencies \(n_s x_n^{(l)} = kR\), where the order number \(l\) gives the number of radial maxima of the field inside the sphere. High-Q, quasi-bound (i.e., weakly radiating) modes require that \(n > n_s k R / n_a\). The small-\(l\) modes are the whispering gallery modes, and for large spheres \((R >> \lambda_0)\) their resonance frequencies can be obtained using expansions of the Bessel functions for large order \(n\),

\[
x_n^{(l)} = n + \frac{1}{2} - \frac{1}{6} \left(\frac{n + 1/2}{2}\right)^{1/3} - \frac{mp}{\sqrt{m^2 - 1}} + \frac{1}{2} \frac{10(n + 1/2)^{1/3}}{2(n^2 - 1)^{2/3}} \left(\frac{n}{2\pi}\right) \sin^{1/3}(\omega t) \text{ for TM and TE resonances, respectively.}
\]

where \([x_n^{(l)}, p] = [a_n^{(l)}, 1/m^2], [\delta_n^{(l)}, 1] \) for TM and TE resonances, respectively. \(\zeta_l\) denotes the \(lth\) zero of the Airy function \(Ai(\zeta_l)\), and \(m = n / n_a\). Here only the first five terms are relevant, and they can be given a simple interpretation. The \(l\)-independent terms result from the approximate vanishing of the mode's electric field at the surface of the sphere, \(j_n(n_s x_n^{(l)}) \approx 0\) when in resonance. The mode number \(n\) is nearly equal to the number of wavelengths \(\lambda_0 / n\), that fit around the sphere's circumference, and the free spectral range follows as

\[
\nu_f = \frac{c}{2\pi R} [x_{n+1}^{(l)} - x_n^{(l)}] = \frac{c}{2\pi n_a R}.
\]

This is the free spectral range expected for a wave that propagates along the inner surface of the sphere that is continuously internally reflected. The total-internal-reflection (TIR) process also gives rise to the frequency shift between the nearly identical TM and TE spectra,

\[
\nu_p = \frac{a_n^{(l)} - \delta_n^{(l)}}{\nu_f} \approx \frac{\sqrt{m^2 - 1}}{m},
\]

because of the relative phase shifts experienced by the fields on TIR.

While the free spectral range and the frequency shift are relatively insensitive to the precise value of \(m\), the relative positions of the modes of each polar-
Fig. 1. Setup for the study of modes of spherical resonators. FC1, FC2, thermal and piezoelectric frequency control inputs. D1, D2, alternate positions for the photodetectors that register scattering and outcoupling of the mode field, respectively.

The mode number is obtained by fitting to 18 of the 22 lowest-order modes, \( n = 1.45008 \pm 0.00005 \).

The measured \( \nu_f = 1.726 \pm 10 \) MHz and polarization shift \( \nu_p = 0.729 \) also compare well with the values of 1731 MHz and 0.724 predicted from Eq. (2) and the fitted index. Temperature drift accounts for the slight discrepancy (tuning coefficient \( d\nu/dT = -2 \) GHz/K).

To determine accurately the linewidths of the resonances we have studied the derivative line shapes by piezoelectrically frequency modulating the laser and lock-in detecting the equatorially scattered light. The smallest linewidth (FWHM) measured was 3.0 MHz, which implies a resonator finesse \( \mathcal{F} = 570 \). Modification of the coupling strength by changing the gap width, thereby changing the line intensities over 2 orders of magnitude, did not result in any appreciable change in linewidth. Thus the finesse is due to bulk and surface losses rather than output coupling losses.

With careful material choice and surface polish, fused-silica TIR resonators may reach submegahertz linewidths. This suggests their use as monolithic resonators for frequency stabilization, provided that their operating temperature is tightly controlled. To demonstrate this, the laser was frequency locked to one particular whispering gallery mode, with the derivative signal used as an error signal fed back to the piezoelectric frequency control.
The relative intensities of the whispering gallery resonances depend on their excitation probability, i.e., their spatial overlap with the Gaussian laser beam, and their losses. For the scattering spectrum this is a function of the mode's intensity distribution as well as the sphere's scattering loss distribution. The electric-field amplitudes [relations (1)] for the TM and TE whispering gallery modes can be approximated in the large-\(n\) case by

\[
E(r) \sim A_i \left[ \zeta_i + \left( \frac{2A_i}{R} \right)^{1/3} (R - r) \right].
\]  

(5)

This Airy mode structure has been observed in bent metallic waveguides.\(^6\) Relation (5) implies that the radial depth of the whispering gallery field distribution is \(\Delta r = 5\zeta \mu m\) in the present case. Since the modes spread out further radially with increasing order number, a decreasing excitation probability is expected for an input beam launched at grazing incidence into the sphere, in agreement with Figs. 2(a) and 2(b), where only the lowest orders appear. With increasing beam launch angle, we observed a growing number of modes as well as strong changes in their intensities.

Similarly we can deduce that the angular distribution must be described by a spherical harmonic peaked near \(\theta = \pi/2\), i.e., with \(q\) on the order of \(n\), to provide good spatial overlap with the laser beam. Also, according to relations (1), \(E_{\theta}\) and \(E_{\phi}\) are then of the same magnitude, as observed. The presence of the narrow equatorial radiation ring confirms this reasoning (see Refs. 4 and 8).

In conclusion, we have reported the observation and identification of whispering gallery modes of spherical optical resonators using a single-frequency laser. The distinctiveness of the resonances allows laser frequency locking, which is necessary for the implementation of spherical gyroscopes\(^4\) or injection-seeded spherical lasers as well as for the study of nonlinear-optical interactions in resonators such as quantum nondemolition\(^8\) and squeezing effects.\(^7\) Optical spheres are one realization of TIR resonators with evanescent wave coupling. The ability of such resonators to operate over a wide wavelength range with variable input–output coupling promises important applications.

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References

11. For \(a_n^{(s)}b_n^{(i)} << n\) and \(a_n^{(s)}b_n^{(i)} << n/m\), expressions (93.8), (93.12), and (93.23), (93.27) in Ref. 12 are appropriate.
14. Consider a plane wave propagating along a closed equatorial polygonal ring path with \(N\) total internal reflections. Addition of the individual phase shifts (given by the Fresnel formulas) that occur during one round trip leads to a relative frequency shift given by relation (4) in the limit of large \(N\).