Power and spectral characteristics of continuous-wave parametric oscillators: the doubly to singly resonant transition

S. T. Yang, R. C. Eckardt, and R. L. Byer
E. L. Ginzton Laboratory, Stanford University, Stanford, California 94305

Received March 18, 1993; revised manuscript received April 16, 1993

The output power and spectral characteristics of optical parametric oscillators (OPO's) as they evolve from the doubly resonant to the singly resonant regime are examined. By assuming a high-Q signal cavity, we derive approximate analytical solutions of pump depletion and conversion efficiency for OPO's with arbitrary idler Q. Using this set of solutions, we examine in detail amplitude and frequency instabilities of doubly resonant oscillators with low signal loss and arbitrary idler loss. We find that OPO's with weak idler feedback, when operated at a few times above threshold, exhibit improved spectral stability. Experimental results are presented for LiNbO$_3$ and KTP OPO's operating with a fixed low signal loss and various idler losses, including the limiting case of the cw singly resonant oscillator with no idler feedback.

1. INTRODUCTION
Continuous-wave doubly resonant optical parametric oscillators (DRO's) were first demonstrated more than 25 years ago.$^{1,2}$ The low-threshold-power advantage of the DRO's however, was offset by the necessity for overlap of signal- and idler-cavity resonances that placed severe tolerance limits on cavity length and pump-frequency fluctuations.$^{3,4}$ It was not until recently, with the availability of single-axial-mode pump sources such as diode-pumped solid-state lasers and stable cavity constructions, that cw DRO's were operated in a single axial mode with narrow linewidth.$^{5,6}$ Continuous tuning of DRO's, however, remained problematic because of cluster effects and the need to tune two variables simultaneously.$^9$

It is well known that, by resonating only the signal or the idler in an optical parametric oscillator (OPO), both output power stability and the tuning range are improved.$^{10}$ Unfortunately the threshold for a singly resonant OPO (SRO) is typically a few hundred times greater than DRO thresholds. The high threshold power has precluded demonstration of the cw SRO until recently.$^{11}$ Given the difficulties in achieving the high cw SRO threshold, we were motivated to examine the DRO-to-SRO transition to discover whether one might obtain the benefits of SRO operation at reduced threshold from a DRO that operates with higher loss for one of the waves. This question was previously addressed by Falk,$^{12}$ who analyzed near-threshold behaviors of OPO's with arbitrary signal and idler loss. From this analysis Falk concluded that, near threshold, the OPO stability is not expected to improve until the feedback from either the signal or the idler wave is eliminated.

In this paper we reexamine this issue by investigating the well-above-threshold behavior of DRO's with a high-finesse signal cavity and an arbitrary-finesse idler cavity. In Section 2 we present approximate analytical solutions of pump depletion and conversion efficiency for OPO's with a high-Q signal cavity and an arbitrary-Q idler cavity. The solutions are then used to examine the amplitude and frequency stability of OPO's at different pumping levels and with various idler losses. Section 3 complements the theoretical discussion by presenting experimental results of LiNbO$_3$ and KTP OPO's operating with low signal loss and arbitrary idler losses ranging from near zero to 100%. Finally some conclusions concerning practical realization of cw SRO's are presented in Section 4.

2. THEORY

A. Pump Depletion and Conversion Efficiency
In this section we examine pump depletion and conversion efficiency for OPO's with low signal loss and arbitrary idler feedback. Previous plane-wave analyses of OPO's have treated low-loss DRO's in which both signal and idler amplitudes are assumed to be spatially invariant.$^{13}$ Here we are interested in examining cases in which the idler feedback approaches zero, the SRO condition. With large idler-output coupling, the constant idler-field approximation is not valid. However, we continue to assume that the signal cavity has high Q, and therefore the signal field is spatially invariant. By assuming a constant signal field, we can reduce the set of three coupled-mode equations to two equations, one for the pump and the other for the idler. With this reduced set of equations, approximate analytical solutions for pump depletion and conversion efficiency can be derived. The derivation is identical to the approach used by Colucci et al.$^{14}$ We repeat a simplified version of their derivation in this section, using notations that are consistent with previous studies on DRO's and SRO's.$^{15}$

We start with the familiar coupled-wave equations$^{15}$:

$$\frac{\partial E_j(z)}{\partial z} = i\kappa_j E_j(z) E_i^*(z) \exp(i\Delta k z), \quad (1a)$$

$$\frac{\partial E_i(z)}{\partial z} = i\kappa_i E_i(z) E_s^*(z) \exp(i\Delta k z), \quad (1b)$$

$$\frac{\partial E_p(z)}{\partial z} = i\kappa_p E_i(z) E_i(z) \exp(-i\Delta k z). \quad (1c)$$

In these equations $\kappa_j = \omega_j d/(n_j c)$, $\kappa_i = \omega_i d/(n_i c)$, and...
\( \kappa_p = \omega_p d/(\eta_p c), \) where \( \omega_s, \omega_i, \) and \( \omega_p \) are the angular frequencies for the signal, idler, and pump, respectively, \( n_s, n_i, \) and \( n_p \) are the refractive indices for the signal, idler, and pump, respectively, \( d \) is the effective nonlinear coefficient, and \( c \) is the speed of light. The momentum mismatch is given by \( \Delta k = k_p - k_i - k_s. \) Assuming a small loss at the signal wavelength, we set \( E_s(z) = E_{s0} = \) constant. This permits us to ignore Eq. (1a), and we can then proceed to solve Eqs. (1b) and (1c). Subject to the boundary conditions that \( E_s(z = 0) = E_{s0} \) and \( E_p(z = 0) = E_{p0}, \) where \( E_{s0} \) and \( E_{p0} \) are complex numbers, solutions for \( E_s(z) \) and \( E_p(z) \) are
\[
E_s(z) = (E_{s0} \cos(\Gamma z) - E_{s0} [i \Delta k/(2 \Gamma)] \sin(\Gamma z)) + (i \kappa_p/\Gamma) E_{p0} E_{s0}^* \exp(i \Delta k z/2), \tag{2a}
\]
\[
E_p(z) = (E_{p0} \cos(\Gamma z) + E_{p0} [i \Delta k/(2 \Gamma)] \sin(\Gamma z)) + (i \kappa_p/\Gamma) E_{s0} E_{p0}^* \exp(-i \Delta k z/2), \tag{2b}
\]
where \( \Gamma^2 = \kappa_p |E_{s0}|^2 + (\Delta k/2)^2. \) Equations (2) express \( E_s(z) \) and \( E_p(z) \) as functions of initial field amplitudes, \( E_{s0}, \) \( E_{s0}, \) and \( E_{p0}. \) We assume here that the pump passes through the crystal once per round trip. Therefore \( E_{p0} \) is equal to the incident pump field. To solve for \( E_{s0}, \) \( E_{s0}, \) and \( E_{p0}, \) we invoke self-consistency equations for the signal and idler fields. For the idler field, self-consistency implies that \( E_i(z = 0) = E_i(z = L) = 0, \) exp(i \( \Psi_i), \) where \( \Psi_i \) represents the round-trip phase shift for the idler given by \( \Psi_i = -4 \pi n_i l_s / \lambda_s - 4 \pi (L - l_s) / \lambda_i, \) where \( l_s \) is the crystal length and \( L \) is the one-way cavity length for a standing-wave cavity. Also, \( r_i \) is the idler-field reflectivity. In the following sections \( r \) denotes field-reflection coefficients, whereas \( R \) indicates power-reflection coefficients. Imposing self-consistency on the idler field, we find that the initial field \( E_{s0} \) is given by
\[
E_{s0} = \frac{\kappa_i}{\Gamma} E_{p0} E_{s0}^* \sin(\Gamma l_s) r_i \exp(i \Delta k l_s/2 + i \Psi_i)}{1 - \left[ \cos(\Gamma l_s) - i \Delta k/2 \sin(\Gamma l_s) \right] r_i \exp[i \Delta k l_s/2 + \Psi_i]}. \tag{3}
\]

The solutions for \( E_s(z) \) and \( E_p(z), \) obtained with the assumption that \( E_s(z) \) is constant, do not differ significantly from those obtained from a full numerical treatment. To improve the accuracy of the signal-field solution, we substitute \( E_s(z) \) and \( E_p(z), \) derived above, into the signal-field derivative and integrate over crystal length \( l_c. \) The resultant \( E_s(z = l_c) \) is
\[
E_s(z = l_c) = 1 + NCF* \sin(x) \times \left[ 1 + \frac{\sin(2x)}{2} + i \Delta k l_c \sin^2(x) - \frac{(\Delta k l_c)^2}{4x^2} \left[ 1 - \frac{\sin(2x)}{2} \right] \right] + \frac{NCF* \sin(x)}{x} \left[ 1 + \frac{\sin(2x)}{2} + i \Delta k l_c \sin^2(x) - \frac{(\Delta k l_c)^2}{4x^2} \left[ 1 - \frac{\sin(2x)}{2} \right] \right], \tag{4}
\]

where \( x = \Gamma l_c \) and \( N = |E_{p0}|^2/|E_{p0}|^2 \) is the pumping level relative to threshold at parametric gain center,\(^{13} \) which is \[|E_{p0}|^2 = (2/\kappa_p \kappa_i l_s)^2 C, \] where \( C = [(1 - r_s)(1 - r_i)]/(r_s + r_i). \) \( F^* \) denotes the complex conjugate of \( F, \) which is defined by
\[
F = \frac{r_i \exp(i \Delta kl_s/2 + \Psi_i)}{1 - [\cos(x) - i \Delta kl_s/(2x) \sin(x)] \exp[i \Delta kl_s/2 + \Psi_i]}. \tag{5}
\]
Finally self-consistency is imposed on the signal field: \( E_s(z = l_c) = E_{s0} r_s \exp(i \Psi_s), \) where \( \Psi_s = -4 \pi n_s l_c / \lambda_s - 4 \pi (L - l_s) / \lambda_i, \) and \( r_s \) is the signal-field reflectivity coefficient. Given the pumping level \( \eta, \) mirror-field reflectances for signal and idler \( r_s \) and \( r_i \) round-trip idler phase shift \( \Psi_i, \) and phase mismatch \( \Delta k l_s, \) we can calculate the signal-field intensity |\( E_{s0} |^2, \) which is embedded in \( x^2, \) from the real part of the signal self-consistency equation:
\[
\frac{1}{r_s} \exp(-i \Psi_s) = 1 + NCF* \sin(x) \times \left[ \frac{\sin(x)}{2} + i \Delta k l_c \sin^2(x) - \frac{(\Delta k l_c)^2}{4x^2} \left[ 1 - \frac{\sin(2x)}{2} \right] \right]
+ \frac{NCF* \sin(x)}{x} \left[ 1 + \frac{\sin(2x)}{2} + i \Delta k l_c \sin^2(x) - \frac{(\Delta k l_c)^2}{4x^2} \left[ 1 - \frac{\sin(2x)}{2} \right] \right] + \frac{NCF* \sin(x)}{x} \left[ 1 - \frac{\Delta k l_c}{2x} \right] \left[ 1 - \frac{\sin(2x)}{2} \right]. \tag{6}
\]
The corresponding round-trip signal phase shift \( \Psi_s \) is obtained from the imaginary part of Eq. (6).

B. Low-Loss DRO and SRO Limits
For comparison with previous analyses of low-loss DRO’s and SRO’s, we assume that \( \Delta k l_s = 0 \) and that signal and idler frequencies coincide with cavity resonances (i.e., \( \Psi_s = 2m \pi, \Psi_i = 2n \pi, \) where \( m \) and \( n \) are integers). With these assumptions, Eq. (6) reduces to
\[
\frac{1}{r_s} = 2 - \frac{1}{r_i} N \left[ \frac{\sin(x)}{2} \right] + \frac{1}{2} \sin^2(x) \left[ 1 - \frac{\sin^2(x)}{[1 - \cos(x)]^2} \right]. \tag{7}
\]
One finds the equation that governs pump depletion by substituting \( E_{s0}, \) given by Eq. (3), into Eq. (2b), which yields
\[
E_s(z = l_c) = E_{s0} \cos(\Gamma z) - \frac{\sin(x) \sin(2x)}{1 - \cos(x) r_i} \sin(\Gamma l_s). \tag{8}
\]
The depleted pump intensity after a single pass through the crystal is then given by
\[
\frac{|E_p(z = l_c)|^2}{|E_{p0}|^2} = \left[ \cos(x) - \frac{\sin^2(x)}{1 - \cos(x) r_i} \right]^2. \tag{9}
\]
One can also derive the photon-conversion efficiency for the signal field by starting from Eq. (6). With some algebra, photon-conversion efficiency for the signal field is given by
\[
\frac{\omega_s n_s |E_{s0}|^2}{\omega_s n_s |E_{p0}|^2} = 2 r_s x^2 \left[ \frac{\sin(x) \sin(2x)}{1 - \cos(x) r_i} + \frac{1}{2} \sin^2(x) \left[ 1 - \frac{\sin^2(x)}{[1 - \cos(x) r_i]^2} \right] \right], \tag{10}
\]
round-trip phase shift for the signal and idler fields $\Psi_s$ and $\Psi_i$. According to Falk's\textsuperscript{15} approach, we define the detuning from coincidence of signal- and idler-cavity resonances by the phase-detuned parameter $\Psi_0$:

$$\Psi_s + \Psi_i = \Psi_0 + 2\Delta k l_c,$$

which is the SRO threshold condition previously derived by Kreuzer.\textsuperscript{16} If we set $r_1 = 0$, Eq. (9) reduces to

$$\left| E_{p}(x = l_c) \right|^2 = \cos^2(x),$$

which again agrees with the SRO-pump depletion result obtained previously.\textsuperscript{15}

Similarly, in the limit of low signal and idler loss, Eqs. (9) and (10) converge to become Bjorkholm's solutions for DRO's with high-Q signal and idler cavities.\textsuperscript{13} Note that the above derivation is based on the plane-wave approximation, so it does not lend itself to quantitative comparisons with the experiments except in the near-field limit. However, we expect that the qualitative behaviors of OPO's will not change significantly with focusing. Also, we assumed that each field (pump, signal, and idler) oscillates in single longitudinal modes.

Figure 1 shows signal-photon-conversion efficiency calculated from Eq. (10) for OPO's with $R_i = 100\%$ and for $R_i$'s ranging from 99.9\% (DRO) to 0\% (SRO). As the idler-power reflectivity $R_i$ decreases toward zero, the OPO behaves more like a SRO. In particular, the pumping level at which 100\% signal-photon-conversion efficiency is achieved moves continuously from $N = 4$ to $N = (\pi/2)^2$ as $R_i$ decreases from 99.9\% to 0\%. This transition, however, is nonlinear with respect to change in idler reflectance. For example, the pumping level at which 100\% signal-photon-conversion efficiency occurs does not move beyond the halfway point between $N = 4$ and $N = (\pi/2)^2$ (i.e., DRO and SRO limits) until $R_i$ has dropped to below 22\%. Pump depletion follows the same behavior.

**C. Amplitude Stability**

Away from exact overlap of signal- and idler-cavity resonances, the OPO output intensity drops. One can examine this intensity variation by taking into account the phase-detuning parameter $\Psi_0$.

The definitions in Eqs. (13) apply for a standing-wave cavity. $\Psi_0$ for a ring cavity is one half of $\Psi_0$ for the standing-wave cavity given by Eqs. (13). For fixed pump frequency, note that $\Psi_0$ depends only on cavity length $L$. As the cavity length changes by half of the pump wavelength, $\Psi_0$ varies from 0 to $2\pi$.

One can calculate the intracavity signal intensity for any detuning from coincidence of signal- and idler-cavity resonances (i.e., $\Psi_s$, $\Psi_i \not= \text{integer multiples of } 2\pi$), using Eq. (6). For simplicity we consider the case of $\Delta k l_c = 0$ in this subsection. Figure 2 is a plot of the intracavity signal intensity normalized to unity at its peak versus phase detuning for OPO's with different signal and idler reflectances. The solid curves represent cases in which $R_i = 99\%$, whereas the single dashed curve denotes the case in which $R_i = R_s = 89.1\%$. One performs all the calculations assuming pumping at three times above the respective thresholds. Without normalization, intensity values are larger for OPO's with higher idler losses since the pumping power required for threshold to be reached is larger in those cases. As Fig. 2 shows, the tolerance range for operation away from coincidence of signal- and idler-cavity resonances, as represented by the half-peak width of $\Psi_s + \Psi_i$, becomes larger as the idler reflectance is lowered. In particular, as $R_i$ decreases to 4\%, the OPO remains above threshold even as signal- and idler-cavity resonances are shifted apart by as much as $\pm \pi \text{ rad}$, assuming no competi-

**Fig. 1.** Theoretical signal-photon-conversion efficiency plotted as a function of pumping intensity relative to threshold. Calculation is done for DRO's with fixed round-trip signal-power reflectance of 100\% and various round-trip idler-power reflectances $R_i$. Note that the 100\% conversion-efficiency point moves continuously from $N = 4$ to $N = (\pi/2)^2$ as the OPO's evolve from the DRO to the SRO regime.

**Fig. 2.** Calculated normalized signal intensity versus detuning for OPO's with different signal and idler reflectances. The solid curves represent cases in which $R_s = 99\%$, whereas the single dashed curve denotes the case in which $R_s = R_i = 89.1\%$. One performs all the calculations assuming pumping at three times above the respective thresholds. Without normalization, intensity values are larger for OPO's with higher idler losses since the pumping power required for threshold to be reached is larger in those cases. As Fig. 2 shows, the tolerance range for operation away from coincidence of signal- and idler-cavity resonances, as represented by the half-peak width of $\Psi_s + \Psi_i$, becomes larger as the idler reflectance is lowered. In particular, as $R_i$ decreases to 4\%, the OPO remains above threshold even as signal- and idler-cavity resonances are shifted apart by as much as $\pm \pi \text{ rad}$, assuming no competi-
Yang phase matching is assumed for these calculations. The calculation is done for DRO's with fixed round-trip signal-power reflectance of 99% and idler-power reflectance of 25%. The phase-detuning range over which the OPO remains above threshold increases as the pumping level is raised. Perfect phase matching is assumed for these calculations.

...tion from adjacent axial-mode pairs. In contrast, the tolerance range for OPO's with \( R_s = 99\% \) is only \( \pm 0.015 \) rad for pumping at three times above threshold. In addition, the output amplitude modulation versus detuning decreases as \( R_s \) is decreased. In the limit when \( R_s = 0 \), the output amplitude remains flat regardless of phase detuning. Of particular interest is the tolerance range difference between the cases in which \( R_i = R_s = 89.1\% \) and in which \( R_s = 99\% \) and \( R_i = 25\% \). These two cases have the same threshold, and yet the OPO with lower idler \( Q \) has a much wider phase-detuning tolerance range.

Figure 3 shows that, for fixed signal and idler losses, the phase-detuning tolerance range widens as the pumping level is increased. Here the parametric gain \((\Omega / t)^2\) is calculated for an OPO with \( R_s = 99\% \) and \( R_i = 25\% \) and at pumping levels \( N = 1.1, 1.5, 3, \) and \( 4 \). As pump power is increased, the phase-detuning tolerance range increases rapidly.

D. Frequency Stability

The frequency stability of an OPO with arbitrary signal and idler feedback was examined previously by Falk.\(^{12}\) On the basis of detailed studies of OPO operation near threshold, Falk concluded that the nonresonant idler feedback must be eliminated to prevent frequency instabilities similar to that for DRO's with high-Q cavities. In this subsection we show that Falk's conclusion is valid only near threshold. When the OPO with slight idler feedback is pumped at a few times above threshold, the frequency instabilities resulting from cavity perturbations decrease substantially.

According to Falk's\(^{12}\) approach, we assume that the pump frequency is sufficiently stable that OPO-frequency instability is due primarily to cavity-length fluctuations. As the cavity length drifts randomly over one pump wavelength, \( \Psi_0 \) changes by \( 2\pi \). A measure of OPO-frequency stability is given by the maximum signal- or idler-frequency excursion from parametric-gain center as one varies \( \Psi_0 \) by \( 2\pi \). For DRO's with high signal and idler feedback, the maximum frequency excursion from parametric-gain center is given by half of the cluster spacing.\(^3,4,9\) This implies that, for DRO's in a standing-wave cavity in which the pump passes through the crystal once but in which signal and idler traverse the crystal twice per round trip, the extreme OPO operating points occur at \( \Delta k l_c = \pm \pi/2 \). For a SRO, the maximum frequency excursion is half of the cavity free-spectral range since the SRO oscillation occurs on the cavity mode nearest parametric-gain center.

To calculate OPO operating point at each phase detuning \( \Psi_0 \) corresponding to a particular cavity-length setting, one solves Eq. (6) for \( \Delta k l_c \) ranging from zero to \( 2\pi \) at the fixed value of \( \Psi_0 \). The value of \( \Delta k l_c \) corresponding to the maximum signal intensity is then considered to be the operating point. Figure 4 is a plot of the OPO oscillation point versus \( \Psi_0 \) calculated at four different pumping levels for OPO's with signal-power reflectance of 99% and idler-power reflectance of 1%. The solid curves represent calculations in which we assumed that the OPO oscillation frequency coincides with the maximum signal output point. We calculated the dashed curve, using Falk's prescription in which the minimum threshold point is considered to be the operating point. Falk's calculation shows that, near threshold, as \( \Psi_0 \) increases from 0 to \( \pi \), the OPO operating point shifts linearly from \( \Delta k l_c = 0 \) to \( \Delta k l_c = 1.32 \) rad. At \( \Psi_0 = \pi \), a cluster hop occurs at which the operating point jumps from \( \Delta k l_c = -1.32 \) rad to 1.32 rad. For pumping close to threshold and to as high as 1.5 times above threshold, we see that the maximum signal output points coincide with the minimum threshold points, and the OPO may operate as far as \( \pm 1.32 \) rad from the \( \Delta k l_c = 0 \) point. However, at pumping levels between 2.15 and 3 times above threshold, the maximum operating-point excursion from parametric-gain center decreases. For example, at pumping 2.5 times above threshold the maximum signal output point does not deviate from the parametric-gain center by more than \( \pm 0.1 \) rad, which means that the OPO essentially functions as a SRO.

At pumping 2.64 times above threshold, 100% pump depletion occurs at the parametric-gain center \( \Delta k l_c = 0 \), and any further increase in pumping results in conversion of generated OPO waves back into the pump. As a result,
We conducted a series of experiments to complement the theoretical calculations and to ascertain the feasibility of having frequency-stable OPO operation. It is not necessary, therefore, to eliminate idler feedback for DRO's with two high-Q cavities. For these threshold measurements we used a bow-tie ring resonator with two curved mirrors (each with a 10-cm radius of curvature) is 11 cm. The incident angle on all the mirrors, and the space between the two mirrors (each with a 10-cm radius of curvature) are spaced 45 cm apart between the two flat mirrors. For operation as a DRO with two high-Q cavities.

A. SRO-to-DRO Threshold Studies

We measured the OPO threshold progression from the doubly resonant to the singly resonant regime by increasing the idler loss of OPO's with a fixed low signal loss. For these threshold measurements we used a bow-tie ring resonator with two curved mirrors (each with a 10-cm radius of curvature) and two flat mirrors as shown in Fig. 6. One of the flat mirrors was mounted upon a piezoelectric transducer (piezo; PZT) for fine cavity-length control. We chose a ring cavity to minimize loss through the crystal per round trip. For operation as a DRO with two high-Q cavities.

3. EXPERIMENTS

We conducted a series of experiments to complement the theoretical calculations and to ascertain the feasibility of cw SRO operation. We operated OPO's with low signal loss and various idler output couplings. Figure 6 shows the experimental setup. In all the experiments the pump source that we used was a multiwatt injection-locked cw Nd:YAG laser that routinely produces 12-W output power at 1064 nm with a linewidth in the range of a few tens of kilohertz. The single-frequency output of the Nd:YAG laser is resonantly doubled in an external enhancement cavity containing a 6-mm-long LiB<sub>3</sub>O<sub>5</sub> crystal. With 12 W of pump input as much as 4 W of green power is routinely produced in a diffraction-limited beam. We used both MgO:LiNbO<sub>3</sub> and KTP crystals. The LiNbO<sub>3</sub> crystal that we used has 4.9% magnesium-ion-concentration doping. The MgO:LiNbO<sub>3</sub> crystal, which is 2 mm × 2 mm × 12.5 mm, was cut for type-I noncritical phase matching along the crystallographic x axis. A two-layer broadband antireflection coating was deposited onto the crystal surface. The coating has residual-power reflection of 0.2%/surface for wavelengths ranging from 940 to 1070 nm.

The 3 mm × 3 mm × 8.6 mm KTP crystal was cut along the crystallographic axes with the long dimension along the z axis, the signal wavelength of 1090 nm and the idler wavelength at 1039 nm are polarized along the y and z axes (i.e., the ordinary and extraordinary axes), respectively, by use of type-II noncritical phase matching.

Fig. 6. DRO experimental setup. The two flat mirrors of the bow-tie ring cavity are spaced 45 cm apart between the two flat mirrors, and the space between the two mirrors (each with a 10-cm radius of curvature) is 11 cm. The incident angle on all the mirrors is 3 deg. To introduce loss preferentially for the idler in a LiNbO<sub>3</sub> OPO, we replace one of the flat mirrors with a Ti:sapphire mirror. For KTP OPO one or two Brewster plates are introduced into the long leg of the cavity to couple out the p-polarized idler waves preferentially.
cavities, all four mirrors are highly reflecting (>99.9%) in the range between 970 and 1160 nm and highly transmitting (99%) at 532 nm. The cavity layout is designed so that, at a signal wavelength near 1 μm, a 31-μm beam waist radius is produced midway between the two curved mirrors.

For studies that use the MgO:LiNbO$_3$ OPO, the crystal is placed in an oven and is heated to the phase-matching temperature of 107°C. We confirm the temperature uniformity of the crystal by measuring the phase-matching temperature bandwidth when doubling a Nd-doped gadolinium gallium garnet laser operating at 1.062 μm. The observed FWHM temperature-acceptance bandwidth of 0.65 deg is close to the theoretical bandwidth of 0.59 deg for a 12.5-mm-long crystal. At the degenerate wavelength of 1064 nm, the OPO-cavity finesse is measured to be 785, implying a round-trip power loss of 0.8%. Also at degeneracy, the signal beam waist has a l/e radius of 31 μm. With a 12.5-mm-long crystal length and refractive index of n = 2.232, the signal- and idler-beam confocal parameters are equal to the crystal length (i.e., $\xi_s = \xi_i = 1$). To maximize the spatial-mode coupling between the pump and the OPO waves, the pump confocal parameter is also set equal to the crystal length. Assuming a round-trip signal- and idler-field power loss of 0.8% and that the Boyd-Kleinman$^{21}$ focusing parameter $\tilde{h}(B = 0, \xi = 1) = 0.8$, we calculate the threshold at degeneracy to be 7.3 mW for a pump-focused spot radius of 22-μm inside the crystal, this represents a damage intensity of 816 kW/cm$^2$ at 532 nm.

The damage occurred at the center of the crystal and appeared as a blackened track. The damage mechanism is thermal in origin, since chopping the pump to produce 28-μs pump pulses at 100-Hz repetition rate increases the damage intensity to 1 kW/cm$^2$. The low cw damage threshold prevented us from reaching the cw SRO threshold when we were using the 4.9%-doped MgO:LiNbO$_3$ crystal.

To operate the LiNbO$_3$ OPO with low signal loss and high idler loss, we replaced one of the flat mirrors in the cavity by a Ti:sapphire laser mirror that is highly reflecting between 870 and 1130 nm and has a 93% transmission peak centered at 1135 nm. To increase idler output coupling, we raised the crystal temperature so that the phase-matched idler wavelength tunes toward 1135 nm. Energy conservation then dictates that the signal wavelength moves toward 1001 nm. However, since all the mirrors remain highly reflecting at 1001 nm, the round-trip signal loss remains small. Thus the OPO operates continuously in the regime between high-Q DRO and SRO depending on the crystal temperature.

The threshold measurement was made with the pump beam chopped to produce pulses of 28-μs duration at a 100-Hz repetition rate. The 28-μs pulse duration was long enough compared with the OPO rise time that steady state was reached during each pulse. By chopping the pump beam we considerably reduced the thermal-focusing effects. During measurement the PZT-mounted flat mirror was continuously scanned, and the output was sent to a 1-m spectrometer with the slit width set for a resolution of 1 Å. We recorded the signal and idler wavelengths and the corresponding threshold at each temperature. The Ti:sapphire mirror transmission was measured at each idler wavelength. Figure 7 shows the measured threshold at each wavelength correlated with the matching idler loss. With 4 W of pump power, the OPO remained above threshold until the idler loss reached 83%. The curve shown in Fig. 7 was calculated with the formula

$$\frac{P_{\text{DROth}}}{P_{\text{highth}}} = \frac{2(1 - r_i)}{\alpha_i(1 + r_i)}$$

where $P_{\text{DROth}}$ is the threshold of the DRO with low signal- and idler-field losses, $\alpha_s$ and $\alpha_i$, and $P_{\text{highth}}$ is the OPO threshold with the same signal-field loss $\alpha_i$ but with arbitrary idler-field reflectance $r_i$. We find that, if we assume that $P_{\text{highth}} = 17$ mW and $\alpha_i = 0.4\%$, the calculated threshold agrees well with the measured values. By extrapolating to 100% idler loss, we find that cw MgO:LiNbO$_3$ SRO has a threshold of 8.5 W. This is within the range of high-power cw argon-ion lasers if the crystal can withstand the high average pump power. Unfortunately, when the pump beam was not chopped, the MgO:LiNbO$_3$ crystal was damaged at 3.1 W of input pump power. With a pump-focused spot radius of 22 μm inside the crystal, this represents a damage intensity of 816 kW/cm$^2$ at 532 nm. The damage occurred at the center of the crystal and appeared as a blackened track. The damage mechanism is thermal in origin, since chopping the pump to produce 28-μs pump pulses at 100-Hz repetition rate increases the damage intensity to 1 kW/cm$^2$. The low cw damage threshold prevented us from reaching the cw SRO threshold when we were using the 4.9%-doped MgO:LiNbO$_3$ crystal.

The experiment with LiNbO$_3$ prompted us to look for a nonlinear crystal that can withstand the high pump intensity necessary for cw SRO threshold. Surveying the available nonlinear crystals, we found KTP to be the most promising material since it has both a high damage threshold and a relatively high nonlinear coefficient of 3 pm/V.$^{20,22,23}$ To determine cw SRO threshold by use of KTP, we conducted threshold experiments similar to those for LiNbO$_3$. For these threshold experiments we used the same bow-tie cavity as before, except that in this case all four mirrors were highly reflecting in the range between 970 and 1160 nm.

![Fig. 7. Measured LiNbO$_3$ DRO threshold as a function of round-trip idler-power loss. We observed minimum threshold of 29 mW. The curve is calculated by the threshold-ratio formula [Eq. (14)], assuming a 17-mW threshold for round-trip signal- and idler-power loss of 0.8%. The extrapolated SRO threshold is 8.5 W.](image-url)
Theoretical threshold, as calculated from the Boyd-Kleinman\textsuperscript{24} threshold formula, is given by $P_{th} = a_2 a_i \times 2897 \text{ W}$, where $a_2$ and $a_i$ are round-trip signal- and idler-power losses. In the threshold calculation, we assumed that the focusing parameter $h(B = 0, \xi = 0.86) = 0.72$ and that $d_{eff} = 3 \text{ pm/V}$ (Refs. 20, 22, and 23) for KTP crystal.

Since the KTP OPO does not operate at degeneracy when it is noncritically phase matched at room temperature, we cannot assume that the signal and idler losses are the same (i.e., $a_2 \neq a_i$). To determine the ratio of round-trip signal to idler losses, we measured the orthogonally polarized signal and idler output power ratio that leaked out from the highly reflecting cavity mirrors. Using the measured power ratio of 2.29, one can obtain the loss ratio from the formula\textsuperscript{24}

$$\frac{P_s}{P_i} = \frac{\gamma_s (\gamma_i + \mu_i)}{\gamma_i (\gamma_s + \mu_s)}, \quad (15)$$

where $\gamma_s$ and $\gamma_i$ are the output coupling loss of the signal and idler waves, respectively, and $\mu_s$ and $\mu_i$ are other spurious signal and idler losses resulting from crystal absorptions, etc. In our case, since we are operating not too far from degeneracy and also since the angle of incidence upon the mirror in the bow-tie cavity is near normal, $\gamma_i = \gamma_s$. In addition, since the round-trip loss of the OPO cavity with the crystal removed is measured to be less than 0.2%, we can assume that $\gamma_s \ll \mu_i$ and $\gamma_i \ll \mu_s$. On the basis of these assumptions, $\mu_s = 2.29 \mu_i$. This is expected because the narrow-band antireflection coating is centered at a signal wavelength of 1090 nm. When the pump beam was focused into a 22-\mu m beam waist inside the crystal, we measured a minimum threshold of 30 mW. The threshold ratio between the signal and idler-power losses is 0.7% and 1.6%, respectively.

Since the signal and idler waves are orthogonally polarized, they can be discriminated by insertion of polarizing elements, such as Brewster plates, inside the OPO cavity. We introduced one or more 3-mm-thick BK7 Brewster plates into the cavity to produce additional loss for the idler while adding minimal loss for the signal wave. The refractive index of BK7 glass is 1.517 at the signal wavelength of 1.09 \mu m, and the Brewster angle is 56.6 deg. Each passage through the Brewster plate introduces a 28.6% loss for the $S$-polarized idler waves. We inserted one and two BK7 Brewster plates into the cavity, and they introduced 29% and 49% loss, respectively, for the idler beams. The OPO threshold power with one and two Brewster plates in the OPO cavity was 707 mW and 2.79 W, respectively. The threshold ratio between the low-loss DRO and OPO with one Brewster plate agrees well when we assume a fixed round-trip signal-power loss of 0.7%. However, with two Brewster plates inserted, the measured threshold is considerably higher than expected. We believe this is due to the difficulty in aligning two Brewster plates for minimum signal loss. Extrapolating from the measured threshold for the DRO without Brewster plates and the DRO with one Brewster plate, we found the threshold for a KTP cw SRO is 7.8 W. In these threshold measurements for KTP OPO the full cw pump power was incident upon the crystal. With maximum pump power of 4 W focused to a beam waist radius of 22 \mu m, we observed no crystal damage.

The KTP DRO threshold experiments revealed the expected cw SRO threshold, and also they showed that the KTP crystal can withstand high average pump power. The extrapolated threshold of 7.8 W, however, exceeds available pump power from the resonantly doubled Nd:YAG laser. To reduce the pump threshold, we chose a standing-wave SRO cavity such that the pump as well as the idler is nonresonantly reflected back through the KTP crystal. As was pointed out earlier by Bjorkholm et al.\textsuperscript{25} and as explained in Appendix A, the SRO threshold can be reduced by as much as a factor of 4 when both the pump and the idler are nonresonantly reflected back through the crystal. The cavity design with pump and idler feedback is shown in Fig. 8. The SRO cavity is a three-mirror standing-wave cavity that consists of two curved mirrors (each with a 5-cm radius of curvature) and one flat mirror. All three mirrors are highly reflecting from 970 to 1160 nm and highly transmitting at 532 nm. Since it is difficult to obtain a mirror coating that is highly reflecting at both the pump and the idler wavelengths, we used a separate mirror that is highly reflecting at the pump wavelength and is situated outside the OPO cavity to reflect the pump back through the crystal. Singly resonant operation is achieved by insertion of an intracavity LiNbO$_3$ Brewster prism that spatially separates the signal and idler beams. The prism is designed such that the $P$-polarized signal beam at 1090 nm is incident upon the prism at Brewster's angle and with ordinary polarization in the LiNbO$_3$ prism, whereas the $S$-polarized idler beam at 1039 nm traverses the prism with extraordinary polarization. Because of the large refractive-index difference between the ordinary and extraordinary axes of LiNbO$_3$, the signal and idler beams emerge from the prism separated by 8 deg. The large separation permits the idler beam to be completely rejected from the cavity. This ensures SRO operation.

The KTP crystal that we used is similar to that used in the DRO experiment, except that the length was increased.

![Fig. 8. Cw KTP SRO experimental setup. Mirrors M1 and M2 each have a 5-cm radius of curvature. Mirror M3 is flat. Mirrors M2 and M3 and the pump mirror are mounted on PZT's. The pump and idler waves are reflected back through the crystal. The LiNbO$_3$ Brewster prism rejects the idler wave when one achieves true SRO operation.](image-url)
Yang et al.

Fig. 9. Generated SRO-idler power versus pump power. The threshold is 1.07 W. At 3.2 W of 532-nm pump power, the nonresonant idler output power is 1.1 W for an idler conversion efficiency of 33%.

to 1 cm. With a measured pump beam waist radius of 21 μm and a calculated signal beam waist radius of 31 μm, both the signal and the pump have confocal parameters that are equal to the crystal length. The finesse of the OPO cavity that was measured along the ordinary axis of KTP was 743 at 1064 nm, implying a round-trip power loss of 0.85%. The actual loss at signal wavelength of 1090 nm is expected to be less since the crystal antireflection coating is centered at 1090 nm. Assuming a 0.85% round-trip power loss and confocal focusing of pump and signal, we calculated a SRO threshold of 7.2 W, using the SRO threshold expression given in Ref. 26. By nonresonantly reflecting both the pump and idler back through the crystal, one can reduce the SRO threshold by 4 times to 1.8 W. We experimentally achieved a threshold of 1.4 W at optimum relative phase among the three waves. Figure 9 is a plot of the SRO-idler output power versus input pump power. We extrapolated the plotted idler power from actual measured values after the beam emerged from the prism, taking into account the 62% reflection loss that the S-polarized idler wave incurred on the LiNbO₃ prisms two surfaces. This reflection loss can, in principle, be eliminated, and the total idler power that is generated can be outcoupled. With 3.2 W of pump power, the maximum idler power generated was 1.07 W. The corresponding signal output from each of the three mirrors is 12 mW. This represents a combined signal-plus-idler output power conversion efficiency of 34.6%.

B. Observed OPO Amplitude and Frequency Stability

In conjunction with the threshold experiments, we also investigated the effect of increasing idler loss on the amplitude and frequency stability of OPO's. By lowering the idler-cavity Q, we expect to reduce the intensity modulation of the OPO output as the cavity length changes. This is confirmed in the case of KTP OPO with increasing idler loss. Figure 10 shows the output intensity of the KTP OPO with three different idler losses as one scans the cavity length by driving the PZT-mounted cavity mirror. The ramp voltage driving the PZT is superimposed in each graph. The tuning rate of the PZT is 8.3 nm/V. With 100–200 V applied to the PZT in each case, the cavity length is scanned over 1 μm. As is shown in Fig. 10(a), the low-loss KTP OPO output intensity varies by as much as 75% when the OPO is pumped at 1.38 times above threshold. With the insertion of one Brewster plate the peak modulation depth is reduced to 43% for pumping at 1.35 times above threshold as shown in Fig. 10(b). In the limit of 100% idler loss, Fig. 10(c) shows the SRO output when the flat mirror, M3 (see Fig. 8), is driven. Since the idler beam is rejected by the LiNbO₃ prism and emerges from the cavity before it reaches the flat mirror, only the resonant signal wave is reflected by mirror M3. In this case we would expect a constant SRO output intensity regardless of cavity-length setting. This is partially confirmed in Fig. 10(c), which shows the SRO output for pumping at 1.3 times above threshold. As the figure shows, amplitude fluctuation of the OPO output as a re-

Fig. 10. OPO temporal output recorded versus cavity-length change. The superimposed ramp is the voltage drive for the PZT. With 100 V applied to the PZT, the cavity length changes by 0.9 μm. (a) Output intensity from a high-Q KTP DRO. As the cavity length is scanned, the output intensity modulation reaches a peak of 75% for pumping at 1.38 times above threshold. (b) Output intensity fluctuation of a KTP OPO with one Brewster plate inserted to discriminate against the idler wave. Note the reduced intensity modulation of 43%. (c) SRO output intensity the position of mirror M3 is scanned (see Fig. 8). Fast amplitude fluctuations have disappeared in this case. Slow amplitude variations are due to pump-mirror position fluctuations.
result of a nanometer change in cavity length disappears. However, output power fluctuates at a millisecond time scale. These slow amplitude changes are due to instability in the feedback-pump-mirror position, which in turn results in fluctuating SRO thresholds as discussed in Appendix A.

When PZT scanning is stopped, the cavity length continues to fluctuate by a few tens of angstroms at a 120-Hz rate because of acoustic pickups. The effect of such a fast cavity-length perturbation is to cause power instabilities on the OPO output accompanied by random jumps in oscillation frequency. This is especially severe for a DRO with high-finesse signal and idler cavities because of the small tolerance range. We observed almost 100% intensity modulation in the output of the low-loss KTP DRO even when the OPO was pumped at 3.6 times above threshold. Corresponding to the rapid intensity fluctuation, we saw the signal- and idler-oscillation frequency hop every few milliseconds. In contrast, when the Brewster plate is inserted into the OPO cavity, the free-running output amplitude becomes more stable. For pumping at 2.36 times above threshold, the free-running output intensity modulation is reduced to 32%. Also, the signal and idler frequency remain at one particular axial mode pair for a much longer time, as we observed with a Fabry–Perot interferometer. Typically, for pumping at 2.36 times above threshold, single-axial-mode operation was stable for hundreds of milliseconds. Note that the cavity-length fluctuation is the same with and without insertion of the Brewster plate. The much stabler output from the OPO with an intracavity Brewster plate is due entirely to the lowering of the idler-cavity Q.

We also found that, as the pumping level is increased for OPO's with one Brewster plate, output intensity and frequency fluctuation decrease. This behavior is illustrated in Fig. 11, which shows the output power of the KTP DRO with one Brewster plate and with increasing pump inputs. The fluctuation decreases from 100% to 23.1% as the pumping level is increased from near threshold to 2.5 times above threshold. The steplike change in the OPO amplitude, as shown in Fig. 11(c), corresponds to axial-mode hops. On average, single-axial-mode operation can be maintained for a few hundred milliseconds for pumping at 2.5 times above threshold.

With insertion of an additional Brewster plate to lower the idler-cavity Q, axial mode hops become more frequent. When the limit of 100% idler loss is reached, single-axial-mode operation is consistently observed, even though small-cavity-length fluctuation persists. This is expected since, as Smith pointed out, the SRO will hop to an adjacent axial mode when the resonant-signal-cavity resonance is shifted to $\pm 1/2$ free-spectral range away from the parametric-gain center. For a standing-wave cavity this implies an axial-mode-hop tolerance of half of the resonant signal wavelength, which is orders of magnitude greater than the tolerance range for DRO's with high-Q signal and idler cavities. A typical idler spectrum as observed with a 300-MHz-free-spectral-range scanning Fabry–Perot interferometer is shown in Fig. 12. The measured idler linewidth of a few megahertz is Fabry–Perot-instrument resolution limited. The actual idler linewidth is potentially as narrow as the pump linewidth of 20 kHz.

In addition to having a larger axial-mode-hop tolerance to cavity-length fluctuation, the SRO is also less susceptible to pump-frequency fluctuation. To determine tolerance range for pump-frequency fluctuation, we note that, as the pump frequency is tuned, the signal frequency in a

![Fig. 11](image1.png)

![Fig. 12](image2.png)
SRO remains fixed at cavity resonance, whereas the idler frequency is free to vary to satisfy the energy-conservation condition. An axial mode hop will occur when the signal frequency is \( \pm 1/2 \) free spectral range away from the parametric-gain center. The pump-frequency change required for the parametric-gain peak to be shifted by \( \pm 1/2 \) free spectral range from the signal-cavity resonance is acquired for the parametric-gain center. The pump-frequency change required for the parametric-gain peak to be shifted by \( \pm 1/2 \) signal-cavity free spectral range. When the pump frequency was recorded as we varied \( \Delta T \) between zero and \( 2\pi \), we observed an axial-mode hop will occur when the signal frequency was recorded as we varied \( \Delta T \) between zero and \( 2\pi \). We observed that, at 2.6 times above threshold and beyond, the tuning range asymptotically approached 0.76 nm, which is close to the calculated tuning width of 0.85 nm.

4. CONCLUSION

We have theoretically and experimentally examined the behavior of OPO's with low signal loss and arbitrary idler loss. By assuming a longitudinally invariant signal field, we calculated pump depletion and conversion efficiency for OPO's with arbitrary idler feedback. In the limits of near-unity idler reflectance and no idler reflectance, the derived expression agrees with previous expressions for DRO's and SRO's. In particular, the 100\% signal-photon-conversion efficiency point is seen to shift slowly from pumping at four times above threshold to pumping at \( (\pi/2)^7 \) times above threshold as idler loss is increased. This shift is more pronounced as \( R_d \) approaches zero.

Using the approximate analytical solutions, we examined amplitude and frequency stability of OPO's with a high-Q signal cavity an arbitrary-Q idler cavity. We showed that, for a given pumping level, the range over which an OPO remains above threshold as signal- and idler-cavity resonances are detuned from each other increases as idler loss is increased. For fixed idler reflectance, the tolerance range also widens as one increases the pumping level. The larger phase-detuning tolerance range is accompanied by reduced modulation of the OPO output amplitude when the signal and idler cavity resonances are detuned from exact overlap.

Using the approximate analytical solutions for signal power, we also examined the frequency instabilities of OPO's at different pumping levels and with various idler reflectivities. We found that, for residual idler-power reflectivity of less than 1\% and pumping at 2.5 times above threshold, frequency jumping of OPO's in the presence of cavity-length fluctuation is substantially reduced. This has important implications for practical realizations of cw SRO's, since it implies that extraordinary effort need not be made to eliminate all residual idler reflectivity to achieve the frequency stability of the SRO. For example, one can readily construct a DRO with weak idler feedback by use of a four-mirror ring cavity in which each mirror has 90\% power transmission at the idler wavelength. After the idler power traverses once through the cavity, only 0.01\% of it is fed back. Figure 5 shows that when this DRO is pumped at two times above threshold it will have the same frequency stability as SRO.

Complementing the theoretical discussion, we conducted experiments in which LiNbO\(_3\) and KTP OPO's operated as DRO's with a fixed low signal loss and increased idler loss, including the SRO case with 100\% idler loss. Careful threshold measurement confirmed the expected DRO threshold progression as the SRO limit is approached, and by observation of OPO temporal and spectral output we confirmed the improved stability of the cw OPO as idler loss is increased toward the SRO limit.

APPENDIX A: DERIVATION OF THE SRO THRESHOLD EQUATION FOR NONRESONANT PUMP AND IDLER REFLECTION

Here we present the derivation of the SRO threshold with both the pump and the idler fields nonresonantly re-
flected. Previously, Bjorkholm et al. treated both the SRO and the DRO cases when only the pump field was nonresonantly reflected. They mentioned the possibility of nonresonantly reflecting both the pump and the idler fields for the SRO but did not present analytic threshold expressions. We show below that the threshold for the case of nonresonant reflection of pump and idler waves is identical to that of the DRO case with pump reflection when the reflection coefficients for pump and idler are equal to unity.

The starting point is again the coupled-wave equations for the pump and idler fields, assuming that the resonant signal field is spatially invariant:

$$\frac{\partial E_i(z)}{\partial z} = i\kappa_i E_p(z) E_{s*}(z) \exp(i\Delta k z), \quad (A1a)$$
$$\frac{\partial E_p(z)}{\partial z} = i\kappa_p E_i(z) E_{s*}(z) \exp(-i\Delta k z). \quad (A1b)$$

Since we are dealing with SRO’s, the initial idler field is zero. Using this boundary condition, we find that the solutions for the forward-going idler and pump fields are

$$E_{i+}(z) = k_i \frac{E_p(0) E_{s*}(0) \sin(\Theta_0)}{i} \exp(i(\Delta k z/2 + \Delta \varphi^+)), \quad (A2a)$$
$$E_{p+}(z) = E_p(0) \left[ \cos(\Theta_0) + \frac{i\Delta}{2\Gamma} \sin(\Theta_0) \right] \exp(-i\Delta k z/2), \quad (A2b)$$

where $+$ denotes forward-going waves and $\Delta \varphi^+$ represents the initial phase difference among the three waves. $\Delta \varphi^+ = \varphi_{s+}^+ - \varphi_{i+}^+ - \varphi_{s-}^+$, where $\varphi_{s+}^+$, $\varphi_{i+}^+$, and $\varphi_{s-}^+$ are initial phases for the pump, signal, and idler at $z = 0$, and $\Gamma^2 = \kappa_i \kappa_p |E_{s0}|^2 + (\Delta k/2)^2$. For the backward-going waves, the initial conditions are

$$E_{i-}(z' = 0) = \sqrt{R_i} \exp(i\Delta \varphi^-) E_{i+}(z = l_i),$$
$$E_{p-}(z' = 0) = \sqrt{R_p} \exp(i\Delta \varphi^-) E_{p+}(z = l_i), \quad (A3)$$

where $z' = l_i - z$, $R_i$ and $R_p$ are mirror-power reflection coefficients of the idler and pump, respectively, and $\Delta \varphi^- = \Delta k l_i + \Delta \varphi^+ + \Delta \varphi$ and $\Delta \varphi = \varphi_{p+} - \varphi_{i+} - \varphi_{s-}$ represent the phase-shift differences incurred on reflection from the mirrors.

With these boundary conditions the solutions for the backward-going idler and pump fields are

$$E_{i-}(z') = \left\{ \begin{array}{l}
\frac{i\sqrt{R_i}}{i} \sin(\Theta_0) \left[ \cos(\Theta_i') - \frac{i\Delta k}{2\Gamma} \sin(\Theta_i') \right] \\
+ i\kappa_i \frac{\sqrt{R_i}}{1} \left[ \cos(\Theta_i) + \frac{i\Delta k}{2\Gamma} \sin(\Theta_i) \right] \\
\times \sin(\Theta_i') \exp i\Delta \varphi \\
\times E_{s0} E_{p0} \exp(i(\Delta k z'/2 + \Delta k l_i/2 + \Delta \varphi^+)),
\end{array} \right. \quad (A4a)$$
$$E_{p-}(z') = \left\{ \begin{array}{l}
\sqrt{R_p} \left[ \cos(\Theta_i) + \frac{i\Delta k}{2\Gamma} \sin(\Theta_i) \right] \\
\times \left[ \cos(\Theta_i') + \frac{i\Delta k}{2\Gamma} \sin(\Theta_i') \right] \\
- \frac{k_i \kappa_p \sqrt{R_i} |E_{s0}|^2 \sin(\Theta_i) \sin(\Theta_i') \exp -i\Delta \varphi \\
\times E_{p0} \exp -i\Delta k z'/2 + \Delta k l_i/2),
\end{array} \right. \quad (A4b)$$

Energy conservation dictates that

$$n_p |E_{p0}|^2 = (1 - R_p) |E_{s+}^*(z = l_i)|^2 - |E_{p-}(z' = l_i)|^2$$
$$= (1 - R_i) n_i |E_{i+}(z = l_i)|^2 + n_i |E_{i-}(z' = l_i)|^2 + 2a_s n_i |E_{s0}|^2, \quad (A5)$$

where $2a_s$ is the round-trip signal-power loss. Substituting the solutions for forward- and backward-traveling pump and idler fields into the energy-conservation equation above, we obtain

$$n_p |E_{p0}|^2 \left[ \sin^2(\Gamma l_i) - R_s \sin^2(\Gamma l_i) \right.$$  
$$+ (R_i + R_p) \cos^2(\Theta_i) \sin^2(\Theta_i) + (R_i + R_p) \left( \frac{\Delta k}{2i} \right)^2$$
$$+ 2\sqrt{R_i R_p} \cos^2(\Theta_i) \sin^2(\Theta_i) \cos(\Delta \varphi)$$
$$- 2 \left( \frac{\Delta k}{2i} \right)^2 \sqrt{R_i R_p} \cos^2(\Theta_i) \cos(\Delta \varphi)$$
$$- 2 \Gamma \sqrt{R_i R_p} \cos(\Theta_i) \sin(\Theta_i) \sin(\Theta_i') \sin(\Delta \varphi)$$

$$= n_p 2a_s \frac{\Gamma^2}{k_i \kappa_i} \cdot \quad (A6)$$

Near threshold $E_{p0} = 0$, which implies that $\Gamma l_i = \Delta k l_i/2$. Substituting these approximations into Eq. (A6), we find that the threshold equation becomes

$$|E_{p0}|_{th}^2 = \frac{2a_s}{k_i \kappa_i} \left( \frac{\Delta k l_i/2}{2} \right)^2 \sin^2(\Delta k l_i/2) \times \left[ \frac{1}{1 + R_i + 2\sqrt{R_i R_p} \cos(\Delta \varphi) + \Delta \varphi} \right]. \quad (A7)$$

When $R_i = R_p = 1$, the threshold-reduction expression [the bracketed quantity in Eq. (A7)] is identical to that of a DRO with nonresonant pump reflection. Assuming that $R_p = R_i = 1$, we find that the maximum-threshold-reduction factor is 4 and the minimum-threshold-reduction factor is 2.1. For a $\Delta \varphi$ change of $\pm \pi$, the parametric-gain peak shifts between $\pm 2.33$ rad.

REFERENCES AND NOTES


19. The MgO:LiNbO₃ crystal is from Crystal Technology, Inc., Mountain View, Calif.


