

Texture Based Classification Of Seismic Image Patches Using Topological Data Analysis

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Oral Presentation Details

Abstract Number 640

Deep Learning and Data Analytics – Methods and Applications I

Thursday, June 6, 2019

9:20 AM - 9:45 AM

Room 11

Abbreviations

The following abbreviations will appear in this talk in various places.

TDA: Topological Data Analysis

PH: Persistent Homology

I will explain them in this talk.

ML: Machine Learning

SVM: Support Vector Machines

RF: Random Forest

NN: Neural Network

CNN: Convolutional Neural Network

These are machine learning specific terminologies. I'll assume working knowledge of these methods.

TDA applications in geosciences / oil & gas

- “Topological data analysis and machine learning for recognizing atmospheric river patterns in large climate datasets”, Muszynski et al., 2019, Geoscientific Model Development
 - “Topological Data Analysis of Oil and Gas Petrophysical Data”, Andrea Cortis, 2015, Ayasdi
 - “Topological Data Analysis to Solve Big Data Problem in Reservoir Engineering: Application to Inverted 4D Seismic Data”, Alfaleh et al., 2015, SPE Annual Technical Conference and Exhibition
-
- The first first paper uses the *same topological technique as us (persistent homology)*, but in a very different way.
 - The last two papers use a *different topological technique (mapper)* for exploratory data analysis.

Our contribution & modest claim

- This is quite possibly the first application of TDA based methods that use **persistent homology** for a seismic imaging application.

More generally...

- This is quite possibly the first application of TDA based methods that use **persistent homology** for a problem relevant to the oil and gas industry.

Seismic textures

- In a seismic image, different lithologies often have very different “visual appearances”.
- For example, a salt bodies appear different from sedimentary sections.
- The trained human eye of seismic interpreters can easily detect these differences.

Seismic interpreter’s job (simplistic viewpoint)

Segment seismic images based on a combination of

- Seismic texture
- Historical memory
- Geological knowledge

Related work

Pre-ML age

- **Seismic attributes:** Developing attributes for easier seismic interpretation.

Post-ML age

- **Seismic texture based ML:** Semi to full automation of seismic interpretation, some directly using textural attributes.

Most relevant

- **Image segmentation based on seismic patch classification:** Chevitarese et al. (2018)[‡] perform seismic image segmentation using 2D patches, and CNNs.

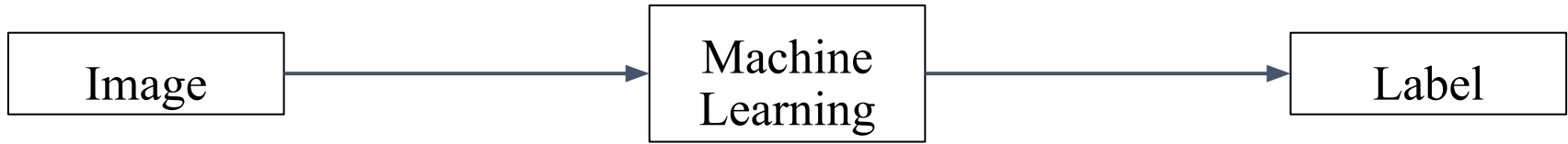
[‡] Chevitarese, D.S., Szwarcman, D., Brazil, E.V. and Zadrozny, B, “Efficient Classification of Seismic Textures”, 2018, International Joint Conference on Neural Networks, IEEE, 1–8.

Seismic texture classification

What we want



A popular strategy



Our roadmap



ML challenges — texture classification

Machine learning with images

- Treating each pixel as a feature is not well-suited for classification tasks.
- Relationship between neighboring pixels matter.
- Most existing methods use derived features (either hand-crafted or learned).

Challenges of texture classification

- Areas with similar “look and feel”. This can be hard to quantify.
(Think: I know it when I see it, but can’t describe exactly what I’m seeing.)
- Repetitive / recurrent (but not necessarily periodic).
- Scale, translation, rotation, deformation invariant.
- What kind of features can capture these properties?

Why topology?

Features of “algebraic topology”

- Study of topological spaces up to homotopy equivalence (continuous deformation).
- Identifies quantities that are **scale**, **translation**, **rotation**, and **deformation** invariant.

Topological data analysis

- Tools to understand topology in data.
- Turns topological information into features (real numbers), that computers can process.
- Adapts tools from algebraic topology to study discrete point cloud data.



Continuous deformation of a coffee mug to a donut

Simplicial Complex

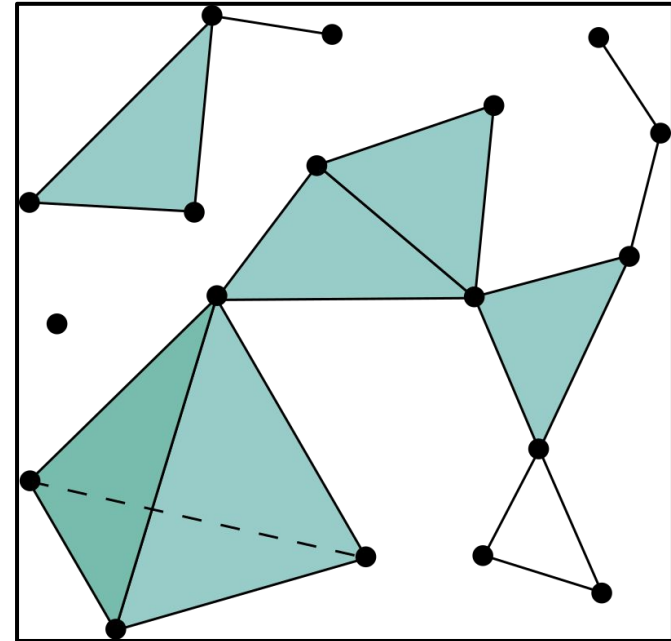
The key topological object (relevant to our work) is a **simplicial complex**. Abstractly this is a triangulation of a topological space.

Definition of a simplicial complex

A set of simplices* (points, lines, triangles, and higher dimensional objects) that satisfy the following two properties:

- Every face of a simplex is also a simplex.
- Intersection of any two simplices is a face of each simplex.

A simplicial complex



Source: Wikipedia

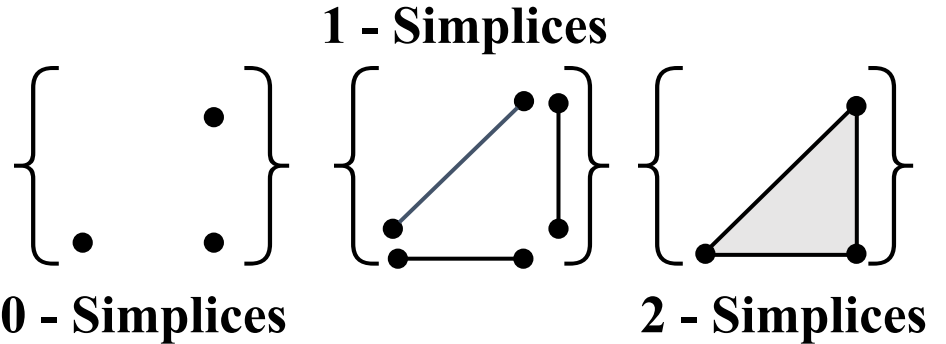
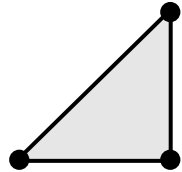
* “Simplices” is the plural of the word “simplex”.

Simplices of a simplicial complex

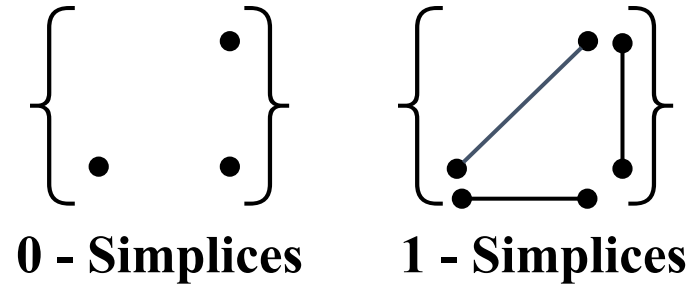
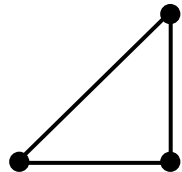
Topological space

Simplicial complex

Filled triangle



Triangle with a hole



Homology of a simplicial complex

Consider formal linear combinations of vertices / edges / triangles in a simplicial complex X of dimension 2. This produces a set of vector spaces $C_k(X)$ ($k = 0$ for vertices, $k = 1$ for edges...). There are linear boundary maps $\partial_k : C_k(X) \rightarrow C_{k-1}(X)$

$$\partial(\bullet \longrightarrow \circ) = (\circ) - (\bullet)$$

$$\partial\left(\begin{array}{c} \bullet \\ \diagdown \quad \diagup \\ \bullet \quad \circ \end{array}\right) = \bullet \longrightarrow \circ + \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array} + \begin{array}{c} \bullet \\ \diagup \\ \circ \end{array} = \begin{array}{c} \bullet \\ \diagdown \quad \diagup \\ \bullet \quad \circ \end{array}$$

with the property that $\partial \circ \partial = 0$.

The k^{th} **homology group**, and the k^{th} **Betti number** are defined as

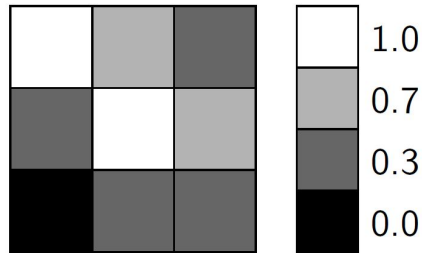
$$H_k(X) = \ker(\partial_k) / \text{img}(\partial_{k+1}), \quad \beta_k = \dim H_k(X).$$

- β_0 counts clusters that are not connected (called **connected components**).
- β_1 counts cycles that are not boundaries (called **holes**).

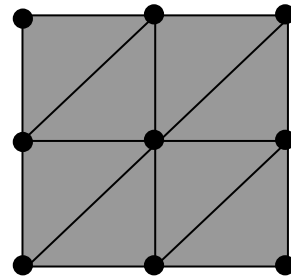
Turning an image into a topological space

One way to do this is to form a simplicial complex as follows:

- Pixels become points in the space
- Adjacent pixels are connected by an edge
- Diagonal edges added by **Freudenthal triangulation**
- 3 adjacent pixels are spanned by a triangle



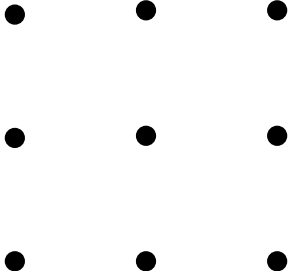
3 x 3 image



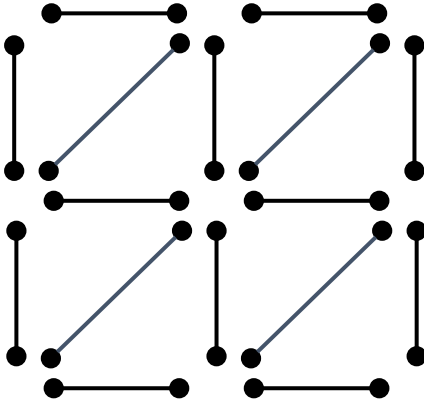
Freudenthal triangulation

Resulting simplicial complex

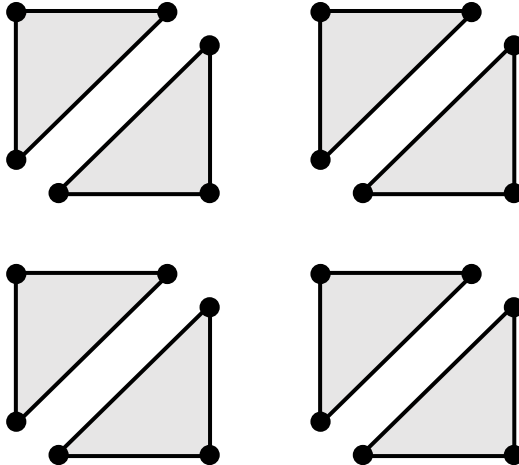
0 - Simplices



1 - Simplices

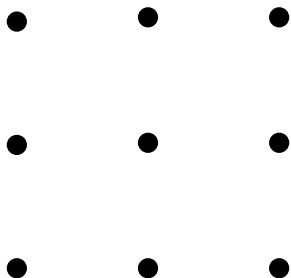


2 - Simplices

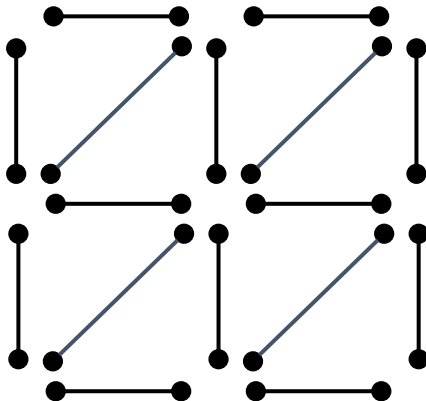


Need for filtered topological spaces

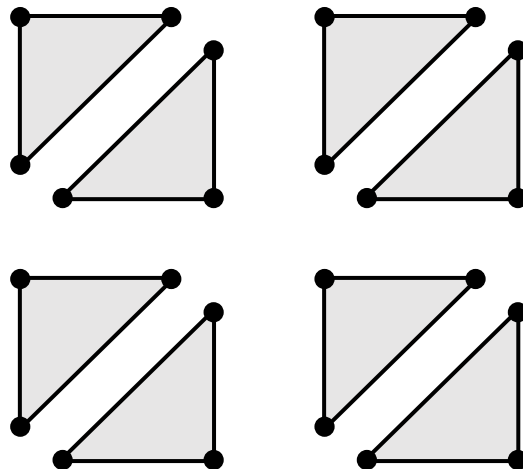
0 - Simplices



1 - Simplices



2 - Simplices

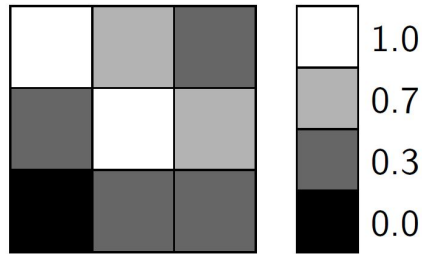


Problem: Topological spaces created from all pixels in the image always generate exactly the same simplicial complex — useless for classification.

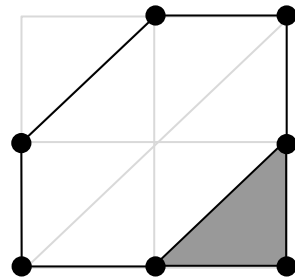
Filtered topological spaces

A more interesting topological space:

- Choose some pixel value w .
- Only points with pixel values $\leq w$ are used.
- Only edges with both endpoints are included.
- Only triangles with boundary edges are included.



3 x 3 image



Topological space at $w = 0.7$

Filtration and persistence

Key ideas

- Create a sequence of nested topological spaces.
- Track homology changes across the topological spaces.
- Turn this information into quantifiable numbers.

Nested topological spaces or Filtration

We use a *sublevel set filtration*.

- Vary pixel value w from minimum to maximum pixel value.
- For each w , we construct a filtered topological space X_w .
- Property: $u \leq w \Rightarrow X_u \subseteq X_w$.

Persistent homology

Persistent homology is the tool that quantifies how homology changes across a filtration.

Input: A filtration $\{X_w\}_w$.

Output: A collection of pairs of real numbers for each homology dimension k , calculated as

$$PH_k(\{X_w\}_w) = \{(b_j, d_j)\}_j.$$

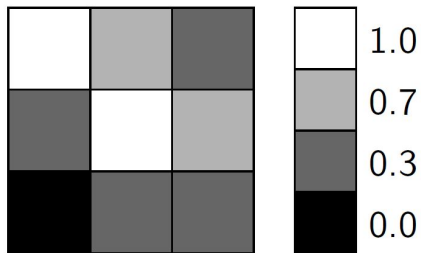
These are called **birth-death pairs**, and track how homology changes over the filtration.

Properties:

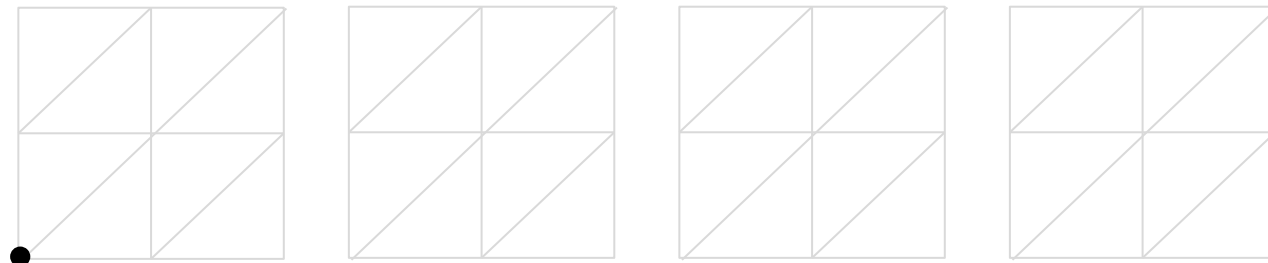
- Homotopy invariant (deformation, rotation, translation).
- Stable to perturbations of pixel values.

Example of how a filtration is built

Example Image



Corresponding Filtration

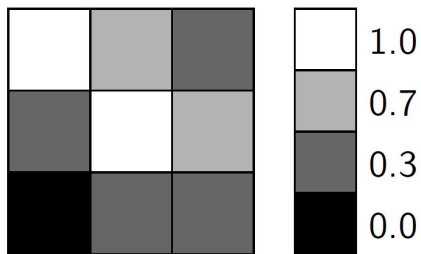


$$w = 0$$
$$\beta_0 = 1, \beta_1 = 0$$

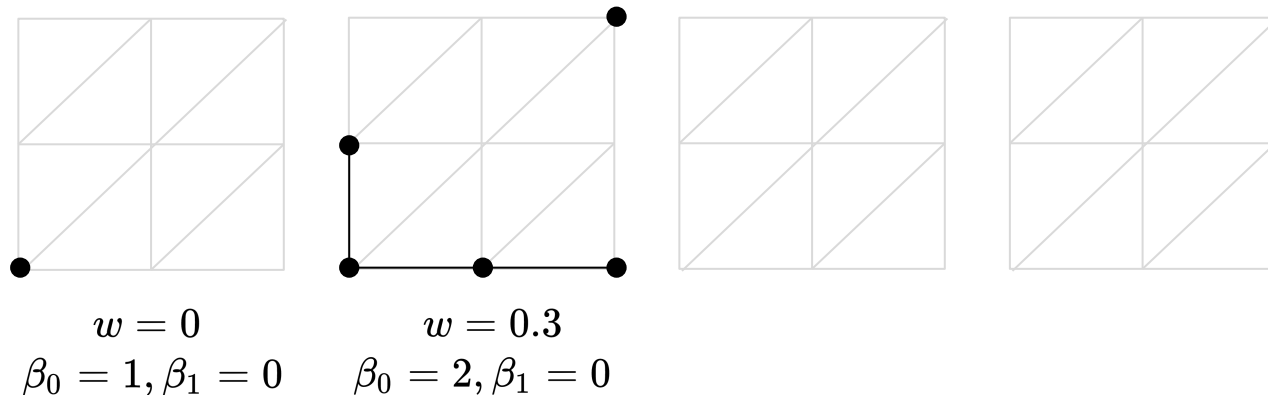
At $w = 0$, a single point appears, and H_0 homology is born.

Example of how a filtration is built

Example Image



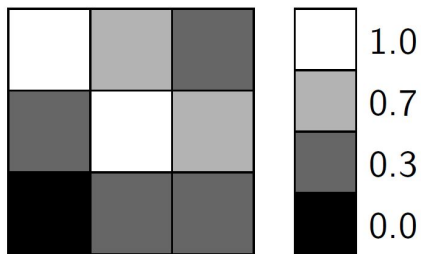
Corresponding Filtration



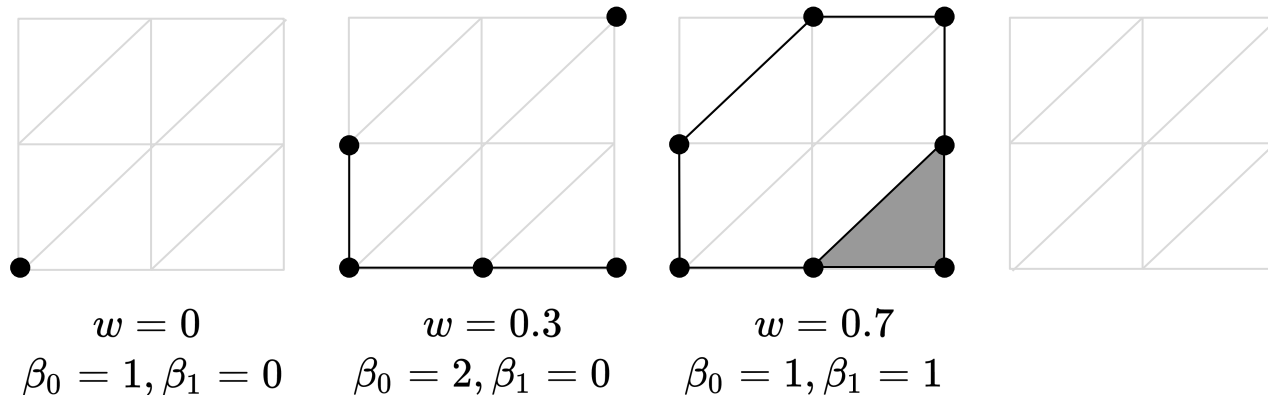
At $w = 0.3$, several points connect to the first point, and a new component emerges. H_0 homology is born one more time.

Example of how a filtration is built

Example Image



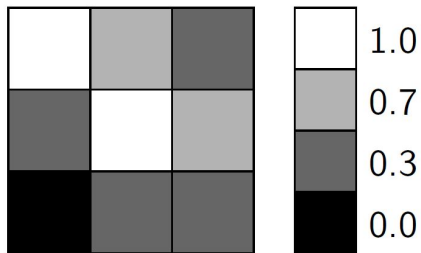
Corresponding Filtration



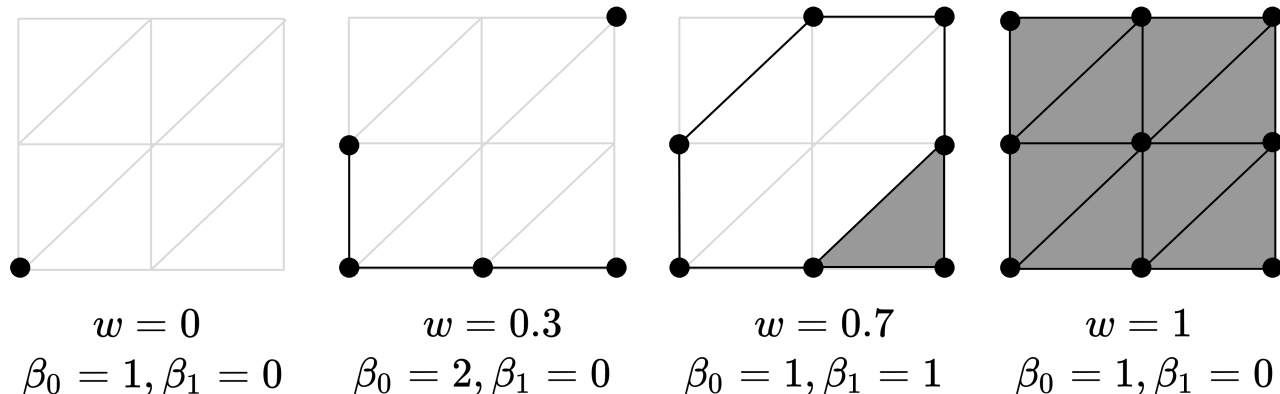
At $w = 0.7$, the two components join, and a hole appears. We also see our first triangle. So H_0 homology has died, while H_1 homology is born.

Example of how a filtration is built

Example Image



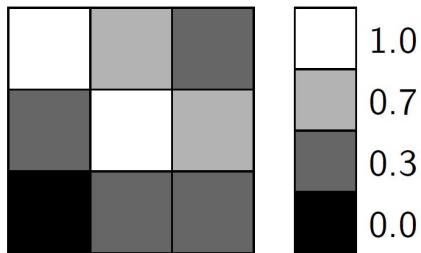
Corresponding Filtration



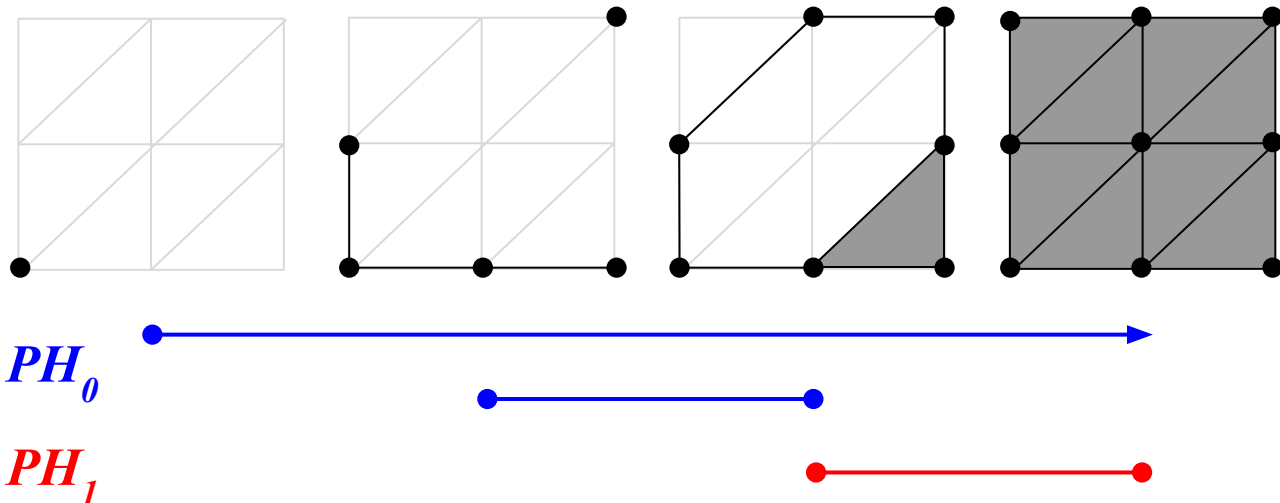
At $w = 1$, all points are now present, and all edges and triangles fill in the space. The hole has now disappeared, and so H_1 homology has died.

Example of how a filtration is built

Example Image



Corresponding Filtration



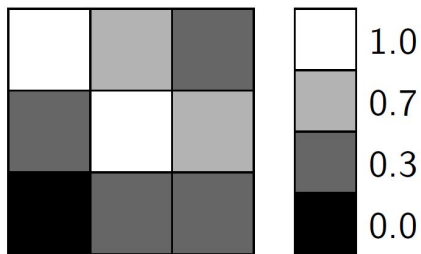
Persistence Barcode:

Information about how components appear and merge is encoded in PH_0 .

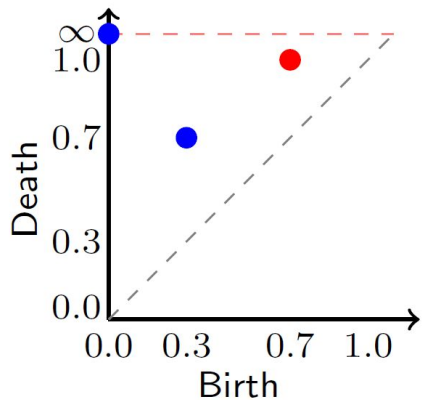
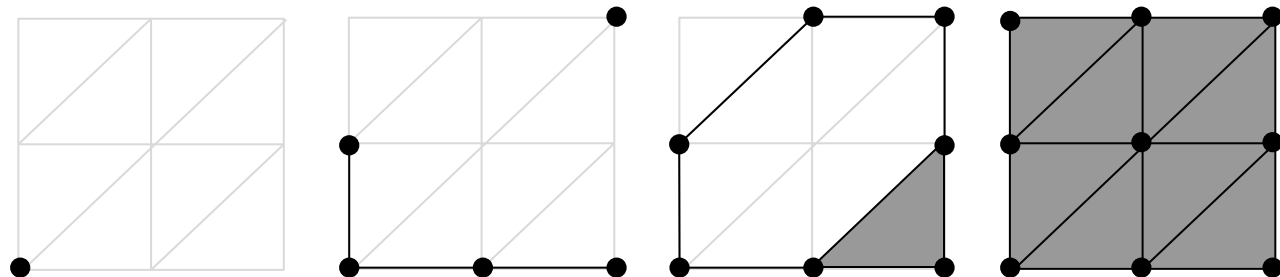
Information about how 1D holes appear and fill is encoded in PH_1 .

Example of how a filtration is built

Example Image



Corresponding Filtration



PH_0

PH_1

Persistence Diagram:

The start and endpoints of the barcode are plotted in the plane.

Each point is referred to as a **birth-death** pair.

Applications on a real 2D dataset

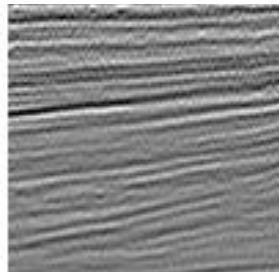
For the rest of this talk we will use the LANDMASS[†] dataset to demonstrate the workflow and our results. This is a publicly available dataset of **two sets** of labeled 2D seismic image patches, each with **4 classes**.

	LANDMASS-1	LANDMASS-2
Image Size (pixels)	99 x 99	150 x 300
Class Names	Number of Images	Number of Images
1. Horizons	9385	1000
2. Chaotic Horizons	5140	1000
3. Fault Patches	1251	1000
4. Salt Domes	1891	1000

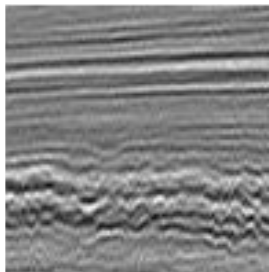
[†][Alaudah, Y., Wang, Z., Long, Z. and AlRegib, G. \[2015\] LANDMASS Seismic Dataset.](#)

Sample images (images not to scale)

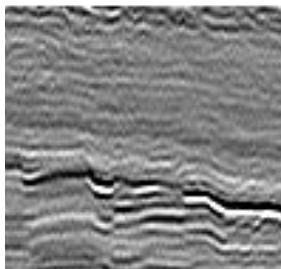
LANDMASS-1



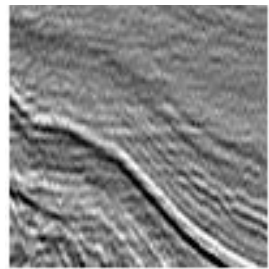
Horizons



Chaotic Horizons

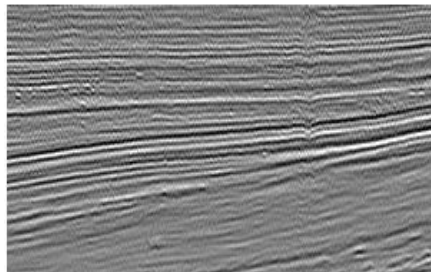


Fault Patches

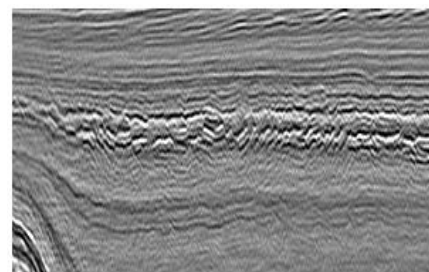


Salt Domes

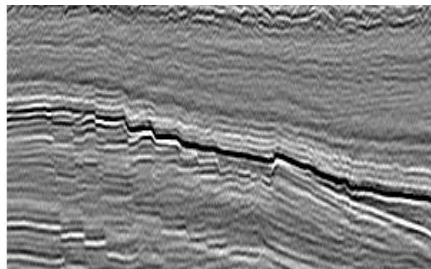
LANDMASS-2



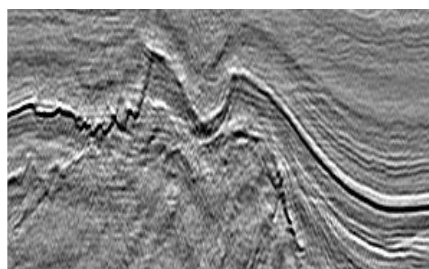
Horizons



Chaotic Horizons



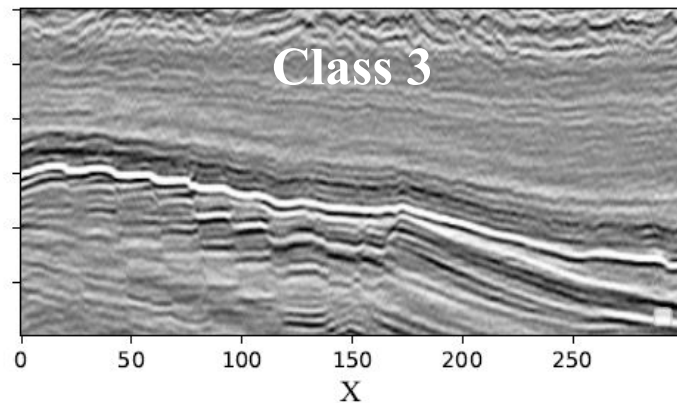
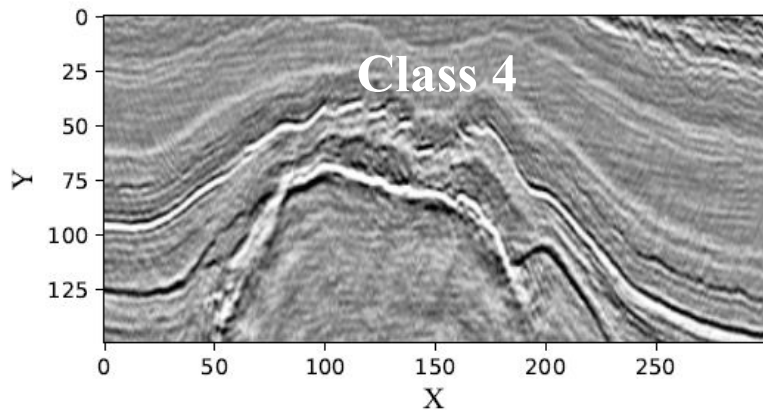
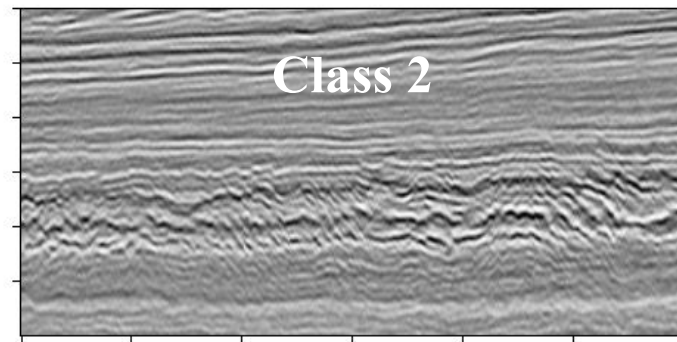
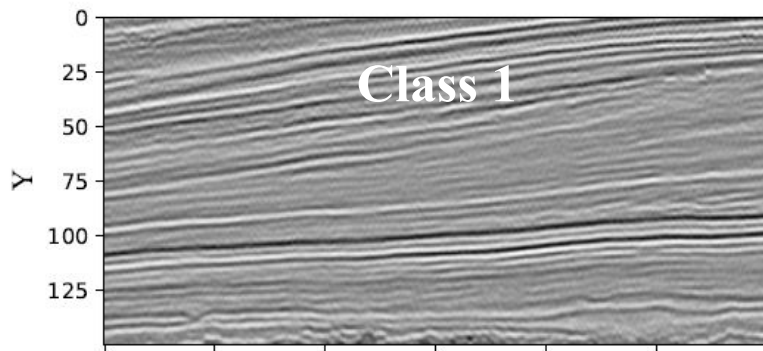
Fault Patches



Salt Domes

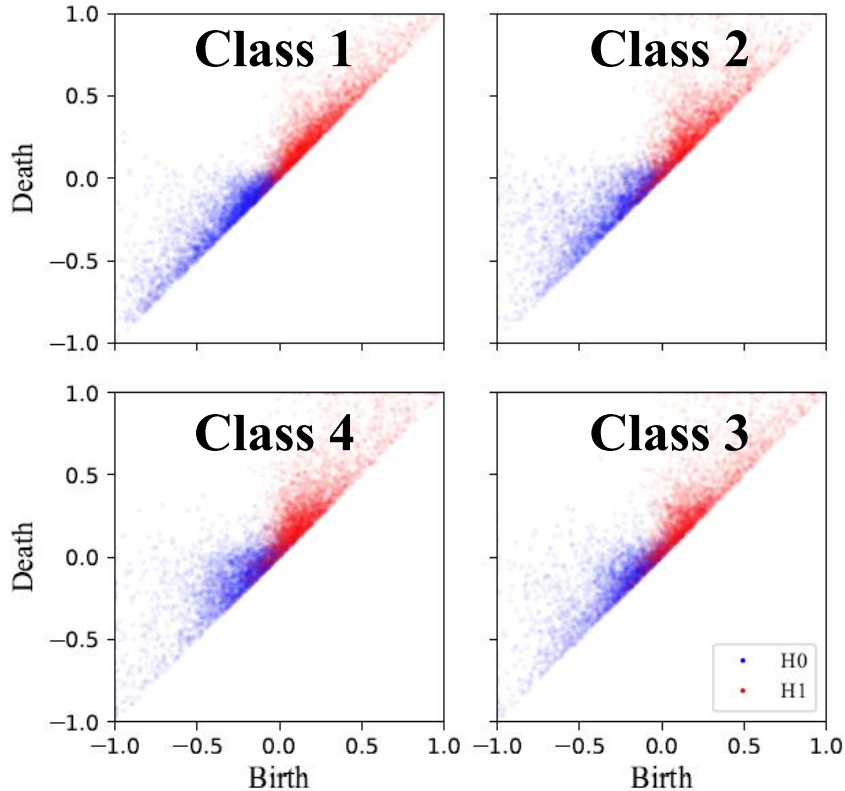
Persistence diagram results (LANDMASS-2)

Sample Images



Persistence diagram results (LANDMASS-2)

Persistence Diagrams



Subtle differences between the persistence diagrams.

To train a classifier we need:

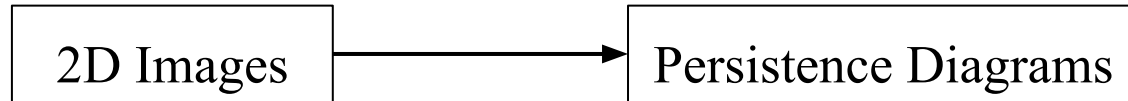
- Statistically significant **intra-class similarity**.
- Statistically significant **inter-class dissimilarity**.

Currently working on how to make this more precise, and generate metrics.

Need for featurization of persistence diagrams

We want to use a machine learning (ML) approach for training a classifier based on the persistence diagrams.

So far:



Key points about the persistence diagrams:

- Every image produces a different number of birth-death pairs.
- We want a *standard number of features* for a ML workflow.

Polynomial featurization

One approach is based on polynomial functions[†], which we adopt in our work:

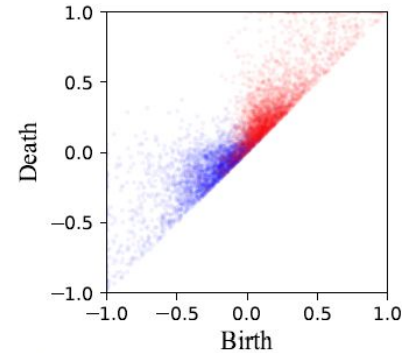
$$p(\alpha; \{b_i, d_i\}_{i \in J}) = \frac{1}{|J|} \sum_{i \in J} \sum_{j,k} \alpha_{j,k} (d_i - b_i)^j (d_i + b_i)^k.$$

For both homology dimensions 0 and 1 we choose:

$$\alpha_{j,k} = \delta_{j=j_0, k=k_0}$$

where $(j_0, k_0) \in \{0, 1, 2, 3\}^2 - \{(0, 0)\}$.

This gives us a total of $15 \times 2 = 30$ features per persistence diagram.



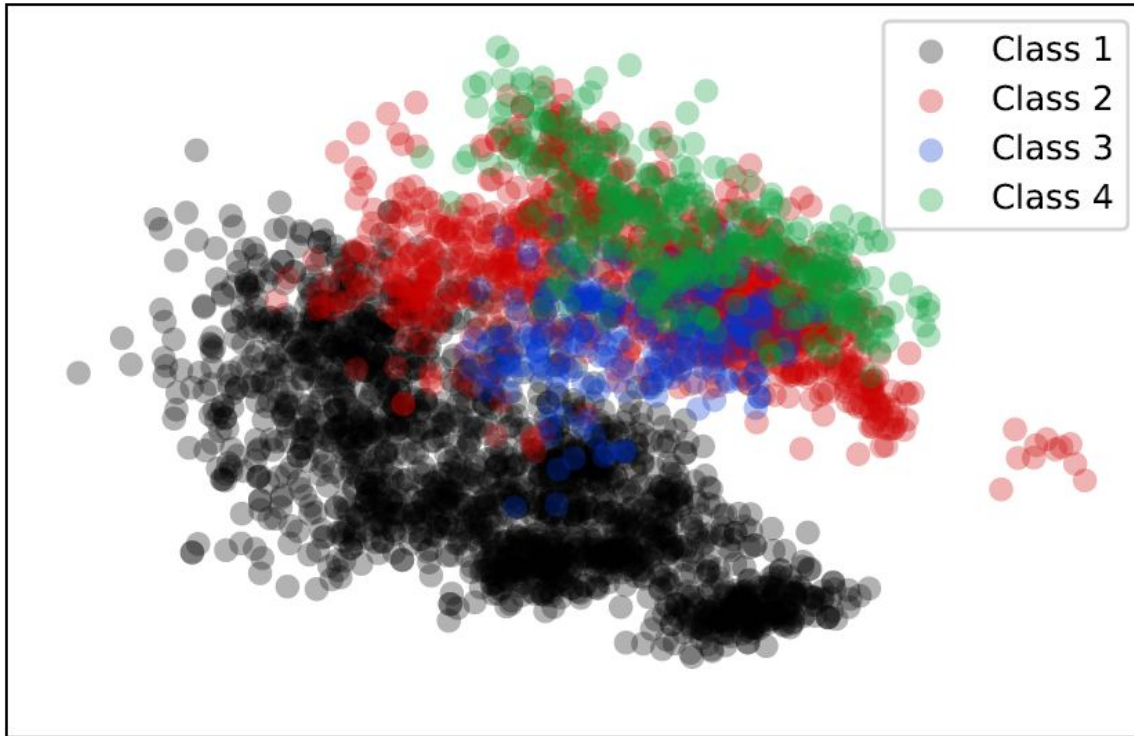
Featurization



[†] A. Adcock, E. Carlsson, G. Carlsson. The ring of algebraic functions on persistence barcodes. *Homology, Homotopy and Applications*. 18(1) 2016.

LANDMASS-1 features

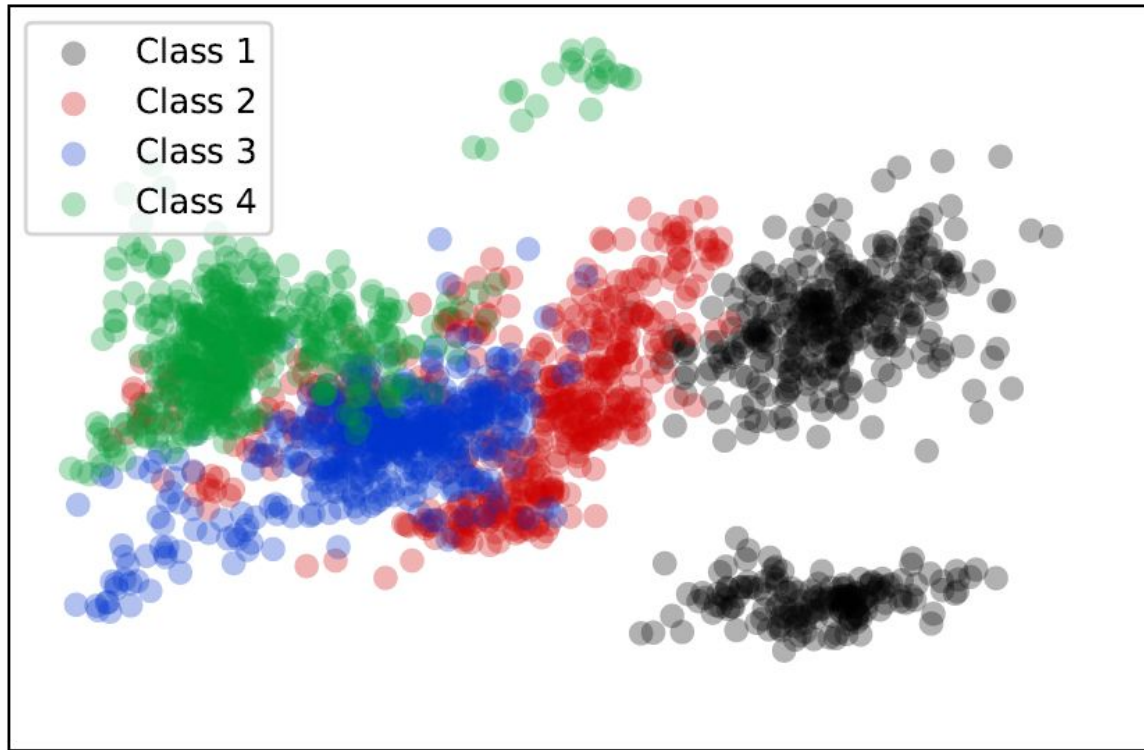
Projection of polynomial features into top two principal components. Each point is an image in the LANDMASS-1 dataset.



- Class 1 separates nicely from the other classes.
- With 2 principal components, classes are not well separated.
- More components are needed.

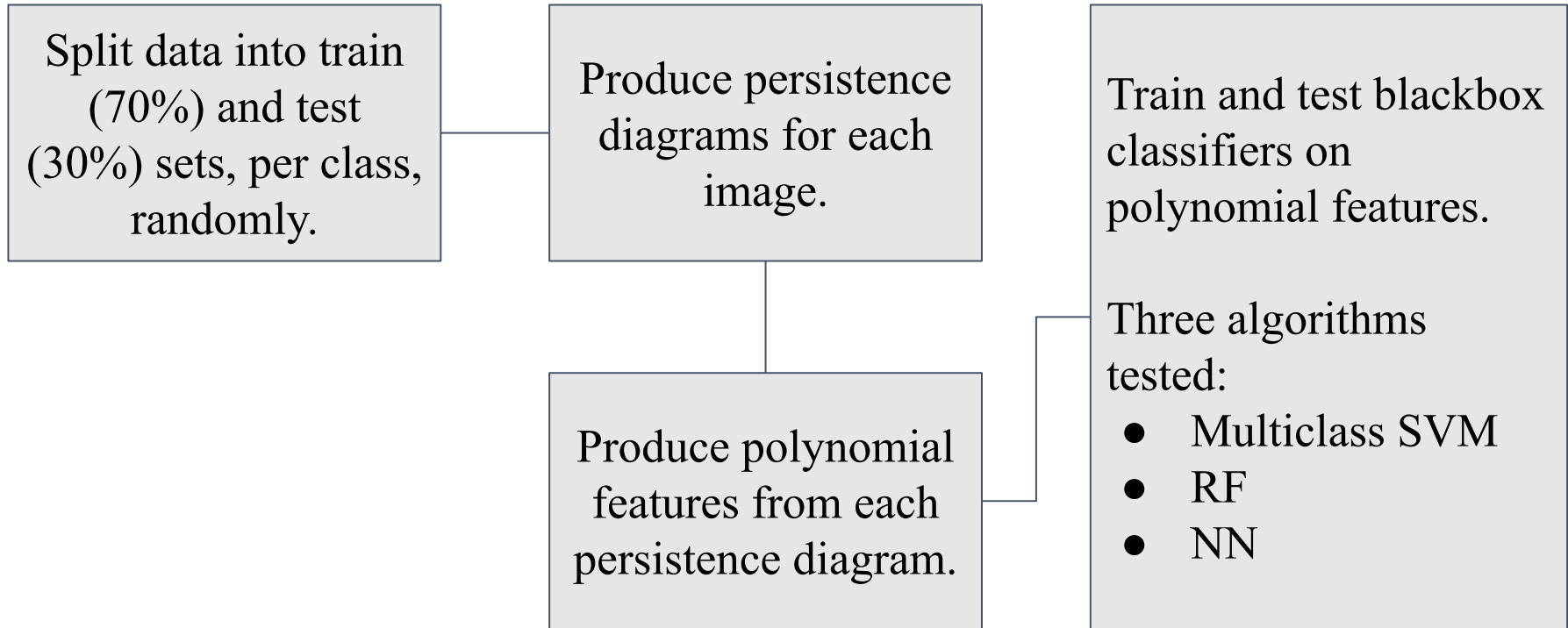
LANDMASS-2 features

Projection of polynomial features into top two principal components. Each point is an image in the LANDMASS-2 dataset.

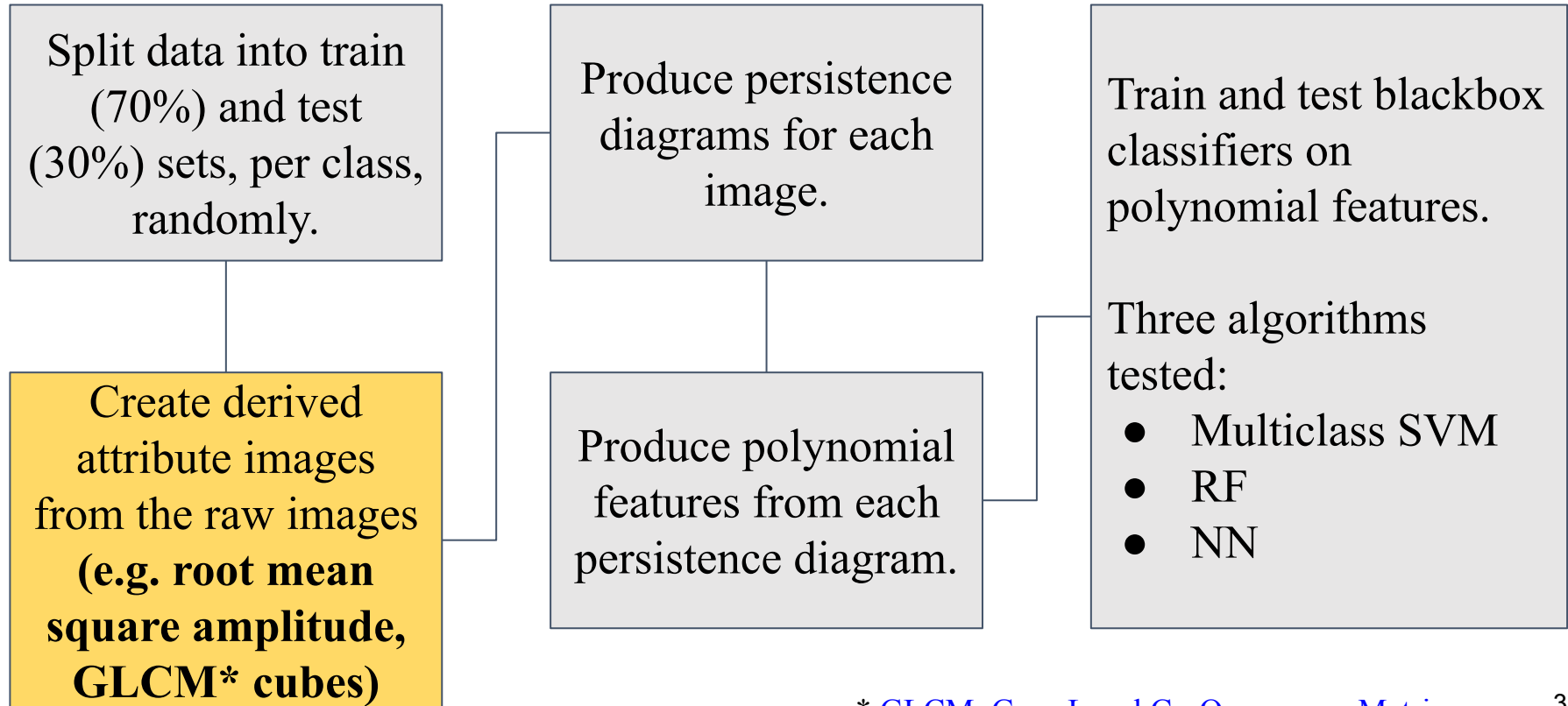


- Classes reasonably well separated with just top 2 principal components.
- Equal class sizes help classification.

Machine learning workflow



Derived attribute image based ML workflow



* GLCM: Gray-Level Co-Occurrence Matrix

Classification results: RF classifier

Attribute	RF Accuracy
Raw	99.9 / 98.6 / 95.2 / 93.3
Image	100.0 / 98.0 / 100.0 / 96.3
GLCM	99.9 / 97.9 / 82.1 / 93.3
Mean	100.0 / 97.0 / 97.3 / 91.7
RMS	99.3 / 96.1 / 88.0 / 82.0
Amplitude	99.7 / 96.0 / 96.0 / 91.7
GLCM	99.3 / 94.9 / 80.8 / 91.2
Correlation	99.7 / 93.7 / 92.0 / 97.0
GLCM	98.5 / 95.7 / 96.3 / 74.0
Variance	99.0 / 95.3 / 96.7 / 89.7

Class 1 / Class 2 / Class 3 / Class 4

Top Row: **LANDMASS-1**

Bottom Row: **LANDMASS-2**

Classification accuracy of **raw image, and best 4 attributes** with respect to RF classifier.

➤ Nonlinear classifiers do much better.

Classification results: NN classifier

Attribute	NN Accuracy
Raw	100.0 / 99.6 / 99.7 / 98.4
Image	100.0 / 100.0 / 99.0 / 95.0
GLCM	100.0 / 97.8 / 92.8 / 97.0
Mean	100.0 / 96.0 / 95.7 / 96.3
RMS	99.5 / 99.1 / 96.3 / 91.5
Amplitude	99.7 / 99.0 / 93.7 / 91.3
GLCM	99.8 / 93.6 / 87.7 / 96.7
Correlation	100.0 / 95.7 / 93.7 / 98.3
GLCM	99.3 / 98.3 / 98.1 / 87.3
Variance	99.7 / 99.0 / 99.3 / 95.0

Class 1 / Class 2 / Class 3 / Class 4

Top Row: **LANDMASS-1**

Bottom Row: **LANDMASS-2**

Classification accuracy of **raw image, and best 4 attributes** with respect to RF classifier.

➤ Nonlinear classifiers do much better.

Classification results: Multiclass SVM classifier

Attribute	SVM Accuracy			
Raw	99.8	75.2	0.0	0.0
Image	100.0	55.0	88.3	74.3
GLCM	100.0	18.6	34.1	29.3
Mean	62.7	19.0	4.0	100.0
RMS	100.0	1.0	0.0	0.0
Amplitude	74.7	85.7	71.3	61.7
GLCM	100.0	0.0	0.0	0.0
Correlation	64.7	32.0	89.3	32.3
GLCM	96.6	94.1	92.8	67.7
Variance	97.3	93.3	91.7	87.0

Class 1 / Class 2 / Class 3 / Class 4

Top Row: **LANDMASS-1**

Bottom Row: **LANDMASS-2**

Classification accuracy of **raw image, and best 4 attributes** with respect to RF classifier.

- Linear classifiers like SVM perform poorly.
- Need nonlinear decision boundaries.

Conclusions

- TDA derived features perform well for texture classification in seismic images.
- Nonlinear decision boundary classifiers are necessary for good classification accuracy.
- These features could augment existing ML workflows for similar tasks.

Current and future work:

- Develop metrics to better quantify differences in persistence diagrams.
- How many polynomial features do we need?
- Extend the workflow to 3D labeled data.

Software used in this study

- **GUDHI**^[1] in Python — persistent homology calculations.
- **Scikit-learn**^[2] in Python — SVM and RF classifiers.
- **Tensorflow**^[3] in Python — NN classifier.

[1] C. Maria, “Filtered Complexes, GUDHI User and Reference Manual”, http://gudhi.gforge.inria.fr/doc/latest/group_simplex_tree.html, 2015.

[2] F. Pedregosa et al., “Scikit-learn: Machine Learning in Python”, *Journal of Machine Learning Research* 12, 2011.

[3] M. Abadi et al., “TensorFlow: Large-Scale Machine Learning on Heterogeneous Systems”, Whitepaper, <https://www.tensorflow.org/>, 2015.

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Questions

Thank you for listening!

Questions?

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