1 Uniform Taxation

(a) Show that the uniform taxation rule derived in class can be generalized as follows: suppose preferences can be written as $U(G(c_1, \ldots, c_n), l)$ with $U_{IG}(.) = 0$ and $G(.)$ is a homothetic function. Recall that $G(.)$ is homothetic if and only if

$$G(c_1, \ldots, c_n) = k(K(c_1, \ldots, c_n))$$

and $K(.)$ is homogeneous of degree $\rho$, $k'(.) \neq 0$. Then all consumption goods $c_1, \ldots, c_n$ are taxed at the same rate at the optimum.

(b) Show that the uniform taxation rule can be generalized to heterogeneous agents in the following sense: suppose preferences can be written as $U^j(G(c_1, \ldots, c_n), l)$ where $j = 1, \ldots, J$ is the household index. If $U^j_{IG}(.) = 0$ for all $j$ and $G(.)$ is homothetic and identical across all $j$, then all consumption goods are taxed at the same rate at any Pareto optimum.

2 Gender-Based Taxation

This problem asks you to consider the taxation of various labor supply activities. Suppose utility is given by

$$U^j(G^c (c_1, c_2, \ldots, c_N), G^Y (Y_1, Y_2, \ldots, Y_M))$$

where $(c_1, c_2, \ldots, c_N)$ denotes a vector of consumption goods and $(Y_1, Y_2, \ldots, Y_M)$ a vector of labor supply activities. Technology is linear with unit prices (ignoring government consumption)

$$\sum_j \left( \sum_{i=1}^N c^j_i - \sum_{i=1}^M Y^j_i \right) \leq 0$$

where $j = 1, 2, \ldots, J$ is the household index.

(a) Suppose that taxation is restricted to imposing linear taxes on $c_i$ and $y_i$. Under what conditions (if any) is it efficient to just set a tax on total income with no tax on consump-
tion goods, so that the budget constraint becomes
\[ \sum_{i=1}^{N} c^j_i \leq t \sum_{i=1}^{M} Y^j_i. \]

(b) Interpret your results in terms of a household consisting of a husband and wife supplying labor services \( Y_h \) and \( Y_w \), respectively. Can this framework accommodate a high labor supply elasticity for \( Y_w \) and low for \( Y_h \)? Is “gender-based taxation,” which suggests taxing the less elastic labor supply at a higher rate, efficient?

3 **Budget Balance and Chamley-Judd**

Consider the infinite-horizon representative agent Ramsey setup reviewed in class (which assumes no lump sum component \( T = 0 \)). In that model the government is assumed to be able to issue bonds in an unrestricted way. In this problem, you consider the opposite extreme: the government must balance the budget in each period. For simplicity, consider the case without uncertainty.

(a) Write down the government budget constraint for each period \( t \), assuming that initial bonds are zero, \( B_0 = 0 \), and that the government cannot issue bonds.

(b) Suppose private individuals cannot buy or sell bonds. Write down their budget constraint for each \( t \) and set up the individual’s maximization problem. Show that if the individual’s budget constraint holds with equality and the resource constraint holds with equality, then the government’s budget constraint also holds with equality.

(c) Define a competitive equilibrium.

(d) Use the first order conditions for the consumer to derive a sequence of implementability conditions that must hold at each \( t \). Prove that if these conditions are satisfied and if the resource constraint holds with equality at all dates there exist taxes and prices that support the allocation as a competitive equilibrium.

(e) Use your result in (d) to study the planning problem. Show that if a steady state exists then at the steady state capital income is not taxed.