1 Unemployment Insurance Design with Biased Beliefs

Consider the following variant of the static Baily/Chetty framework discussed in class (following Spinnewijn, 2013). A risk-averse worker is employed with exogenous probability $p$ and loses his job otherwise. If unemployed, he exerts unobservable search effort at utility cost $e$. As a result, he gets reemployed with probability $\pi(e)$ and remains unemployed with probability $1 - \pi(e)$. The government provides unemployment insurance $b$, financed by a tax $\tau$. Consumption is therefore $w - \tau$ for the employed, $b$ for the unemployed and $w$ for the reemployed.

The key difference to the standard model is that we allow the unemployed to have biased beliefs about the probability to find a job as a function of search effort. Formally, let $\hat{\pi}(e)$ be the perceived probability of getting reemployed when exerting search effort $e$. We call an individual baseline-optimistic (pessimistic) if $\hat{\pi}(e) - \pi(e) \geq (\leq)0$ for all $e$. An individual is called control-optimistic (pessimistic) if $\hat{\pi}'(e) - \pi'(e) \geq (\leq)0$.

(a) Start with the standard case in which the worker has unbiased beliefs, so that $\hat{\pi}(e) = \pi(e)$ for all $e$ and (both the perceived and true) expected utility is

$$pu(w - \tau) + (1 - p)[\pi(e)u(w) + (1 - \pi(e))u(b) - e]. \quad (1)$$

Write down the FOC for the individual’s incentive constraint and show that the unemployed worker’s optimal search effort only depends on the unemployment insurance benefit $b$, so we can write $e^*(b)$. Find the tax $\tau^*(b)$ that finances this benefit.

Write down the government’s welfare maximization problem. Using the envelope theorem, show that, at the optimum, the following Baily/Chetty formula has to hold:

$$\frac{u'(b) - u'(w - \tau^*(b))}{u'(w - \tau^*(b))} = \varepsilon_{1-\pi(e^*(b)),b}$$ \quad (2)

where $\varepsilon_{1-\pi(e^*(b)),b}$ is the elasticity of the unemployment probability $1 - \pi(e^*(b))$ with respect to the benefit $b$. Interpret.
(b) Now consider the case with biased beliefs. In particular, the individual’s perceived expected utility is now

\[ pu(w - \tau) + (1 - p)[\hat{\pi}(e)u(w) + (1 - \hat{\pi}(e))u(b) - e], \tag{3} \]

where the perceived probability \( \hat{\pi}(e) \) is no longer necessarily equal to the true probability \( \pi(e) \).

For a given policy \((b, \tau)\), the worker will now choose search effort \( e \) to maximize (3) rather than (1). Write down the corresponding FOC, which implicitly determines optimal effort \( \hat{e}(b) \). Find the tax rate \( \hat{\tau}(b) \) that is required to finance benefits \( b \) when the agent is behaving according to \( \hat{e}(b) \), but the government budget constraint is in terms of the true probability \( \pi \).

Suppose the government is paternalistic, i.e. it maximizes the “true” expected utility (1) subject to the incentive constraint and the budget constraint. Show that, in this case, the following modified Baily/Chetty formula holds:

\[
\frac{u'(b) - u'(w - \hat{\tau}(b))}{u'(w - \hat{\tau}(b))} = \varepsilon_{1-\pi(\hat{e}(b)),b} \left[ 1 + \frac{\pi'(\hat{e}(b)) - \hat{\pi}'(\hat{e}(b))}{\pi'(\hat{e}(b))} \frac{u(w) - u(b)}{bu'(w - \hat{\tau}(b))} \right]. \tag{4}
\]

(c) Argue that the standard formula (2) overestimates the socially optimal level of unemployment benefits if the unemployed are control-pessimistic. Interpret.

Explain intuitively why only control bias affects the modified formula, whereas baseline bias does not change the paternalistic optimum. How would this result change if insurance was provided by private insurance companies which compete to attract insurees, rather than a paternalistic government?

2 TIMING OF UNEMPLOYMENT INSURANCE BENEFITS

Consider the following variant of the dynamic model of unemployment insurance (UI) discussed in class (Shimer/Werning, 2008). Time is continuous and runs from 0 to \( \infty \). A unit mass of agents all start out unemployed at \( t = 0 \) and receive job offers with Poisson arrival rate \( \alpha \) and iid wage draws \( w \sim F(w) \). Agents decide whether to accept or reject job offers. If they accept a job offer with wage \( w \), they become employed at that wage forever. Otherwise, they stay unemployed and continue to receive new job offers. Preferences are given by

\[ \mathbb{E} \int_0^\infty e^{-rt}u(c(t))dt, \]
where $r$ is the interest rate and $c(t)$ is consumption at $t$.

Consider a “flat benefits” UI system where the unemployed get a constant benefit $\bar{b}$ (independent of the duration of unemployment) financed by a flat tax $\tau$ on the employed. Individuals can freely save and borrow at the interest rate $r$.

(a) Argue that the intertemporal budget constraint of the employed is

$$\dot{a}(t) = ra(t) + w - \tau - c(t).$$

Based on this, show that the remaining lifetime utility of an employed agent who accepts a job with wage $w$ and has assets $a$ is

$$V_e(a, w) = \frac{u(ra + w - \tau)}{r}.$$

(b) Provide the intuition for why the remaining lifetime utility of an unemployed agent who has assets $a$, $V_u(a)$, must satisfy the HJB

$$rV_u(a) = \max \left\{ u(c) + V_u'(a)(ra + \bar{b} - c) \right\} + a \int_0^\infty \max \{ V_e(a, w), V_u(a), 0 \} dF(w).$$

Use this and the result from (a) to show that the agent’s job acceptance decision is determined by a reservation wage $\bar{w}(a)$.

(c) From now on, assume that $u(c) = -e^{-\gamma c}$ and that the reservation wage is in fact independent of $a$, i.e. $\bar{w}(a) = \bar{w}$. Use this and the HJB to show that

$$V_u(a) = \frac{u(ra + \bar{w} - \tau)}{r} \quad \text{and} \quad c_u(a) = ra + \bar{w} - \tau,$$

where $c_u(a)$ is the policy function for consumption determined by the HJB.

(d) Guess and verify that $V_u(a)$ and $c_u(a)$ indeed solve the HJB if the reservation wage $\bar{w}$ satisfies

$$\gamma(\bar{w} - \bar{b} - \tau) = \frac{\alpha}{r} \int_{\bar{w}}^\infty [1 + u(w - \bar{w})] dF(w).$$

*Hint:* Use the property of the CARA utility function that $u(a + b) = -u(a)u(b)$, $u(a - b) = -u(a)/u(b)$ and $u(0) = -1$.

(e) Show that, even though the UI benefit $\bar{b}$ is constant, consumption of the unemployed decreases over the unemployment spell, and jumps upward discretely once
they accept a job with a wage $w > \bar{w}$. Relate this to the optimal time path of UI benefits in the Hopenhayn/Nicolini (1997) model.