Optimal Social Insurance Design: Timing of UI Benefits

1 No Borrowing/Saving

- how should UI benefits vary over the unemployment spell?
- Shavell/Weiss (JPE 1979), Hopenhayn/Nicolini (JPE 1997)
- dynamic moral hazard model where unemployed workers exert an unobserved search effort in each period that increases their probability of finding a job
- risk averse preferences

\[ \mathbb{E} \sum_{t=0}^{\infty} \beta^t [u(c_t) - a_t], \]

where \( c_t \) and \( a_t \) are consumption and search effort in period \( t \)
- probability of finding a job in period \( t \) is \( p(a_t) \), increasing, concave
- workers cannot borrow or save, so risk neutral government can directly control their consumption
- workers start out unemployed at \( t = 0 \), search for jobs
- once they find a job, it is permanent with a constant wage \( w \) (no multiple unemployment spells, no heterogeneous wages)
- if effort \( a_t \) was perfectly observable, optimal policy would involve constant consumption stream (independent of whether employed or unemployed) and constant effort across all periods
- not incentive compatible with hidden effort (agent would put zero search effort)
- study second best problem with unobservable search effort using recursive formulation
- let \( V \) be the expected discounted utility at time \( t = 0 \) to the (unemployed) worker

optimal policy specifies a current consumption level $c^u$ and an effort $a$ and promises future expected discounted utility conditional on having found a job in $t = 0$, $V^e$, and remaining unemployed, $V^u$

it must hold that

$$V = u(c^u) - a + \beta [p(a)V^e + (1 - p(a))V^u]$$

the agent’s incentive constraint is

$$a \in \arg\max_{a'} u(c^u) - a' + \beta [p(a')V^e + (1 - p(a'))V^u]$$

necessary and sufficient FOC

$$\beta p'(a)(V^e - V^u) = 1$$

clearly, positive search effort requires $V^e > V^u$

once worker found a job, incentive problem disappears, so optimal to provide constant consumption from then on with

$$V^e = \frac{u(c^e)}{1 - \beta}$$

net transfer is $c^e - w$ per period, with NPV (assuming interest rate $r = 1 - \beta$)

$$W(V^e) = \frac{c^e - w}{1 - \beta} = \frac{u^{-1}((1 - \beta)V^e) - w}{1 - \beta} \Rightarrow W'(V^e) = \frac{1}{u'(c^e)}$$

note: $w - c^e$ is the constant tax on the employed to finance the unemployment benefits $c^u$, so $W(V^e) < 0$

next consider situation when agent is unemployed

let $C(V)$ be the expected discounted cost for the principal associated with the optimal contract when the agent is unemployed with a continuation value $V$

$C(V)$ satisfies Bellman equation

$$C(V) = \min_{c^u,a,V^e,V^u} c^u + \beta [p(a)W(V^e) + (1 - p(a))C(V^u)]$$
subject to the promise-keeping constraint (1) and the incentive constraint (2)

- form Lagrangian

\[
L = c^u + \beta \left[p(a)W(V^e) + (1 - p(a))C(V^u)\right] \\
+ \xi \left[V - u(c^u) + a - \beta \left[p(a)V^e + (1 - p(a))V^u\right]\right] \\
+ \eta \left[1 - \beta p'(a)(V^e - V^u)\right]
\]

- the FOC for \(c^u\) implies

\(\xi = \frac{1}{u'(c^u)}\)

- FOC for \(a\) (after using (2)):

\(p'(a) \left[W(V^e) - C(V^u)\right] = \eta p''(a)(V^e - V^u)\)  \(\text{(6)}\)

- FOC for \(V^u\):

\[C'(V^u) = \frac{1}{u'(c^u)} - \eta \frac{p'(a)}{1 - p(a)} \quad \text{(7)}\]

- FOC for \(V^e\):

\[W'(V^e) = \frac{1}{u'(c^u)} + \eta \frac{p'(a)}{p(a)} \quad \text{(8)}\]

- envelope condition

\[C'(V) = \xi = \frac{1}{u'(c^u)} \quad \text{(9)}\]

- moreover, adding \(p(a)\) times (8) and \(1 - p(a)\) times (7) yields

\[
\frac{1}{u'(c^u)} = p(a)W'(V^e) + (1 - p(a))C'(V^u) = \frac{p(a)}{u'(c^e)} + \frac{1 - p(a)}{u'(c^u)}, \quad \text{(10)}
\]

where \(\tilde{c}^u\) is next period consumption if still unemployed

- Inverse Euler equation, implies that (comparing with Euler equation) optimal UI policy needs to be able to prevent agent from saving, similar to what we’ve seen with capital taxation

- key result: optimal unemployment benefit is decreasing over time while the worker remains unemployed, i.e. \(c^u > \tilde{c}^u\)
proof: from (7) and (8),

\[ W'(V^e) - C'(V^u) = \eta p'(a) \left[ \frac{1}{p(a)} + \frac{1}{1 - p(a)} \right] = \frac{\eta p'(a)}{p(a)(1 - p(a))} \]

- can show \( \eta > 0 \), so \( W'(V^e) > C'(V^u) \)

- then (9) and (10) together imply

\[ W'(V^e) > C'(V) > C'(V^u) \]

- the last inequality and the envelope condition imply

\[ \frac{1}{u'(c^u)} > \frac{1}{u'(\tilde{c}^u)} \Rightarrow c^u > \tilde{c}^u \]

by concavity of \( u(c) \), q.e.d.

- ever decreasing benefits over the unemployment spell, principal interprets long unemployment duration as evidence for low search effort

- however, benefits never completely (and abruptly) terminate, instead fall smoothly

2 Access to Borrowing/Saving: Liquidity vs. Insurance

2.1 Setup

- suppose agent has free access to financial markets, i.e. saving and borrowing in a risk-free asset

- Shimer and Werning (AER 2008)

- allows to distinguish two components of optimal UI policy:

  - insuring workers against uncertainty in the prospect of finding a job (duration risk)
  
  - providing workers with liquidity to smooth consumption while unemployed

- if workers can save and borrow, UI system only needs to accomplish the former (see e.g. policy proposal of UI savings accounts, Feldstein, AER 2005, for achieving the latter)
what would be the optimal UI policy in this scenario?

show that optimal policy is constant benefits if preferences are CARA, and roughly constant benefits otherwise

intuition: benefits are constant, but workers’ own consumption-savings decision makes sure that their consumption falls with the duration of the unemployment spell, as in Hopenhayn/Nicolini (1997)

use a different model of sequential job search, going back to McCall (QJE 1970)

continuous time, preferences

\[ \mathbb{E} \int_0^\infty e^{-\rho t} u(c(t)) dt \]

special case: CARA with

\[ u(c) = -e^{-\gamma c} \]

again, worker starts unemployed, receives job offers at Poisson arrival rate \( \alpha \)

job offers vary in their wage \( w \) with cdf \( F(w) \)

worker privately observes the wage and decides whether to accept or reject it

if accepts, becomes employed at that wage forever

if rejects, continues to be unemployed and to get new wage offers

government can’t observe the job offers that the agent received, only whether employed or unemployed (and for how long)

moral hazard here comes from the fact that, as UI benefits rise, worker becomes too picky about accepting wage offers

interest rate \( r = \rho \)

### 2.2 Constant Benefits Policy

consider constant benefits policy: worker gets \( \bar{b} \) as long as unemployed, pays constant tax \( \bar{t} \) forever once employed
• perfect access to riskless asset with return $r$, intertemporal budget constraint

$$\dot{a}(t) = ra(t) + \bar{b} - c(t)$$

while unemployed and

$$\dot{a}(t) = ra(t) + w - \tau - c(t)$$

while employed at wage $w$

• this is a very restrictive policy: benefit does not depend on duration, nor does the tax upon employment

• consider a worker who is employed at wage $w$ and has assets $a$

• since $\rho = r$ and no remaining uncertainty, perfectly smoothes consumption and thus keeps assets constant:

$$c = ra + w - \tau$$

• remaining lifetime utility

$$V^e(a, w) = \frac{u(ra + w - \tau)}{r}$$

• consider unemployed worker with assets $a$ and let $V^u(a)$ denote expected lifetime utility

• satisfies HJB

$$rV^u(a) = \max_c \left\{ u(c) + V^u'(a)(ra + \bar{b} - c) \right\} + \alpha \int_0^\infty \max \left\{ V^e(a, w) - V^u(a), 0 \right\} dF(w)$$

(11)

• first term on RHS: choose consumption to equalize marginal utility from consumption to marginal value of assets

• second term on RHS: accept job with wage $w$ if $V^e(a, w) > V^u(a)$, reject otherwise. If accept, get “capital gain” $V^e(a, w) - V^u(a)$.

• since $V^e(a, w)$ is increasing in $w$, individual chooses reservation wage and accepts all job offers whose wage exceeds the reservation wage

• solution: policy functions for consumption $c^u(a; \bar{b}, \tau)$ and reservation wage $\bar{w}(a; \bar{b}, \tau)$, value function $V^u(a; \bar{b}, \tau)$
• in particular, FOC for $c$ is
\[ u'(c) = V'^u(a) \] (12)
and reservation wage is determined by indifference condition
\[ V^u(a) = V^c(\bar{w}, a) \] (13)

• Proposition: With CARA preferences, the reservation wage policy function $\bar{w}(a; b, \tau) = \bar{w}(\bar{B})$ solves
\[ \gamma(\bar{w} - \bar{B}) = \frac{\alpha}{r} \int_{\bar{w}}^{\infty} [1 + u(w - \bar{w})] dF(w) \] (14)
with $\bar{B} \equiv \bar{b} + \tau$ (the total “subsidy to unemployment”). $c^u(a; b, \tau)$ and $V^u(a; b, \tau)$ satisfy
\[ c^u(a) = ra + \bar{w} - \tau \] (15)
\[ V^u(a) = \frac{u(ra + \bar{w} - \tau)}{r} \] (16)

• note in (14): LHS is increasing in $\bar{w}$, RHS is decreasing, so unique solution
• then (16) just follows from indifference condition (13)
• (15) follows from differentiating (16):
\[ V'^u(a) = \frac{ru'(ra + \bar{w} - \tau)}{r} = u'(c) \]
by (12), which implies $c = ra + \bar{w} - \tau$
• reservation wage only depends on $\bar{B} = \bar{b} + \tau$ rather than $\bar{b}$ and $\tau$ separately (could think of model where the worker always has to pay $\tau$, but gets $\bar{b} + \tau$ while unemployed: no change if there are no wealth effects, as is the case with CARA)
• moreover, reservation wage does not depend on assets $a$ (again since no wealth effects from CARA) and is thus constant over the unemployment spell
• hence, worker faces constant hazard rate of accepting a job $H(\bar{B}) \equiv \alpha(1 - F(\bar{w}(\bar{B})))$
• moral hazard effect from (14): $\bar{B} \uparrow \rightarrow$ LHS $\downarrow \rightarrow \bar{w} \uparrow \rightarrow H \downarrow$, longer duration
• but this is not necessarily a bad thing here (Acemoglu and Shimer, JPE 1999): compared to risk-neutral case, risk aversion leads to lower reservation wage $\bar{w}$ (workers become less picky to accept a job). But risk neutral case maximizes present value
of income (output): a small introduction of UI benefits with risk averse workers increases output, no tradeoff between incentives and insurance in the range of small UI benefits.

- Shimer and Werning (QJE 2007): (15) and (16) imply that welfare effects of changes in $\bar{b}, \tau$ on unemployed workers only depend on $\bar{w} - \bar{c}$

- net reservation wage is “sufficient statistic” for welfare

- in particular, we can test the efficiency of the UI system with flat benefits by looking at $d\bar{w}/d\bar{B}$ and $d\bar{c}/d\bar{B}$ (through the government budget constraint). Clearly, the system is inefficient whenever

$$\frac{d\bar{w}}{d\bar{B}} > \frac{d\bar{c}}{d\bar{B}}$$

since then an increase in benefits, financed by the required tax increase, makes both the unemployed and the employed better off

- to prove the proposition, we only need to prove (14), since (15) and (16) follow from it, as shown above

- proof strategy: guess and verify (15) and (16) in the HJB (11) (see Problem Set 2)

- note that

$$\dot{a} = ra + \bar{b} - c = ra + \bar{b} - ra - \bar{w} + \tau = \bar{b} - \bar{w} < 0$$

from (14) (since the RHS of (14) is positive), so worker decumulates assets over the unemployment spell

- by (15),

$$\dot{c}^u = ra < 0,$$

so we get decreasing consumption over the unemployment spell even though benefit is flat

- if $w > \bar{w}$, worker gets discrete upward jump in consumption upon reemployment

- the next step is to choose the flat benefits policy $\bar{b}, \tau$ optimally: the UI agency chooses the policy to maximize the worker’s expected discounted utility given available resources and an initial asset level. Equivalently, consider dual problem of minimizing total resource cost, equal to benefits net of taxes plus initial assets, of delivering a given utility to the agent.
• again using the CARA specification and thinking of the model where \( \tau \) is always payed and unemployment benefit is \( \bar{b} + \tau \), the cost of policy \( \bar{b}, \tau, a_0 \) is

\[
C \equiv -\frac{\tau}{r} + \frac{\bar{b} + \tau}{r + \alpha (1 - F(\bar{w}))} + a_0,
\]

since \( \alpha (1 - F(\bar{w})) \) is the hazard rate of becoming employed (so need to discount the cost of unemployment benefits with this in addition to \( r \))

• initial utility is

\[
V_0 = V^u(a_0) = \frac{u(ra_0 + \bar{w} - \bar{\tau})}{r} \Rightarrow \frac{u^{-1}(rV_0)}{r} = a_0 + \frac{\bar{w} - \bar{\tau}}{r}
\]

and hence the optimal flat benefits policy solves

\[
C(V_0) = \min_{\bar{B}, \bar{w}} \left\{ \frac{\bar{B}}{\frac{1}{r + \alpha (1 - F(\bar{w}))} + \frac{u^{-1}(rV_0) - \bar{w}}{r}} \right\}
\]  

(17)

s.t. \( \bar{w} = \bar{w}(\bar{B}) \) from equation (14)

• since (14) implies a one-to-one relationship between \( \bar{w} \) and \( \bar{B} \), we can also use it to solve for \( \bar{B} \) as a function of \( \bar{w} \):

\[
\bar{B} = \bar{w} - \frac{\alpha}{\gamma r} \int_{\bar{w}}^{\infty} (1 + u(w - \bar{w})) dF(w)
\]

and substitute in (17) to get

\[
C(V_0) = \frac{u^{-1}(rV_0)}{r} - \max_{\bar{w}} \left\{ \frac{\alpha}{\gamma r} \Phi(\bar{w}) \right\}
\]

(18)

with

\[
\Phi(\bar{w}) = \int_{\bar{w}}^{\infty} \frac{1 + \gamma \bar{w} + u(w - \bar{w}) dF(w)}{r + \alpha (1 - F(\bar{w}))}
\]

2.3 Optimal Time-Dependent Benefits Policy

• now consider optimal time-dependent UI policy

• shut down agent’s access to saving/borrowing, so that worker’s consumption is equal to UI benefit if unemployed, net wage if employed

• let \( b(t) \) be the UI benefit to a worker who is still unemployed at \( t \)
• let a worker who finds a job at $t$ pay a tax $\tau(t)$ for the remainder of her life

• then the unemployed worker will choose some time dependent reservation wage policy $\overline{w}(t)$

• strictly more powerful policy instruments:
  
  – can directly control the worker’s consumption
  
  – can implement duration-dependent policy

• key result: cannot do better than with constant benefits and free saving/borrowing for the agent

• given policy $\{b(t), \tau(t)\}$ and worker’s reservation wage sequence $\{\overline{w}(t)\}$, the worker’s utility from $t'$ on is

\[
U\left(t', \{\overline{w}(t), b(t), \tau(t)\}\right) = \int_{t'}^{\infty} e^{-\int_{t'}^{t} \alpha (1 - F(w(s))) ds} \left( u(b(t)) + \alpha \int_{\overline{w}(t)}^{\infty} \frac{u(w - \tau(t))}{r} dF(w) \right) dt \tag{19}
\]

• interpretation: worker is still unemployed at time $t$ with probability

\[
e^{-\int_{0}^{t} \alpha (1 - F(w(s))) ds},
\]

which we therefore add to the discounting (similar to a model with stochastic death)

• if unemployed, worker gets $u(b(t))$ at $t$

• if worker draws a wage above the reservation wage $\overline{w}(t)$, she takes the job and from then on gets $u(w - \tau(t))$ forever, thus total remaining lifetime utility (NPV) is $u(w - \tau(t))/r$

• particular way of “accounting”: think of it as agents dropping out of the model once they find a job, which is why we include the probability of remaining unemployed in discounting, but we account for their entire remaining lifetime utility once they drop out of the model

• again pursue dual approach: UI agency sets $\{b(t), \tau(t)\}$ to minimize the cost of
providing the worker with utility $V_0$:

$$C^*(V_0) = \min_{\overline{w}(t), b(t), \tau(t)} \int_0^\infty e^{-\int_0^t (r + \alpha (1 - F(w(s)))) ds} \left( b(t) - \alpha (1 - F(\overline{w}(t))) \frac{\tau(t)}{r} \right) dt$$

subject to the promise keeping constraint

$$V_0 = U(0, \{\overline{w}(t), b(t), \tau(t)\})$$

and the incentive constraint, requiring that the worker prefers the recommended reservation wage sequence to any other sequence $\{\hat{w}(t)\}$:

$$U(0, \{\overline{w}(t), b(t), \tau(t)\}) \geq U(0, \{\hat{w}(t), b(t), \tau(t)\}),$$

which captures the moral hazard problem

- **Lemma**: With CARA,

$$C^*(V_0) = \frac{u^{-1}(rV_0)}{r} + C_0$$

where $C_0$ is some constant. Moreover, let $\{\overline{w}^*(t), b^*(t), \tau^*(t)\}$ denote the optimum for initial promised utility $V_0 = u(0)/r$. Then

$$\left\{\overline{w}^*(t), b^*(t) + u^{-1}(rV_0), \tau^*(t) - u^{-1}(rV_0)\right\}$$

is optimal for any initial promise $V_0$

- **key**: path for reservation wage is unchanged when promised utility changes. In fact, since promised utility is the state variable, the Lemma implies that the optimal reservation wage path is flat, as we found for the constant benefits policy. Direct consequence of absence of wealth effects from CARA.

- **Proof**: Suppose we add $x$ to $b(t)$ and subtract $x$ from $\tau(t)$ for all $t$. Due to CARA, this just multiplies lifetime utility (19) by $-u(x)$, without affecting reservation wage, i.e.

$$U(t', \{\overline{w}(t), b(t) + x, \tau(t) - x\}) = -u(x)U(t', \{\overline{w}(t), b(t), \tau(t)\}) \quad \forall x.$$  

Let $C_0 \equiv C^*(u(0)/r)$. If the policy $\{\overline{w}^*(t), b^*(t), \tau^*(t)\}$ is optimal for initial promised utility $u(0)/r$, the policy

$$\{\overline{w}^*(t), b^*(t) + u^{-1}(rV_0), \tau^*(t) - u^{-1}(rV_0)\}$$

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is optimal to deliver initial promised utility

\[-u \left( u^{-1}(rV_0) \right) \frac{u(0)}{r} = \frac{u(u^{-1}(rV_0))}{r} = V_0.\]

The cost of this policy is \( u^{-1}(rV_0) / r \) plus the cost of the policy that delivers \( u(0) / r \), q.e.d.

- comparing (18) with (23), we only need to solve for the constant \( C_0 \) in (23) and show that
  \[
  C_0 = -\frac{\alpha}{r\gamma} \max_{\bar{w}} \Phi(\bar{w})
  \]
  to establish that the cost function with optimal, time-dependent UI, \( C^*(V_0) \), is identical to the cost with constant benefits, \( C(V_0) \). This is done by guessing and verifying \( C(V_0) \) in the HJB implied by the optimality problem (20) s.t. (21) and (22).

- this establishes the key theorem: \( C^*(V_0) = C(V_0) \), i.e. the allocation obtained with constant benefits and free borrowing/saving is optimal.

- no gain from restricting the agent’s access to financial markets, or using a duration dependent policy.

- with wealth effects, e.g. CRRA, numerical simulations suggest that
  - optimal UI policy involves almost flat benefits
  - policy with flat benefits is almost optimal