Nominations for Sale

Silvia Console-Battilana* and Kenneth A. Shepsle†

Abstract

Models of nomination politics in the US often find "gridlock" in equilibrium because of the super-majority requirement in the Senate for the confirmation of presidential nominees. A blocking coalition often prefers to defeat any nominee. Yet empirically nominations are successful. In the present paper we explore the possibility that senators can be induced to vote contrary to their nominal (gridlock-producing) preferences through contributions from the president and/or lobbyists, thus breaking the gridlock and confirming the nominee. We model contributions by the president and lobbyists according to whether payment schedules are conditioned on the entire voting profile, the vote of a senator, or the outcome. We analyze several extensions to our baseline approach, including the possibility that lobbyists may find it more productive to offer inducements to the president in order to affect his proposal behavior, rather than trying to induce senators to vote for or against a given nominee.

1 Introduction

Models of preference-based voting in committees and elections have a long and distinguished pedigree. The spatial model, borrowed from Hotelling (1929) and popularized by Downs (1957) and Black (1958), is now well established in the political economy lexicon. In its most essential form, the space of alternatives is the unit interval and agents are assumed to possess symmetric, strictly single-peaked preferences on [0,1] with \( x_i \) the \( i^{th} \) agent’s ideal point. In its simplest application, two candidates (or motions) are pitted against each other and the one securing the most votes from agents wins.\(^1\) The well-known equilibrium result for this class of problems – Black’s Median Voter Theorem – states that the alternative closer to \( x_m \), the ideal point of the median

\*Stanford University, silviacb@stanford.edu. Console-Battilana acknowledges the support of the Stanford Institute of Economic Policy Research and the Stanford Freeman Spogli Institute.

\†Harvard University, kshepsle@iq.harvard.edu. Shepsle acknowledges the research support of the National Institute of Aging (RO1-AG021181) and the hospitality of the Hoover Institution.

\(^1\)All of these features may be complicated (multidimensional space, multicandidate contests, plurality or supermajority rules, etc.), but we will begin with the stripped-down arrangement and add some complications shortly.
voter, will win the contest. As a consequence, movers of motions or nominators of candidates who want to win will converge in their proposals to $x_m$.

Institutions, however, often possess additional features constraining the operation of pure majority rule. In the present paper we wish to take up one of the more prominent examples – the use of supermajority procedures in legislatures like the U.S. Senate. From the seminal work of Krehbiel (1998) it is well known that when a motion requires a supermajority to pass, it may not be possible to alter an existing status quo.\(^2\) If, for example, 60 votes are required in a 100-person legislature to pass a particular motion, then any coalition of 41 may block this motion. For some status quo positions, there may exist no motion able to overcome this obstacle. Krehbiel calls the gridlock region the range of prospective status quo points which cannot be dislodged when a specific supermajority rule is in effect. We are interested in what happens when gridlock is imminent. Are there ways in which enough agents can be induced to vote contrary to their nominal preferences to break out of the gridlock?

1.1 Context

Although this question may be posed in general settings, we have a specific one in mind. In the constitutional order of the United States, the Supreme Court is one of the three branches of government that makes policy, not through legislation or executive edicts but rather through its rulings. On a case before the Court, each justice makes two decisions. The first, called the vote on the merits, is a decision on whether to affirm or reverse a ruling from a lower court. A plurality in favor of reversal is decisive; a tie or smaller vote affirms the earlier ruling (in effect sustaining the status quo). The vote on the merits affects only the parties in dispute and, for this reason, is often of little importance for public policy.\(^3\) The second decision is each justice’s opinion, giving the constitutional rationale for his or her vote on the merits. In principle, each justice may write a separate opinion. Often, groups of justices sign a common opinion after having bargained over opinion language. The Court’s rationale becomes binding on lower courts, affecting their disposition of similar cases in the future and hence taking on broader public policy significance, if a majority signs the same opinion. Thus, the strategic policy process on the Court is one in which the question arises of whether there exists a majority consensus on moving the status quo to some new policy. Inasmuch as the Court is a nine-person body, if policy may be represented as unidimensional, and if justices have single-peaked preferences, then the policy preference of the median justice (that is, his or her ideal language and rationale for a majority opinion) will prevail.

Imagine, now, a death or retirement of a justice. The eight-person Court continues to function, but without a unique median. Rather, bargaining takes place among justices with an outcome forecast to lie between the ideal of the fourth and fifth justices. If the status quo policy lies in this interval, it cannot be revised since no coalition of five justices will agree to a change; if it lies outside this interval, on the other hand,

\[^2\]When a simple-majority procedure is in use, $x_m$ is the only status quo for which this statement holds.

\[^3\]However, when the United States is a party to the suit, a decision on the merits can have significance, even if a Court majority does not agree on a constitutional rationale.
bargaining is assumed to bring it inside the interval (Snyder and Weingast, 2000; Krehbiel, 2004, 2005; Rohde and Shepsle, 2006).

While this is the forecast for the surviving eight-person Court, the departure of a justice is a nomination inducing event in which the president may propose a new justice to the Court who, if confirmed, will generate a newly defined median in the full-complement, nine-person Court. Confirmation requires the "advice and consent" of the Senate. Nominally, this is a simple majority requirement. But the U.S. Senate has an unusual procedure known as the principle of unlimited debate. In order to end debate on a motion – in this case the motion to confirm a presidential nominee – and move directly to a vote, cloture must be secured, and this requires an absolute supermajority of sixty votes. Any group of 41 senators may keep the Senate from voting on confirmation by blocking cloture. This leads to the possibility of gridlock in which any nominee preferred by the president (because of the policy forecast for the full nine-member Court) is opposed by at least 41 senators, and the status quo outcome of the eight-member Court is preferred by the president to any nominee 60 or more senators would support (if any).

There are several models of this strategic interaction between president, Senate, and Court. We shall elaborate one by Rohde and Shepsle (2006) shortly. Many of these models find that gridlock obtains under a wide range of conditions. A fortiori, as politics in America has grown more polarized (in a manner that will be made precise), the set of circumstances in which gridlock prevails has grown wider. In the present paper we explore a set of options available to the president and special interest groups to offer inducements to senators to vote contrary to their nominal preferences, thereby cutting the Gordian knot and breaking the gridlock.

1.2 Model of Supreme Court Appointments

Rohde and Shepsle (2006) begin with a policy space, [0,1], along which are arrayed the ideal points of the one hundred senators, $S_i$. The ideal point of the president, $P$, is placed at an extreme location, mainly for ease of presentation. (All results, appropriately adjusted, apply for a more moderate president.) The senators and the president possess symmetric, strictly single-peaked utility functions on [0,1]. Some of their ideal points are displayed in Figure 1. In particular, $S_{41}$ and $S_{60}$ define the gridlock region. For any status quo located in $[S_{41}, S_{60}]$, no alternative is preferred to it by 60 or more senators. If $r$ is such a status quo (or reversion point), then any move to the left is opposed at a minimum by all senators at or to the right of $S_{60}$ – forty-one in all – and any move to the right is opposed at a minimum by all senators at or to the left of $S_{41}$.

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4This rule (Senate Rule 22) has been in effect since 1975. Between 1917 and 1975 cloture was obtained with the support of two-thirds of those present and voting. Before 1917, there was no rule to end debate short of unanimous consent.
A Court resignation or death – a nomination-inducing event – leaves an eight-person Court in place. Let \( r \) be the commonly anticipated policy position of this Court (the result of bargaining the details of which we suppress here). Any nomination by the president, if confirmed by the Senate, produces a nine-person Court with a well-defined median whose ideal point will be the new Court policy position. Label this \( E \). In Figure 1, \( E \) is to the right of \( r \), but its exact location is a function of the position of the justice nominated by the president. The cut point between \( r \) and \( E \), labeled \( cp \), partitions senators into those who prefer \( r \) to \( E \) and those who prefer \( E \) to \( r \). Since \( r \) lies in the gridlock region, this nominee cannot secure the sixty votes necessary for confirmation. In effect, in voting on whether to confirm a presidential nominee, senators compare \( r \) and \( E \). They should not be seen as expressing a preference on the nominee’s ideology except as it determines \( E \).

From the analysis in Rohde and Shepsle (2006), the conditions for gridlock would appear to be a commonplace. As American politics has become more polarized – this is reflected in a "stretching" of the gridlock region with \( S_{41} \) pulled to the left and \( S_{60} \) to the right – situations like that depicted in Figure 1 become even more common.

But presidents are not limited to proposing nominees. In addition, they may be thought to possess an *inducements budget* consisting of divisible, targetable payments to senators in exchange for their support. (We have in mind here earmarked appropriations for state-specific projects, campaign contributions, presidential endorsements of incumbents up for reelection, presidential support for pet bills of senators, etc.) And the president is not the only agent with an inducements budget and an interest in influencing support for (or opposition to) a nominee. Special interests with resources valuable to senators are active in the process. Indeed, in light of the availability of inducements, the nomination itself is an endogenous product of more than the initial policy preferences of senators and the reversion point of the eight-person Court following a departure of a justice. Protection, in the spirit of Grossman and Helpman (1994), is not the only thing for sale.

The contribution of the current paper is to extend recent work that uncovers a wide range of circumstances in which gridlock prevails. As a comparative static, the gridlock potential has in all likelihood been exacerbated as the gridlock region has increased with the growing polarization of representative institutions. We propose an approach, borrowed from models of special interest lobbying, that provides conditions in which the gridlock may be mitigated. Special interests, including the president himself, provide the lubricant that "greases the skids" for successful nomination results.

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1.3 Overview of Results

We introduce a model in which both local lobbies and the president can offer inducements. In particular, we will assume that the president is free to offer inducements to any senator, while lobbies are senator-specific. Thus, the president and local lobbies may be competing against each other. Groseclose and Snyder (1996) had shown that buying a supermajority might be cheaper than buying a strict majority to prevent countervailing lobbying. Taking a different modeling path, applying the methodology of Console Battilana (2005, 2006) and Dal Bo (2000, 2006), we find that when inducements can be made conditional on the entire voting profile, the president can defeat any competing lobbies and secure the confirmation of his nominee by targeting a supermajority of votes, and can do so at no cost, i.e., no contributions are paid in equilibrium. Dal Bo (2000) gives the first intuition for this result. In his model a single lobby is able to create a prisoner dilemma among voters by locking them into an equilibrium in which no one is pivotal, and hence every voter will be willing to vote against her preferred outcome for an infinitesimally small contribution. (Since she is not pivotal and cannot affect the outcome, she would forego the contribution if she did not vote against her preferred outcome.) We then explore other alternatives that still allow the president to overcome the gridlock, but limit his ability to shift the policy outcome towards his preferred point.

A key feature of our results is the importance of the "event" on which inducements may be conditioned. We elaborate this below. In section 2 we describe the conventions and maintained assumptions of our analysis. In section 3 we present our main results in which the president and interest groups may offer inducements to senators to vote in particular ways, where the inducements schedule is conditioned on the entire voting profile. In section 4 we extend these results in three ways. We constrain inducement schedules to those that only may be conditioned on an individual senator’s vote or, alternatively, on the final outcome (rather than the entire voting profile). We also examine the possibility of interest groups focusing inducements on the nomination by the president rather than on the votes of senators. In section 5 we conclude. All proofs of results are found in an appendix.

2 Contributions Models

2.1 Motivation

The motivation for our problem is that there are “too many” instances, given the filibuster, in which pure policy voting by senators leads to gridlock when deciding on Supreme Court nominations. In these instances the policy of the eight-member Court following the departure of a justice from the full Court remains in place since no replacement justice can be confirmed. Since the empirical reality appears quite different, with a reduced Court a temporary circumstance, we want to identify the mechanisms by which gridlock is overcome.\textsuperscript{6}

\textsuperscript{6}Although a reduced court is a temporary aberration in the case of the Supreme Court, it is not at all uncommon for vacancies in the lower federal courts to remain unfilled, sometimes for years at a time, owing to gridlock in the Senate (which must confirm such appointments).
2.2 Conventions

To proceed we use the following notation and conventions:\(^7\):

- Senators are labeled by their ideal points, \( S_s \), and ordered from left to right. The filibuster gridlock region is \([S_{41}, S_{60}]\) – there are 41 senators at or to the left of \( S_{41} \) and there are 41 senators at or to the right of \( S_{60} \).

- The original nine-person Court is described by a left-to-right ordering of the ideal points of the justices, \( \{J_1, ..., J_9\} \). The eight-person Court resulting from a departure of one of the justices is an order-preserving relabeling, \( \{J_1^*, ..., J_8^*\} \).

- \( r \) represents the (reversion) policy of the eight-person Court after the departure of a justice from the original nine-person Court.

- \( J_N \) is the president’s nominee.

- \( E \) is the (equilibrium) policy (forecast) of the nine-person Court if the president’s nominee is confirmed. The upper bound of \( E \) is \( \overline{E} \).

- \( cp \) is the cut point between \( r \) and \( E \): senators to the left of \( cp \) prefer \( r \) to \( E \) whereas senators to the right of \( cp \) prefer \( E \) to \( r \).

- \( s = CP \) is the first senator with an ideal point to the left of or equal to \( cp \), i.e. \( S_{CP} \leq cp \).

- The president is a "he," a senator is a "she," and an interest group is an "it."

In words, there is a retirement on the Court. The eight-member Court remaining is forecast to produce policy at \( r \). If \( r \) is in the filibuster gridlock region, any change from \( r \) (through a new appointment) will be opposed by at least 41 senators – so the filibuster prevents a vote on any presidential nominee. That is, for any nomination by the president, and its equilibrium forecast for the Court, \( E \) (a function of the nominee), this nominee will be opposed by at least 41 senators.

In order for the president to succeed in having the Senate confirm a nominee, producing new policy \( E \), he must induce each senator in \( \{S_{41}, ..., S_{CP}\} \) to vote contrary to her policy preferences. If successful, the president can overcome the filibuster and move the policy outcome to the pivotal judge in the new nine-person Court. As shown in Rohde and Shepsle (2006), regardless of how extreme the nominee is, the most extreme pivotal judge will be the fifth judge in the original eight-person Court. We denote this upper bound as \( \overline{E} \), the furthest a president can move Court policy with a successful appointment.\(^8\)

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\(^7\) Throughout we assume, without loss of generality, that the president’s ideal policy lies to the right of the gridlock region. A symmetric set of conventions may be written for a left-wing president and, with small modifications, for a moderate president. Nothing of substance is sacrificed by restricting things as we do.

\(^8\) Proposition 0 of Rohde-Shepsle (2006) shows that \( E \) will be \( \overline{E} \) if \( J_N \) is to the right of the fifth justice on the eight-member court, and \( J_N \) itself if it is the fifth justice on the new Court.
We introduce two classes of agents who are in a position to attempt to influence senatorial voting. The president, in addition to nominating a candidate of a particular type, may be in a position to offer compensation to any of the one hundred senators. Special interest groups (lobbyists), on the other hand, are assumed to be senator-specific in the sense that each of them may attempt to influence a specified senator only.

The form that their respective offers take, to be made precise below, is a menu of payments to senators (Bernheim and Whinston, 1986). This menu offers payment conditional on various events. We consider several alternatives: payments conditional on the entire voting profile, on the particular vote of a senator, or on the final outcome. The first takes the form "if senator \( s \) votes \( v_s \) and the remaining profile of votes is \( v_{-s} = (v_1, \ldots, v_{s-1}, v_{s+1}, \ldots, v_{100}) \), then her compensation is \( c_s(v) \)." The second takes the form "If senator \( s \) votes \( v_s \) then her compensation is \( c_s(v_s) \)." The final contingency takes the form, "If the nominee is confirmed (rejected), then the compensation for senator \( s \) is \( c_s(E) \) [\( c_r(r) \)]." We organize the analysis in terms of the conditioning event and on whether the president alone, interest groups alone, or the president together with special interest groups offers compensation to senators for their votes. As is seen below, we mainly emphasize conditioning on the entire profile, developing the other possibilities as extensions. We will also explore in the section on extensions the possibility that interest groups offer inducements to the president to nominate in a manner they prefer, instead of bribing senators to vote as they prefer.

2.3 Maintained Assumptions

In order to avoid repeating contextual details of our models, we will maintain the following unless explicitly revised:

- There are three types of agents: a president (P), one hundred senators (\( S_s \)), and one hundred lobbies (\( l_s \)), where \( s \in \{1, \ldots, 100\} \).

- Each lobby is associated with a specific senator (hence we refer to it as a local lobby).\(^9\) It may offer contributions to at most its own senator. (In some models below lobbies are inactive and thus offer no contributions.)

- The president proposes a nominee (\( J_N \)) and may also offer contributions to any senator. (In some models below the president nominates only and may not offer contributions.)

- The policy outcome is assumed to be a point in \( \mathbb{R} \). Each agent derives utility from this outcome according to a symmetric and strictly single-peaked utility function on \( \mathbb{R} \), written \( P(\cdot), S_s(\cdot), \text{and } L_s(\cdot) \), for the president, senator \( s \), and lobbyist \( l_s \), respectively.\(^{10}\)

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\(^9\) In Console Battilana (2005) the "local" lobbies are unique to a particular nation, whereas transnational lobbies are able to influence the representatives of any nation. In the present paper we mean by "local" that a lobbyist is specific to a particular senator’s state. (To simplify our analysis we assume that each of the senators from a state is associated with a distinct lobbyist.)

\(^{10}\) From symmetry it follows that preferences are monotonically decreasing in Euclidean distance from the peak, or ideal point, of the utility function.
Each agent values policy and contributions additively.

Every senator votes for or against the nominee. If sixty or more vote in favor, the nominee is confirmed.

The reversion policy outcome, upon a rejection of the president’s nominee, is \( r \). We normalize utilities so that \( P(r) = S_s(r) = L_s(r) = 0 \). We assume, without loss of generality, that the president’s ideal policy lies to the right of \( r \). For \( E \geq r \), if \( S_s(E) > 0 \) or \( L_s(E) > 0 \), we say that the senator or the lobby prefers \( E \) to \( r \). If \( S_s(E) < 0 \) or \( L_s(E) < 0 \), we say the senator or the lobby prefers the reversion policy. If \( S_s(E) = 0 \) or \( L_s(E) = 0 \), we say the senator or the lobby is indifferent. (We do not assign any indifference breaking rule.)

Unless otherwise noted in the Equilibrium subsection, senators are ranked according to \( S_s(E) \)\( S_s +1 \)\( E \).

An alternative ranking will occasionally be used. It is defined by the mapping \( L S_s (E) = L_s (E) + S_s (E) \). For each \( E \), order senators so that \( L S_s (E) \leq L S_s +1 (E) \). Note, for \( E' \neq E'' \) and two particular senators \( i \) and \( j \), that we can have that \( L S_i (E') < L S_j (E') \) and \( L S_i (E'') > L S_j (E'') \), i.e., the ordering is not necessarily preserved over \( E \). Consider \( L S_{41} (E) \), the function that maps \( E \) to the payoff of the forty-first senator. The identity of the forty-first senator associated with different elements of the domain of \( E \) may vary, so the function \( L S_{41} (E) \) is not necessarily continuous. Furthermore, by the earlier normalization assumption, \( L S_s (r) = 0 \).

We assume that the functions \( P(\cdot), S_s(\cdot), \) and \( L_s(\cdot) \) are common knowledge, as well as the location of the reversion point \( r \) and the policy \( E \) resulting from proposal \( J_N \).

3 Contributions Conditional on Entire Voting Profile

3.1 Neither Lobbies nor the President Offer Contributions

Before we begin our main analysis, we note that the setting in which no contributions are possible from any agent is the original Rohde-Shepsle (2006) model. There the president proposes a nominee to fill a vacancy, and senators vote according to their preferences between \( r \) and \( E \), the latter the median of the new nine-member Court determined by confirmation of the presidential nomination.\(^{11}\) As observed earlier, in this case there is often gridlock – whenever \( r \in [S_{41}, S_{60}] \) there is insufficient support to confirm the nominee.\(^ {12}\) We now determine whether inducements from special interests (possibly including the president) can break the gridlock. We first explore the impact

\(^{11}\)Technically it should be written \( E(J_N) \), since the median of the full nine-member court will depend upon the location of the newly confirmed justice. We will normally not write this out in full, except to avoid confusion, so \( E \) should be understood implicitly as a function of \( J_N \).

\(^{12}\)There are other cases as well in which presidential preferences between \( E \) and \( r \) conflict with those of sixty or more senators for any choice of \( J_N \).
of only local lobbies offering inducements to their respective senators to vote in a particular way. Then we examine the case of only the president offering inducements (in addition to proposing a nominee). Finally, we consider the result of both the president and local lobbies attempting to influence the votes of senators.

### 3.2 Only Lobbies Offer Contributions

#### 3.2.1 Strategy sets

The president proposes a nominee that will result in policy \( E \) if approved. Each senator votes in favor or against the proposal, written \( v_s = 1 \) and \( v_s = 0 \), respectively. We write a voting profile as \( v = \{ v_1, v_2, \ldots, v_{100} \} \), and call the set of all possible voting profiles \( V \). Note that there are \( 2^{100} \) voting profiles. Each lobby \( l_s \) offers contribution \( c_{vl_s} \) to senator \( s \), conditional on the entire voting profile \( v \). Thus \( c_{vl_s} : V \rightarrow \mathbb{R} \) describes the strategy space for contributions.\(^{13}\) The president and each of the lobbyists receive utility from the policy implemented, but must net out the cost from any contributions paid. In this subsection, the president makes nominations only and thus only interest groups must net out the contributions they pay. Each senator maximizes the sum of the contributions from her local lobby and her personal utility from the policy outcome. Her strategy space is \( v_s : \mathbb{R} \rightarrow \{0, 1\} \).\(^{14}\)

#### 3.2.2 Stages of the game

The game unfolds in three stages.

**Stage 1.** Given a reversion policy \( r \), the president proposes a nominee that, if approved, will lead to policy \( E \). This policy is common knowledge.

**Stage 2.** Simultaneously and non-cooperatively each lobby \( l_s \) offers a contribution schedule to its corresponding senator \( s \), conditional on the entire voting profile. Each senator \( s \) observes only the contribution schedule offered by \( l_s \).

**Stage 3.** The senators, observing only the presidential nomination and the contribution schedule offered to them, simultaneously cast their votes. If strictly more than 40 senators vote against the nomination, then \( r \) is imposed; otherwise \( E \) results.

#### 3.2.3 Equilibrium

We look for subgame perfect pure Nash equilibria. We rank senators and lobbies according to \( LS_s \) and establish the following claim:

**Claim 1** In equilibrium the president proposes \( \hat{E} = \max \{ E \text{ s.t. } LS_{41}(E) \geq 0 \text{ and } E \leq \bar{E} \} \). On the equilibrium path of play, senators in \( \{S_{41}, \ldots, S_{100}\} \) ordered according

\(^{13}\)The contribution functions map the set of all possible voting profiles to the real numbers. Suppose for example there were only three senators. Then there would be \( 2^3 = 8 \) possible voting profiles. A lobbyist’s contribution function offers an amount of money to its senator that depends on the full voting profile, hence \( c^s_{vl_s} \) is a vector with eight components.

\(^{14}\)The domain of \( v_s \) is \( \mathbb{R} \times \mathbb{R} \), the cross product of the one-dimensional policy space and the scalar contribution. Since we assume these combine additively, the domain collapses to \( \mathbb{R} \).
to \(LS_n\) play \(v_s = 1\); senators in \(\{S_1, \ldots, S_{40}\}\) play \(v_s = 0\); lobbies in \(\{l_{41}, \ldots, l_{100}\}\) with \(L_s(E) > 0\) pay \(\min\{L_s(E), \max[0, -S_s(E)]\}\) to their senator, and all other lobbies pay zero.

Notice that in equilibrium all senators are pivotal. The proof of this (and other) result(s) is placed in the appendix. Here we offer some intuition. In stage 2 and 3, lobbies and senators are de facto facing a binary alternative between a reversion policy outcome \(r\) and a proposed policy outcome \(E\). Given the solution to this continuation game, the President in stage 1 will propose his preferred policy among those that would be approved, i.e. those such that \(LS_{41}(E) \geq 0\). Therefore, we look directly at the continuation game.

Given a generic proposal \(E\), senators are ordered according to \(LS_n(E)\). In stage 3, each senator will choose the vote that maximizes the sum of the contribution she receives, \(c'_v\), and her personal utility \(S_s(.)\) from the resulting voting profile. In stage 2, each lobby preferring the proposal of the president is willing to contribute no more than its utility \(L_s(E)\). However, each lobby will want to contribute the minimum possible while ensuring the preferred outcome is obtained. Thus, if \(S_s(E) + L_s(E) > 0\), the lobby can either contribute nothing (if the senator already prefers \(E\)) or contribute just \(-S_s(E)\) (if the senator prefers the reversion policy, but can be recruited for less than the entire benefit of policy \(E\) to the lobby). Furthermore, in equilibrium a lobby will give positive contributions only to pivotal senators. Contributions to non-pivotal senators can be saved, because that senator cannot affect the outcome. So the offer will be of the type "If you vote for \(J_N\) and in the equilibrium voting profile you are pivotal, I offer to compensate you for any losses you might incur and give you an additional \(\varepsilon\), up to a maximum contribution of \(L_s(E)\) (my benefit from the proposal). If you are not pivotal and vote for \(J_N\), I give you \(\varepsilon\). For any voting profile in which you vote against \(J_N\), I offer you nothing". The symmetric argument holds for the lobbies preferring the reversion policy. Their offer will be of the type "If you vote against \(J_N\) and are pivotal, I am willing to compensate you for any loss you might incur plus \(\varepsilon\), up to \(-L_s(E)\) (my loss from the other outcome). If you are not pivotal and vote against \(J_N\), I give you \(\varepsilon\). For any voting profile in which you vote for \(J_N\), I give you nothing". Therefore, in stage 3, each senator with \(S_s(E) + L_s(E) \geq 0\) will vote for the proposal. Hence, for any proposal \(E\) such that \(LS_{41}(E) \geq 0\), there is a continuation game in which 60 pivotal senators vote for it.\(^{15}\)

In the first stage, the president would like to move the outcome as much to the right as possible. Therefore, he will propose a nominee leading to the outcome \(E\) he most prefers among those that would be (super)majority approved. This is the policy outcome \(E\) most to the right such that the associated \(LS_{41}(E)\) is weakly positive.

\(^{15}\)Note that other equilibria exist in which strictly more than 60 senators vote for any proposal, no matter how far to the right. In such equilibria, no senator or lobbyist is pivotal, and thus none has an incentive to deviate; even outcome \(E\) could be approved. Since these rely on arbitrary ways of breaking indifference, we put them to one side. In other sections we look at equilibria in which no agent is pivotal; however these are the only equilibria. Furthermore, in the zero probability case in which \(LS_{40} = LS_{41} = 0\), note that there exists a continuation game in which \(v_{40} = 0\) and \(v_{41} = 1\).
3.3 Only the President Offers Contributions

3.3.1 Strategy sets

In this section, only the president offers inducements; thus there are no local lobbyists. The strategies are as follows. The president proposes a nominee that results in policy $E$ if approved. In addition, the president may offer contributions to any senator $s$, conditional on the entire voting profile, which we denote as $c_{ps} : V \rightarrow \mathbb{R}$. Each senator votes in favor or against the proposal, $v_s = 1$ and $v_s = 0$, respectively, i.e., $v_s : \mathbb{R} \rightarrow \{0, 1\}$.

3.3.2 Stages of the game

Stage 1. Given a nomination-inducing event defining the reversion outcome $r$, the president proposes a nominee that produces policy $E$ if approved, and offers a schedule of contributions $c_{ps}$ to senators. The contribution schedule is conditional on the entire voting profile. $E$ and $c_{ps}$ are chosen to maximize the president’s utility net of contributions. $E$ and $r$ are common knowledge.

Stage 2. In addition to $E$ and $r$, each senator observes the contribution schedule offered to her only, and then votes. If strictly more than 40 senators vote against $J_N$, then $r$ prevails. Otherwise, $E$ is the equilibrium.

3.3.3 Equilibrium

We look for subgame perfect pure Nash equilibria, establishing our next claim.

Claim 2 If the president conditions contributions on the entire voting profile, there are multiple equilibria but all share the following characteristic: the president obtains his preferred policy $E$ at zero cost.

We again provide some intuition here and provide the proof in the appendix. The president contributes to multiple senators. To sixty-one senators he says, "I offer to compensate you for any losses you might bear (i.e. give you $-S_s(E)$ if $S_s(E) < 0$) and give you $\varepsilon$ more for any voting profile in which you vote for my nominee and are pivotal. For any voting profile in which you are not pivotal and vote for my nominee, I offer you $\varepsilon$. If you vote against my nominee, I give you nothing." Each senator offered this contribution schedule has a dominant strategy regardless of her personal utility: vote for the nominee. Hence, 61 senators will vote for the proposal. But then, none of them is pivotal in equilibrium, and the president only has to pay them $\varepsilon$. In equilibrium, $\varepsilon$ is vanishingly small. This result relies on the ability of the president to recruit multiple senators at the same time. The president is in effect creating a prisoner dilemma. Senators with $S_s(E) < 0$ would be better off if $r$ were chosen. However, in equilibrium they are not pivotal, and hence they cannot affect the outcome. They can be "recruited" for $\varepsilon$ infinitesimally small. The ability to contribute to multiple senators and at the same time conditioning contributions on the entire voting profile gives the president the possibility of obtaining his first-best outcome for free. Dal Bo (2000, 2006) and Console Battilana (2005, 2006) show a similar result in a different setting.\footnote{Non-uniqueness of this equilibrium arises from the fact that the president can approach any sixty-one or more senators. The result of Dal Bo and Console Battilana is for a group of three committee members instead of a group of sixty-one senators.}
3.4 Both Lobbyists and President Offer Inducements

3.4.1 Strategy sets

In this section we assume that each senator can receive contributions from both a local
lobby and the president, conditional on the voting profile. Therefore, the strategy space
of each lobby is \( c_l : V \rightarrow \mathbb{R} \), and the strategy space of the president is \( c_{ps} : V \rightarrow \mathbb{R} \).
Each senator observes her personal utility, and only the contributions offered to her
by both the local lobby and the president. Her objective is to maximize the sum of
personal utility plus contributions received. Hence, given additivity of policy utility
and contributions, senator s’s strategy space is \( v_s : \mathbb{R} \rightarrow \{0, 1\} \).

3.4.2 The Game

Stage 1. Given a commonly known reversion policy \( r \), the president proposes a
nominee that, if approved, will lead to commonly known policy \( E \).

Stage 2. Simultaneously and non-cooperatively, the president and the local lobbies
offer contributions. The president offers \( c^{ps} \) to each senator, conditional on the
entire voting profile, while each local lobby \( l_s \) contributes only to its corresponding
senator, \( c^l_s \), also conditional on the entire voting profile.

Stage 3. Each senator observes the president’s nominee and the contributions offered
to her and then votes. If more than 40 senators vote against \( J_N \), then \( r \) is
sustained. Otherwise, \( E \) results.

3.4.3 Equilibrium

We focus on subgame perfect pure Nash equilibrium but we employ a refinement:

Equilibrium Refinement: No senator is offered positive contributions if she is not
pivotal, both on and off the equilibrium path.

Claim 3 If local lobbies and the president condition contributions on the entire voting
profile, there exists an equilibrium in which a nominee yielding the policy outcome
preferred by the president, \( E \), is proposed and approved, and the president pays zero
contributions. Furthermore, all equilibria must have these two properties under the
refinement.

As before, we provide the formal proof in the appendix, giving only some intuition
here. Since we are looking for subgame perfect Nash equilibria, we solve the game
backwards. There is always an equilibrium in which at least sixty-one senators vote
in favor of the proposal, each local lobby offers zero for all voting profiles and the
president offers zero contribution to any senator for any voting profile. No senator is
pivotal, hence no senator has an incentive to deviate at stage 3 – the outcome would
not change and her contributions would still be zero. No local lobby can influence the
members. Their equilibrium is unique because it entails a contributions schedule in which the unique
coealition of all three members is approached with an offer. Thus, our result extends theirs to arbitrarily
sized committees.
outcome, since its corresponding senator is not pivotal; hence no local lobby has an incentive to deviate at stage 2. The president is obtaining his preferred policy for free, so the president has no incentive to deviate at stage 2. Thus, in stage 1 the president can propose any nominee that yields $E$ and it is approved. He has no deviation as it would involve him proposing a nominee yielding a policy he prefers no better.

An equilibrium with the property that the president obtains his preferred outcome for free always exists. We now show that all equilibria under the refinement possess these properties. Consider any candidate equilibrium in which the president either pays positive contributions or he does not obtain his preferred policy or both. The president has a deviation from any such equilibrium. He can play a pivot strategy (defined below) to induce strictly more than sixty senators to vote for his proposal, and pay each one of them only a vanishingly small $\varepsilon > 0$. In fact, for any candidate equilibrium, the Equilibrium Refinement implies that each senator is offered no contribution from her local lobby whenever she is not pivotal. Since no senator is pivotal, if the president induces sixty-one senators to vote for the proposal, each one of these senators needs only to be paid $\varepsilon > 0$. If she is offered $\varepsilon$ by the president to vote $v_s = 1$, senator $s$ will do so since she receives $\varepsilon$ more in contributions then if she voted $v_s = 0$, and her personal utility is $S_s(E)$ regardless of her vote. In effect, as long as the president can construct a contribution schedule for $s$ such that $v_s = 1$ is a dominant strategy for any possible voting profile, then he can create a prisoners’ dilemma by offering this schedule to sixty-one senators. Accordingly, the president will prevail, even if all lobbies and all senators are against his proposal.

For an outcome satisfying neither property described in Claim 3, we show that the pivot strategy is a deviation, establishing that this candidate outcome cannot be an equilibrium. The demonstration works as follows: the president targets a group of sixty-one senators. Given any senator $s$, denote $c_{l_s}^{0,v_s}$ as the contribution offered by lobby $l_s$ to senator $s$ in the candidate outcome when $v_s = 0$ and the other ninety-nine senators vote according to profile $v_{-s}$. The president can play the following pivot strategy with sixty-one senators:

<table>
<thead>
<tr>
<th>$v_{-s}$</th>
<th>$v_s$</th>
<th>President’s Contribution $c_{ps}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $v_{-s}$ has strictly less than 59 voting 1</td>
<td>1</td>
<td>$c_{l_s}^{0,v_{-s}} + \varepsilon$</td>
</tr>
<tr>
<td>b) $v_{-s}$ has exactly 59 voting 1</td>
<td>1</td>
<td>$c_{l_s}^{0,v_{-s}} + \varepsilon + \max[0, -S_s(E)]$</td>
</tr>
<tr>
<td>c) $v_{-s}$ has strictly more than 59 voting 1</td>
<td>1</td>
<td>$c_{l_s}^{0,v_{-s}} + \varepsilon$</td>
</tr>
<tr>
<td>d) For any $v_{-s}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Pivot Strategy.

Informally, the president is constructing a schedule that says "No matter what you are receiving to vote 0, I am always going to offer you $\varepsilon$ more to vote 1, and I will also compensate you for your personal outcome-dependent utility loss if you are pivotal." Here is the intuition for each circumstance displayed in the table:

a) If strictly less than 59 other senators vote 1, then senator $s$ is non-pivotal. Regardless of her vote, the outcome will be $r$ and her personal outcome-dependent payoff will be $S_s(r) = 0$. If she votes 0 the contribution offered to her from lobby $l_s$ is $c_{l_s}^{0,v_{-s}}$. The president however offers $c_{l_s}^{0,v_{-s}} + \varepsilon$; hence voting 1 is indicated when strictly
less than 59 others vote 1.

b) If exactly 59 other senators vote 1, senator \( s \) is pivotal. If she votes 0 her payoff is \( S_s(r) = 0 \) plus any contribution she might receive from lobby \( l_s \). If she votes 1, she receives her personal payoff is \( S_s(E) \) plus any contribution she might receive. The president offers \( c_{ls}^{0,v,s} + \max[0, -S_s(E)] + \varepsilon \) for any voting profile with exactly 59 others voting 1. If \( S_s(E) < 0 \), the senator would have a net gain of \( \varepsilon \) if she votes 1 rather than voting 0, while she would have a net gain of \( S_s(E) + \varepsilon > 0 \) if \( S_s(E) \geq 0 \) for voting 1 instead of 0. Hence, when exactly 59 other senators vote 1, senator \( s \)'s best response is to vote 1.

If strictly more than 59 senators other than \( s \) vote 1, i.e. at least 60 senators vote 1, then senator \( s \) is not pivotal, and the outcome is \( E \) regardless of her vote. If she votes 0, senator \( s \) has a personal outcome-dependent utility \( S_s(E) \) and is offered \( c_{ls}^{0,v,s} \) in contributions to vote 0. If she votes 1, the personal outcome utility will still be \( S_s(E) \) and the president additionally offers \( c_{ls}^{0,v,s} + \varepsilon \). Her net benefit will be \( \varepsilon \) higher if she votes 1.

Therefore, given any voting profile, senator \( s \)'s best response is to vote 1. We had started by assuming there was a candidate outcome in which the president either paid positive contributions or the president’s proposal was rejected. Since the president, when deviating from this candidate outcome, plays the pivot strategy with 61 senators, the president will actually have to pay only what he offered in the case in which strictly more than 59 senators vote 1 (row c in the table above). But in row c, the corresponding \( c_{ls}^{0,v,s} \) must be zero by the refinement (otherwise the candidate outcome would not have been consistent with the refinement). Thus, if at least 60 other senators vote 1, senator \( s \) is not pivotal. Therefore, when playing the pivot strategy with 61 senators, the president obtains the approval of any proposed policy at a cost of \( 61\varepsilon \).

But if this deviation by the president is possible, then no candidate outcome in which the proposal of the president is rejected can be an equilibrium. If it were an equilibrium, then there can be no deviation for the president. But, as we just demonstrated, the president can play the pivot strategy with 61 senators and obtain his preferred policy at a vanishingly small cost. Likewise, there can be no equilibrium in which the president pays positive contributions, because the president could again deviate and play the pivot strategy with \( \varepsilon \) small enough to reduce his contributions.

Therefore, even if we allow for the possibility of local lobbies, there always exists an equilibrium in which the president proposes any nominee resulting in his preferred policy \( \overline{E} \) and it is approved at no cost to the president. Any nominee \( J_N \) to the right of the fifth justice in the current eight-member Court produces this outcome. Furthermore, from the Equilibrium Refinement, we obtain that all equilibria possess these properties.

This is a very strong result. Even if all lobbies and all senators dislike the president’s nominee, the president can still manipulate the votes so that no one is pivotal, and hence deny influence to any local lobby or senator. Note also that this is true even in very extreme cases. Consider for example a change from \( r \) to \( \overline{E} \) that improves the president’s utility by 1 and decreases the utility of each local lobby by 100,000,000.
The president will still be able to impose his preferred nominee at no cost. This raises an interesting possibility. It would pay the lobbyists to focus on offering inducements to the president to refrain from offering a nominee who would impose such large costs on them. We address this possibility in the extensions section below.

### 3.5 Ideological Cost

For the case of the president conditioning contributions on the entire voting profile, we have established that he can nominate any candidate, no matter how extreme. For an eight-member court, \( \{J_1^*, \ldots, J_8^*\} \), any nominee-justice \( (J_N^*) \) to the right of this Court’s fifth justice \( (J_5^*) \) establishes \( J_5^* \) as the median of the full nine-member Court and, since \( J_5^* = \bar{E} \), the best equilibrium available to the president is achieved.

However, there may be an ideological cost for senators to support the president’s nominee. That is, quite apart from the outcome \( (J_N^*) \), constituents may disapprove of their agent supporting a nominee not to their liking, even if their senator were not pivotal. When facing re-election a senator whose constituency median is to the left of \( cp \) incurs constituency unhappiness if she votes in favor of an extreme nominee to the right. In effect, constituents assess their agent on the basis of agent actions, so for their assessment it only matters whether she voted in favor of the nominee or not. We call this ideological cost, defining it as \(-I(J_N^* - S_s)\): each senator to the left of \( cp \) (hence with \( S_s(E) < 0 \)) incurs an ideological cost when voting for the nominee. This cost is directly proportional to the distance between the ideal point of the proposed nominee and the ideal point of senator \( s \) (the latter a measure of median constituent preferences).

We argue that recruiting 61 votes by the president will not come for free anymore; the president will have to compensate senators in \( \{S_{40}, \ldots, S_{CP}\} \) for their ideological loss \(-I(J_N^* - S_s)\).

Rather than targeting 61 senators and compensating a few for their ideological loss (if any), the president could also choose to recruit only 60 votes. In this case, each of the 60 senators would be pivotal and thus each of them would have to be compensated for ideological loss (if any), utility loss (if any), and the contributions (if any) offered by local lobbyists for a vote against the proposal (holding the remainder of the voting profile fixed). In this instance, there are multiple equilibria, and their systematic

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17Note that we are assuming a non-binding budget constraint. If a senator is pivotal in equilibrium, the president can credibly commit to offer her more than the local lobby contribution plus her welfare loss in order to vote 1. Each targeted senator plays \( v_s = 1 \) in equilibrium because it is a dominant strategy. It could very well be that more than 60 senators and their respective lobbies would be better off if they could cooperate. However, every single senator has a unilateral incentive to deviate from such cooperation. In effect the president has created a prisoner dilemma.

18We thank Torsten Persson for raising this possibility.

19For tractability we assume that an ideological cost is borne only if a senator votes contrary to his constituency’s preference between \( r \) and \( E \) (and then it is proportional to the distance between the nominee and the constituency ideal); no ideological cost is borne by senators who vote with their constituency on this pairwise decision.

20We assume here that it is up to the president to compensate senators for the ideological costs they bear. (If lobbyists could also do this, there are coordination issues that must be addressed, something beyond the scope of the present paper.)
description is beyond the scope of this paper. Instead, we focus on a (plausible) circumstance:

\textit{Assumption H1:} It is cheaper to compensate the cheapest 61 senators for their ideological cost (if any) rather than compensating 60 senators for their ideological cost, their utility loss, and their contributions loss.

\subsection*{3.5.1 Model}

The setup is the same as in section 3.4, except that now we add the prospect of ideological cost for senators. As before, each senator derives utility from contributions received given the equilibrium voting profile \((c_{pa} \text{ and } c_{ls})\), and from the personal utility which depends on the outcome, \((S_s (E))\). However, in addition senators in \(\{S_1, ..., S_{CP}\}\) face an ideological cost of \(-I (J_N - S_s (E))\) when voting in favor of the proposal.

\subsection*{3.5.2 Equilibrium}

We look for subgame perfect Nash equilibria with the refinement that no contributions are offered to non-pivotal legislators by local lobbies.

\textbf{Claim 4} Under H1, in equilibrium the president proposes \(J_N = \argmax_J [P(E(J)) - \sum_{s=40}^{CP} I(J - S_s (E))]\), and he pays \(I(J_N - S_s (E))\) to senators in \(\{S_{40}, ..., S_{CP}\}\), and zero to all other senators. The proposal is approved. No nominee to the right of \(J^*_N\) will ever be proposed.

There exists an equilibrium in which the president pays \(I(J_N - S_s (E))\) to senators in \(\{S_{40}, ..., S_{CP}\}\) and no one has an incentive to deviate. No senator has an incentive to deviate in stage three: since she is not pivotal, she would have the same personal utility regardless of her vote, receive zero contributions from local lobbies regardless of her vote, and is compensated by the president for her ideological loss if she votes in favor of the proposal. The lobbies have no incentive to deviate in stage two, because their senator is not pivotal and therefore they cannot influence the outcome. The president has no incentive to deviate in stage two. This is because each senator with \(S_s (E) < 0\) incurs an ideological cost from voting \(v_s = 1\) regardless of whether her vote influences the outcome and thus has to be compensated for that loss. The cheapest 61 senators to target are senators \(S_{40}\) to \(S_{100}\), ranked on the basis of \(S_s (E)\) (since their ideological loss is perfectly correlated with this). In previous sections we had established that no one is paid positive contributions if not pivotal. However, in this section, senators face a cost that is dependent only on their vote, not on being pivotal. If the president deviates to offer a payment schedule in which he zeroes out one of the 61 senators (one with \(S_s (E) < 0\)), then that senator would not support the president and thus only 60 senators are voting for the proposal. In this circumstance, every senator would be pivotal, and the president would have to compensate each one of them for ideological loss, contribution loss, and personal utility dependent on the outcome. But then the president would be spending more in contributions by assumption H1.

In stage one, the president has no deviation: he is proposing the policy that maximizes his utility, net of contributions paid. He will never propose a candidate to the
right of $J_5^*$ because that would increase the contributions paid without shifting the outcome any further to the right.

There is no other equilibrium that contradicts claim 4. Suppose there were such a candidate equilibrium. Then the president could deviate by playing the pivot strategy of section 3.4, additionally compensating supporting senators for any ideological loss. The president would make the following offer to 61 senators: "If you are pivotal, and vote in favor of the proposal, I will compensate you for any outcome-dependent utility loss you have, any contributions you would receive if you voted otherwise (in the candidate equilibrium), and any ideological cost you incur. On top of that, I will give you $\varepsilon \geq 0$. If you are not pivotal, and vote for the proposal, I will compensate you for any ideological cost you incur, and give you $\varepsilon$ on top of that. If you vote against my proposal, I will give you nothing". To the remaining 39 senators the president always offers zero. Each of the 61 senators has the dominant strategy of voting for the proposal. Hence, in equilibrium, 61 senators will be voting for the proposal and no one will be pivotal. The president only has to compensate each senator for their ideological loss, if any, plus $\varepsilon$. This deviation by the president establishes that the candidate outcome cannot be an equilibrium.

4 Extensions

In this section we explore several variations on our model. First we examine the possibility of either the president or special interest groups conditioning their contributions on the vote of a senator or on the final outcome. We also explore the possibility of lobbyists make contributions directly to the president in exchange for a nominee they prefer.

4.1 Contributions Conditional on the Vote Only

In this subsection we require contributions to be conditional on the vote of each senator. We look only at the case in which we have both local lobbies and the president attempting to influence senatorial votes (since the instances where one or more of these do not make contributions are special cases). Given that there is a coordination problem between the president and the local lobbies sharing his preference, with resulting multiple equilibria, we focus attention on the equilibria in which in stage 2 the president coordinates with the local lobbies that prefer his proposal to the reversion policy and extracts the full surplus resulting from their cooperation. So, if there is a case in which the president alone is unable to recruit the necessary number of votes, but the president together with the local lobbies with the same preferences could jointly recruit a sufficient number of votes, we assume that the president is capable of inducing the lobbies to offer contributions to obtain the equilibrium the president prefers.

The strategy sets are as follows: the president proposes a nominee, $J_N$, and offers contributions conditional on the vote, $c_{ps} : \{0,1\} \rightarrow \mathbb{R}$. As before, the president can offer a contribution schedule to each senator. Each lobby offers contributions to its senator only, conditional on the vote, $c_{l_s} : \{0,1\} \rightarrow \mathbb{R}$. Each senator observes contributions offered to her only, as well as her personal utility, and votes, $v_s : \mathbb{R} \rightarrow \{0,1\}$. 

17
The game proceeds as follows:

**Stage 1.** The president nominates a candidate, $J_N$, that will result in policy $E$ if successful.

**Stage 2.** The president and the local lobbies simultaneously offer contributions to each senator $s$ conditional on her vote only. While the local lobbies cannot coordinate among themselves, the president can impose coordination among the local lobbies sharing his preferences. The president is free to offer contributions $c_{ps}$ to any senator, while each local lobby $l_s$ can only offer contributions to its corresponding senator $s$.

**Stage 3.** Each senator observes the proposal and the contributions offered to her only and casts a vote. If the number of senators voting in favor of the proposal is sixty or more, $E$ is the final outcome. Otherwise, the reversion policy $r$ results.

Our equilibrium concept, as before, is pure subgame perfect Nash. We refine equilibria to those in which no lobby offers positive contributions for a vote in favor of the alternative associated with its least favorite outcome. We refine equilibria further to allow the president to coordinate local lobbies in order to extract the maximum surplus. That is, using his asymmetric position, the president extracts the full willingness-to-contribute from each lobby that prefers the nominee proposed. Local lobbies that prefer the proposed policy are willing to accept the coordination because they would be (weakly) worse off in its absence. The following claim characterizes our results.

**Claim 5** Order all senators according to $LS_s(E)$. Given a reversion policy $r$ and generic proposal $E$, $(r \leq E \leq \overline{E})$, there exists a continuation equilibrium in which $E$ is approved if $P(E) \geq \sum_{s=41}^{100} \max[0, -LS_s(E)]$. If the president can impose coordination and extract the full surplus from the lobbies, the reversion policy $r$ can never arise when this sufficient condition is satisfied. Then, in the first stage, the president proposes $\arg\max_J \{P(E) - \sum_{s=41}^{100} \max[0, -LS_s(E)]\}$.\footnote{Recall that $E$ depends on $J$.}

The proof of this is in the appendix. We look at the continuation equilibrium given a status quo and a proposal $E \geq r$. In stage 3 there is always an equilibrium in which strictly more than 60 senators vote in favor of the proposal, and no contributions are offered. This is an equilibrium in which no senator is pivotal, hence no senator has an incentive to deviate and no local lobby can influence the outcome. The president obtains his preferred policy for free, so he has no deviation. This equilibrium might be one in which some senators are playing a weakly dominated strategy.

Another potential equilibrium is one in which senators are pivotal. Can the president and the local lobbies (who prefer $E$ over $r$) jointly recruit 60 senators? Absent

\footnote{When writing out contributions offered to a single senator, we order them with the contribution offered for a $v_s = 0$ vote first; for example, $c_{ps} = \{0, 5\}$ means the president offers 0 for $v_s = 0$ and 5 for $v_s = 1$.}
contributions from the president, each senator would vote based on her own preferences and the contributions received by the corresponding local lobby. If a given senator is against the proposal ($S_s(E) < 0$) but the corresponding local lobby is in favor ($L_s(E) \geq 0$) and is willing to offer contributions up to its maximum benefit, then the president needs to contribute only $\max[0, -S_s(E) - L_s(E)]$ to secure senator $s$’s vote. Absent coordination, the local lobby might not be willing to offer any contributions. However, if the president can impose coordination on the lobbies, then each lobby will contribute a positive amount for a vote in favor of outcome $E$, since the contributions offered are not wasted in a coordination failure. For those senators with $S_s(E) + L_s(E) < 0$, the president needs to contribute only $\max[0, S_s(E) + L_s(E)]$ to secure senator $s$’s vote. Absent coordination, the local lobby might not be willing to offer any contributions. However, if the president can impose coordination on the lobbies, then each lobby will contribute a positive amount for a vote in favor of outcome $E$, since the contributions offered are not wasted in a coordination failure. For those senators with $S_s(E) + L_s(E) < 0$, the president needs to contribute $\max[0, S_s(E) + L_s(E)] = LS_s(E)$, which is less than $-S_s(E)$ when the local lobby offers no positive contributions for $E$. In order for such an equilibrium to be sustainable, however, the president needs to be willing to compensate all the cheapest necessary senators that would otherwise vote against the proposal. Given the proposal $E$, senators are ranked according to $LS_s(E)$. Then, senators in $\{S_{41}, \ldots, S_{100}\}$ are the cheapest to recruit. The condition for $E$ to result in equilibrium will thus be $P(E) \geq \sum_{s=41}^{100} \max[0, -S_s(E) - L_s(E)]$. In stage 1, the president applies constrained maximization: chose the best $E$ among those that would be approved: $\arg\max_J \{P(E) - \sum_{s=41}^{100} \max[0, -LS_s(E)]\}$. Recall that the set $\{S_{41}, \ldots, S_{100}\}$ may vary with $E$.

4.2 Contributions conditional on vote and ideological cost

We briefly extend the previous subsection allowing for the possibility of an ideological cost as defined in section 3.5: each senator with $S_s(E) < 0$ faces an ideological cost of $-I(J_N - S_s(E))$ when voting for the president’ s nominee. We introduce a new dummy variable: $d = \begin{cases} 1 & \text{if } S_s(E) < 0 \\ 0 & \text{otherwise} \end{cases}$. The strategy sets and the game are as in the previous subsection.

Intuitively, the difference here is that the president will have to compensate each senator with $S_s(E) < 0$ for her ideological loss in order to recruit her vote. In section 3.5 the president was able to pay only this ideological cost by locking the senators into the voting profile in which no senator was pivotal. Senators thus did not have to be compensated for their personal preference. However, the pivot strategy cannot be played when contributions are conditional on the vote; the president cannot force an equilibrium in which no one is pivotal. Therefore we focus on the equilibrium in which everyone is pivotal because this equilibrium gives us a sufficient condition: if there exist an equilibrium in which everyone is pivotal and a certain outcome $\hat{E}$ is achieved, then no $E < \hat{E}$ can result in any equilibrium. We argue that the following modification of claim 5 holds:

**Claim 6** Order all senators so that $LS_s(E) + dI(J_N - S_s(E)) < LS_{s+1}(E) + dI(J_N - S_{s+1}(E))$. Then, given a reversion policy $r$ and a generic proposal $E$, $(r \leq E \leq \hat{E})$, there exists a continuation equilibrium in which $E$ is approved if
\[
P(E) \geq \sum_{s=41}^{100} \{\max[0, -LS_s(E)] + dI(J_N - S_s(E))\}.
\]

If the president can impose coordination and extract the full surplus from the lobbies, the reversion policy \( r \) can never arise when this condition is satisfied. Then, in stage 1, the president proposes:

\[
\arg\max_J \left\{ P(E) - \sum_{s=41}^{100} \{\max[0, -LS_s(E)] + dI(J_N - S_s(E))\} \right\}.
\]

This claim follows from claim 5 and section 3.5, therefore we provide only an intuitive explanation. If every senator is pivotal, the president has to make sure the legislator is indifferent between voting in favor of the proposed nominee and voting against. In the previous section, absent ideological cost, he had to compensate the senator only for whatever the local lobby failed to contribute: \( \max[0, -S_s(E) - L_s(E)] \).

If the senator bears an ideological cost (i.e. if \( d = 1 \)), the president will also have to compensate her for this loss; therefore an additional \( I(J_N - S_s(E)) \) will have to be paid. The cheapest set of 60 senators is now determined by this double compensation: compensation for the ideological loss, if any, of the senator and compensation of the net effect between local lobbies and the senator. As in section 3.5 we conclude that no nominee to the right of \( J_5^* \) will ever be proposed. We denote the equilibrium outcome of claim 6 as \( \widehat{E} \) in order to make a comparison with other possible outcomes.

As in the previous section we admit the possibility of equilibria in which strictly more than 60 senators vote for the proposal of the president, whatever this might be: since no senator is pivotal, each senator would have to be only compensated for her ideological loss. These equilibria can be greater than \( \widehat{E} \). However, as long as the conditions of claim 6 are satisfied and the president can impose coordination, no policy to the left of \( \widehat{E} \) will result in equilibrium. Even if we are in the gridlock region and even if senators face an ideological cost when voting for the nominee’s proposal, the president can ensure that the policy is moved at least to \( \widehat{E} \).

### 4.3 Contributions conditional on the outcome

In this subsection, we return to the case with no ideological cost and briefly explore the possibility of conditioning the contributions on the outcome. The strategy spaces are as follows. The president proposes a nominee and offers a contribution schedule to each senator \( s \) conditional on the outcome: \( c_{ps} : \{r, E\} \rightarrow \mathbb{R} \). Each lobby offers a contribution schedule to its senator conditional on the outcome, \( c_{ls} : \{r, E\} \rightarrow \mathbb{R} \). Each senator observes her personal utility, as well as contributions offered to her only, and casts a vote, \( v_s : \mathbb{R} \rightarrow \{0, 1\} \). All the remainder of the game is as in section 4.1.

We argue that claim 5 holds only under the refinement that no senator plays weakly dominated strategies. In claim 5 we had shown that the president would pick the preferred nominee among those that can be approved by exactly 60 senators through his and lobbyists’ joint contributions. We now give some intuition for why this equilibrium still exists, and also explain why the president and lobbies cannot guarantee that senators will pick the voting profile that sustains this equilibrium unless we eliminate weakly dominated strategies.
We look at the continuation game, given a reversion policy $r$ and a generic $E \geq r$. If contributions can be made conditional only on the outcome, each senator is exactly indifferent on how to vote whenever she is not pivotal. If contributions were conditional on the vote, as before, a non-pivotal senator could be induced to vote one way or the other. However, this is not possible when contributions are conditional on the outcome. If she is not pivotal, she cannot affect the outcome by her vote and hence the contributions conditional on the outcome cannot break her indifference. Thus, there will always be an equilibrium (from which no senator has an incentive to deviate) in which at least 61 senators vote in favor of any given nominee and no contributions are paid. However, this may involve some senators playing weakly dominated strategies. If they were pivotal, they would be better off switching their vote.

We look at the equilibrium in which every senator voting in favor of the proposal is pivotal. As in section 4.1, the president would have to compensate each senator for her personal outcome-dependent utility loss, if any, and the contributions she might be able to receive if she votes against the proposal (and hence changes the outcome). In section 4.1, whenever the conditions of claim 5 were satisfied ($P(E) \geq \sum_{s=41}^{100} \max[0, -S_s(E) - L_s(E)]$ and coordination), the reversion policy could never have been an equilibrium. However, when contributions are conditional on the outcome, the president cannot make it a dominant strategy for a senator to vote for the proposal. In stage 3, each senator also considers the case in which she is not pivotal and she cannot affect the outcome. Since contributions are conditional on the outcome only, neither the president nor the lobby can break her indifference when she is not pivotal. So, even if the conditions of claim 5 are satisfied, we could have an equilibrium in which strictly more than 60 senators vote for the reversion policy and no one has a deviation. In order to ensure that the reversion policy will never be an equilibrium under the conditions of the claim, we have to add a refinement: no senator plays a weakly dominated strategy.\footnote{If weakly dominated strategies are available, then existence can be established, but not uniqueness of the equilibrium characteristics.}

## 4.4 Lobbies Contribute to the President

When both the president and local lobbies can make contributions conditional on the entire voting profile, we found that the local lobbies have no influence. The president can propose his preferred policy and obtain his preferred outcome for free. What, however, would happen if the local lobbies chose to contribute to the president instead? Given that the president is able to impose his preferred outcome for free, the lobbies might be better off trying to influence the president directly. We expect that the equilibrium policy outcome might be to the left of $E$. There is however a lower bound on how much the policy might be moved to the left. According to Rohde and Shepsle (2006), regardless of the nominee, all possible policy outcomes lie between the preferred point of the fourth and the fifth justice of the eight-member Court. We define $E$ to be this lower bound.

The strategy space is as follows. Each lobby offers contributions to the president, $c_t(E)$, conditional on the equilibrium resulting from his proposal in stage 1, $c_t : \mathbb{R} \rightarrow \mathbb{R}$. If weakly dominated strategies are available, then existence can be established, but not uniqueness of the equilibrium characteristics.
\( \mathbb{R} \). The president offers contributions to senators, conditional on the voting profile, \( c_{ps} : V \rightarrow \mathbb{R} \). Senators vote, after observing the proposal and the contributions offered to them, \( v_s : \mathbb{R} \rightarrow \{0, 1\} \). The timing is as follows:

**Stage 1:** Everyone observes \( r \). Lobbies, simultaneously and non-cooperatively offer a contribution schedule to the president, \( c_l(E) \).

**Stage 2:** The president observes the contribution schedule and proposes a nominee \( J_N \) with resulting policy \( E \), and offers a contribution schedule \( c_{ps} \) to each senator conditional on the vote profile.

**Stage 3:** Senators observe \( E \) and the contributions offered to them only, and simultaneously and non-cooperatively vote. If the number of senators voting in favor of the proposal is sixty or more, \( E \) is the final outcome. Otherwise, reversion policy \( r \) will result in equilibrium.

This model can have multiple equilibria, and describing them is beyond the scope of this paper. However, we are interested in knowing if there exist equilibria in which the lobbies can successfully move the policy to the left of \( E \). We know already that in stage 3 the president’s proposal will be approved for free: he can play the pivot strategy described in section 4.1. Given the continuation game of stage 3, stage 1 and 2 are the description of a first price menu auction. The whole game can be seen as one in which multiple bidders (the lobbies) make offers to a single auctioneer (the president), who has decision power over the final outcome. Since the set \([E, \bar{E}]\) is compact, and the utility functions of lobbies and president are continuous over this set, we can apply Theorem 2 of Bernheim and Whinston [1986].

Theorem 2 tells us that, in any Nash equilibrium in which lobbies play a truthful strategy, the auctioneer will select a policy that maximizes the joint utility of all players. In other words, the equilibrium outcome will be \( E = \arg \max_{[E, \bar{E}]} P(E) + \sum_s L_s(E) \) for any truthful strategy by the lobbies.

Depending on the shape of \( P(.) \) and \( L_s(.) \) and on the relative location of the preferred point of each lobby, the result might be to the left of \( \bar{E} \). For example, suppose preferences were Euclidean, and the president’s preferred point \( P \) was at \( \bar{E} \), while all lobbies had their preferred point at \( \bar{E} \). In other words, \( P(E) = -|E - \bar{E}| \) and \( L_s(E) = -|\bar{E} - \bar{E}| \). Then, in equilibrium, the outcome would be chosen to be equal to \( \arg \max_{[E, \bar{E}]} -|\bar{E} - E| + 100\{-|\bar{E} - E|\} < \bar{E} \). Note that, if the lobbies had chosen to influence the senators instead, as in section 3.4, they could not have affected the outcome in any way. Instead, the lobbies might be able to influence the outcome if contributions to the president are possible.

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**Footnotes:**

24 For the reader familiar with Bernheim and Whinston [1986]: the set of bidders \( \{i\}_{i=1}^{M} \) is our set \( \{l_s\}_{s=1}^{100} \); the set \( S \) is \([E, \bar{E}]\); the payoff function \( g_i \) is our \( L_s(E) \) and the payoff function \( d \) is our \(-P(E)\). The strategy \( f_i \) corresponds exactly to our \( c_{li} \) and the lower bound \( k_i \) is equal to zero.

25 See definition 1 of Bernheim and Whinston [1986]. Intuitively, a contribution profile is truthful if contributions correctly reflect the relative value of preferences over the elements in the set \([E, \bar{E}]\).
5 Conclusion

We started with the observation in Rohde and Shepsle (2006) that a reversion policy, \( r \), in the gridlock region is an equilibrium when no contributions are permitted. This means that no presidential nomination is confirmable, and thus the existing eight-member Court will remain in place. We then examined a world in which political agents (local lobbyists, the president) could offer inducements to legislators to vote contrary to their initial preferences. What would happen if local lobbies offered contributions conditional on the voting profile? We observed that the (conservative, without loss of generality) president would be able to move the policy to the right of \( r \) if there were enough lobbies willing to contribute to do so. We then explored what would happen if only the president could offer contributions conditional on the voting profile, and found a very extreme result: in equilibrium, the president can propose any nominee, no matter how extreme, and his preferred policy \( E \) is approved with zero contributions. Furthermore, this result is unchanged even if we add local lobbies. That is, even if all senators and all lobbies prefer the reversion policy to any nominee to its right, the president is still able to obtain his preferred policy for free. This result is very strong. It relies, on the one hand, on the capability of one party (the president) to influence a group of legislators by creating a prisoner dilemma that locks them into an equilibrium in which none of them is pivotal. It depends also on the assumption that these legislators are unable to cooperate among themselves. Dal Bo (2000, 2006) and Console Battilana (2005, 2006) develop a similar result in very different contexts.

<table>
<thead>
<tr>
<th>Lobbying Conditions</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>No lobbying</td>
<td>( r )</td>
</tr>
<tr>
<td>Only lobbies contribute conditional on ( v )</td>
<td>( \max_{E \leq E} { E \text{ s.t.} LS_4(E) \geq 0 } )</td>
</tr>
<tr>
<td>Only president contributes conditional on ( v )</td>
<td>( E ). Any ( J_N )</td>
</tr>
<tr>
<td>Both contribute conditional on ( v )</td>
<td>( E ). Any ( J_N )</td>
</tr>
<tr>
<td>Both contribute conditional on ( v ) with ideological cost</td>
<td>( J_N = \arg \max { E } P(E) - \sum_{s=40}^{100} I(J_N - S_s(E)) ) ( J_N \leq J^*_5 )</td>
</tr>
<tr>
<td>Both contribute conditional on ( v_s )</td>
<td>( \arg \max_{[E, F]} { E } P(E) - \sum_{s=41}^{100} \max[0, -LS_s(E)] )</td>
</tr>
<tr>
<td>Both contribute conditional on outcome</td>
<td>( \arg \max_{[E, F]} { E } P(E) - \sum_{s=41}^{100} \max[0, -LS_s(E)] )</td>
</tr>
<tr>
<td>( L_s ) contribute to ( P(\text{cond on } E) ) and ( P ) to ( S_s(\text{cond on } v) )</td>
<td>( \arg \max_{[E, F]} { E } P(E) + \sum_s L_s(E) )</td>
</tr>
</tbody>
</table>

Table 2. Lobbying and Equilibrium: Results.

We then explored other possibilities, summarized in Table 2. What if senators suffer an ideological cost for voting against their constituency? In that case, they must be compensated to do so even if they are not pivotal. We found that extreme nominees will no longer be proposed – in particular no nominee to the right of the
fifth justice in the eight-member Court will ever be nominated. Depending on the
shape of the indifference curves and on the distribution of senators' ideal points, the
nominee proposed by the president in the first stage might yield a policy to the left
of $E$. However, local lobbies still have no impact, regardless of their willingness to
contribute.

Conditioning on the entire voting profile is certainly first-best for the president;
however, in the real world, this might be too complex or too implausible. Senators
might not respond favorably to contributions conditional on events outside their con-
trol, such as the votes of other senators. We thus explored the effect of conditioning
contributions on the vote of each senator. While multiple equilibria can arise, we found
that the president can be sure to have his nominee pass the filibuster hurdle as long
as he proposes $\text{arg max } P(E) - \sum_{s=41}^{100} \max[0, -LS_s(E)]$. If the distribution of preferred
points of senators and legislators is sufficiently biased towards the left, the president will
not be able to impose his preferred policy and will have to pay positive contributions
in equilibrium. We noted that there always exists an equilibrium in which no senator
is pivotal and everyone votes for the proposal, regardless of how extreme this is. How-
ever, this is an equilibrium over which the president has no control (it results from each
senator voting in favor of the proposal when indifferent, even if this might be weakly
dominated). If contributions can be conditioned only on the outcome, the president is
worse off; he can not ensure that any policy satisfying $P(E) - \sum_{s=41}^{100} \max[0, -LS_s(E)]$ is
approved, unless we restrict equilibria to the ones in which no senator plays a weakly
dominated strategy. Finally, we inquired whether lobbyists might not have another
strategic option when contributions are conditional on the entire voting profile. Given
that a local lobby never manages to have an impact on the outcome when lobbying its
senator under this assumption, we examined whether the lobby might be better served
by offering inducements to the president instead? In the case in which the president
conditions the contributions to senators on the entire voting profile, we found that the
outcome might be moved to the left of $E$.

We started from a pivotal-politics literature that uncovered a "too much gridlock"
problem. As American politics has become more polarized, this gridlock problem has
been exacerbated. Yet one must ask whether it is really the case that nominees other
than the one lying exactly at $r$ can be consistently blocked when $r$ is in the gridlock
region. This is not consistent with our casual empirical impressions. Our paper has
provided several instances in which, through lobbying, the too-much-gridlock problem
is mitigated.\footnote{In the case of conditioning on the entire voting profile, we may have replaced one anomaly with
another – solving the problem of too much gridlock but producing a world with too much presidential
power. Nevertheless, it is striking empirically how much deference is accorded many presidential
nominees, so perhaps we have in our results the kernel of an explanation for such deference.} The nomination process described here provides a description of politics
more subtle than one limited to presidential proposing followed by senatorial preference
revelation in a super-majority decision making context. The politics of Supreme Court
appointments, like that surrounding policies affecting trade about which Grossman
and Helpman (1994) so elegantly wrote, involves strategic attempts by a variety of
agents, including the president himself, to influence senators faced with accepting or rejecting nominees. We expect further development of models integrating proposing, influencing, and voting to occupy us and others in the future.
References


Console Battilana, S. (2005). Lobbies, legislators, external agenda setters: 0,0,0,1 strategic reasons to be neighborly. Presented at the Stanford Economics Public and Political Economics seminar.


Appendix
Proof of claim 1

We first solve the continuation game (stage 2 and stage 3) given \( r \) and \( E \geq r \), and then apply constrained maximization in stage 1.\(^{27}\)

There are four possible cases for equilibrium actions:

(a) Strictly more than 60 senators vote for the proposal.
(b) Strictly more than 41 senators vote against the proposal.
(c) Exactly 60 senators (all pivotal) vote for the proposal.
(d) Exactly 41 senators (all pivotal) vote against the proposal.

Case (a) and (b) are equilibria over which the president and the lobbies have no control. This follows because in this equilibria every senator is indifferent between voting for or against. Why? First, no senator is pivotal, and therefore her vote cannot affect her outcome-dependent utility. Second, no lobby will make a positive contribution for a voting profile in which its senator is not pivotal, since these contributions could be saved without the outcome changing. Together these imply that each senator is indifferent concerning her vote. Hence, these equilibria arise from an arbitrary resolution of indifference. We will focus attention on the other equilibria.\(^{28}\)

In case (c) there are 60 pivotal senators voting for the proposal. We prove the following:

An equilibrium with exactly 60 pivotal senators voting for the proposal can exist only if \( LS_{41}(E) \geq 0 \).

Assume the contrary. Suppose \( LS_{41}(E) < 0 \), yet there is an equilibrium with exactly 60 senators voting for the proposal. But then, there would be at least one senator \( s \) voting \( \bar{v}_s = 1 \) with \( LS_s(E) < 0 \). But then this could not have been an equilibrium as we now show. There are three instances under which \( LS_s(E) < 0 \): (i) \( L_s(E) > 0 \) and \( S_s(E) \leq 0 \), (ii) \( L_s(E) \leq 0 \) and \( S_s(E) < 0 \) and (iii) \( L_s(E) < 0 \) and \( S_s(E) \geq 0 \). In each of these cases there is a potential deviation. In (i) the senator receives from her lobbyist at most \( L_s(E) \) to vote for the proposal, and her utility would be at most \( LS_s(E) < 0 \) in equilibrium. If she deviates to vote 0, the outcome would change and her utility would be 0, an improvement for her. In (ii) the senator will receive no contributions to vote for the proposal and her utility is \( S_s(E) < 0 \). If she were to deviate, the outcome would change and her utility would be \( S_s(r) = 0 \) plus any contributions she might receive. Therefore she would deviate. In (iii) the senator receives no contributions from the lobby and she would have a utility of \( S_s(E) \geq 0 \). However, the lobby could offer \( S_s(E) + \varepsilon \) to the senator to switch her vote. The senator would deviate in this circumstance, the outcome would change, and the lobby would have gained \( -LS_s(E) - \varepsilon > 0 \). Therefore, an equilibrium with exactly 60 pivotal senators voting for the proposal can exist only if \( LS_{41}(E) \geq 0 \). We show that this equilibrium exists and we furthermore show that equilibrium (d) cannot exist when

\(^{27}\) \( E \leq r \) cannot arise as an equilibrium outcome. If such an \( E \) could be approved in the continuation game, then the president would propose \( r \) (or make no proposal at all) in stage 1 and the result would be \( r \). Of course, if an \( E < r \) could not be approved, then the outcome necessarily would be \( r \) or some \( E > r \).

\(^{28}\) In other parts of the paper we consider equilibria in which no one is pivotal. However, when we do so these are the only equilibria.
LS_{41}(E) > 0\textsuperscript{29}. Constrained maximization in stage 1 implies that the president will pick his preferred \( E \) among those satisfying \( LS_{41}(E) \geq 0 \).

Existence. Given a status quo \( r \) and proposal \( E \geq r \) such that \( LS_{41}(E) > 0 \), the following set of strategies is an equilibrium. At stage 3 each senator votes \( E \) iff the payoff from voting \( E \) is higher than the payoff from voting \( r \). If the payoffs are the same, the senator is indifferent. At stage 2 each lobby with \( L_s(E) \geq 0 \) offers \( \min[L_s(E), \max[0, -S_s(E)]] \) for all \( v \) in which senator \( s \) plays \( v_s = 1 \) and is pivotal (exactly 59 other senators vote for the proposal), and zero for all other voting profiles. Each lobby with \( L_s(E) < 0 \) offers \( \min[-L_s(E), \max[0, S_s(E)]] \) for all \( v \) in which senator \( s \) plays \( v_s = 0 \) and is pivotal (exactly 40 other senators vote against the proposal), and zero for all other voting profiles.

Given these strategies, the following equilibrium actions are played: senators in \{\( S_1, \ldots, S_{40} \}\) vote \( v_s = 0\textsuperscript{30} \), senators in \{\( S_{41}, \ldots, S_{100} \}\) vote \( v_s = 1 \), lobbies in \{\( l_{41}, \ldots, l_{100} \}\) with \( L_s(E) > 0 \) pay \( \min[L_s(E), \max[0, -S_s(E)]] \) to their senator, and all other lobbies pay zero for any voting profile.

No senator has an incentive to deviate. For any senator in \{\( S_{41}, \ldots, S_{100} \}\}, \( LS_s(E) \geq 0 \) by assumption. Therefore, one of the following has to hold: (a) \( S_s(E) \geq 0 \) and \( L_s(E) \geq 0 \), (b) \( S_s(E) > 0 \) and \( L_s(E) < 0 \) or (c) \( S_s(E) < 0 \) and \( L_s(E) > 0 \).

(a) If \( S_s(E) \geq 0 \) and \( L_s(E) \geq 0 \), senator \( s \) would lose \( S_s(E) \) if she deviated. She is offered no contributions from \( l_s \), but her personal utility is at least as high voting for the proposal rather than against. (b) If \( S_s(E) > 0 \) and \( L_s(E) < 0 \), she has a benefit of \( S_s(E) \) from voting for the proposal, and would have zero personal utility and a contribution of \( -L_s(E) \leq S_s(E) \) if she voted against the proposal; therefore she has no deviation. (c) If \( S_s(E) < 0 \) and \( L_s(E) > 0 \), the payoff from voting for the proposal (a combination of personal utility plus lobbyist contribution) would be \( S_s(E) - S_s(E) = 0 \), while the payoff from voting against the proposal would be zero; therefore she has no deviation.

Senators in \{\( S_1, \ldots, S_{40} \}\), on the other hand, have no incentive to deviate either. Since none of them is pivotal, none of them can affect their personal outcome dependent utility, and contributions are always zero when a senator is not pivotal.

Now let us see whether any lobby has an incentive to deviate. Each lobby in \{\( l_1, \ldots, l_{40} \}\) is paying no contribution and cannot affect the outcome, since its senator is not pivotal; therefore they have no deviation. To show that lobbies in \{\( l_{41}, \ldots, l_{100} \}\) have no deviation, we consider two cases: (i) \( L_s(E) < 0 \) and (ii) \( L_s(E) \geq 0 \).

(i) If \( L_s(E) < 0 \), the lobby is not obtaining its preferred outcome. By assumption, \( S_s(E) \geq -L_s(E) \) and \( S_s(E) > 0 \). Therefore, in order to induce \( S_s \) to vote \( v_s = 0 \), lobby \( l_s \) would have to offer weakly more than \( L_s(E) \) (recall that the senator is pivotal). But then this deviation would not be profitable for the lobby.

(ii) Consider now \( L_s(E) \geq 0 \). If \( S_s(E) \geq 0 \), then lobby \( l_s \) is paying no contributions

\textsuperscript{29}When \( LS_{41}(E) = 0 \), senator 41 can be made exactly indifferent between voting for or against the proposal, therefore both continuation equilibrium (c) and (d) could exist. However, if she breaks indifference in favor of \( r \), we have a non existence problem in substage 1 whenever \( LS_{41}(E) = 0 \) for some \( E < E \): for any approved policy \( E \) such that \( LS_{41}(E) > 0 \), the president would like to deviate to propose an \( E \) slightly to the right, so that \( LS_{41}(E) \) is still positive.

\textsuperscript{30}In the zero probability case that \( S_{41}(E) + L_{41}(E) = S_{40}(E) + L_{40}(E) = 0 \), we consider the equilibrium in which senator 41 votes against and senator 40 in favor.
and is obtaining its preferred outcome. If $S_s(E) < 0$, then lobby $l_s$ is paying $-S_s(E)$ in equilibrium and its payoff is $L_s(E) + S_s(E)$, which is weakly positive by assumption. If it deviates to contribute less for the equilibrium voting profile, say $-S_s(E) - \varepsilon$, then senator $s$ has a deviation since payoffs from voting $v_s = 1$ are $-S_s(E) - \varepsilon + S_s(E) < 0$ while payoffs from voting $v_s = 0$ are zero. If the senator deviates, the outcome switches to the reversion policy, and the lobby has a utility of zero. Therefore, the lobby has no incentive to deviate. Obviously, deviations to contribute more would add a cost without any additional benefit. Therefore, lobbies with $L_s(E) \geq 0$ have no incentive to deviate.

Furthermore, there cannot be an equilibrium with exactly 41 senators voting against the proposal if $LS_{41}(E) > 0$. Suppose there were. Then, there would be at least one senator $\bar{s}$ voting $v_{\bar{s}} = 0$ with $LS_{\bar{s}}(E) > 0$. There are three cases: (i) $L_{\bar{s}}(E) > 0$ and $S_{\bar{s}}(E) \leq 0$, (ii) $L_{\bar{s}}(E) \geq 0$ and $S_{\bar{s}}(E) > 0$ and (iii) $L_{\bar{s}}(E) < 0$ and $S_{\bar{s}}(E) > 0$. In each case it is easy to see that there is a profitable deviation. Therefore there cannot be an equilibrium with pivotal senators voting against the nominee when $LS_{41}(E) > 0$.

**Proof of claim 2**
This is a special case of claim 3. After proving this latter claim, we show how claim 2 also follows.

**Proof of claim 3**
*Existence of equilibrium*

The following is an equilibrium. In stage 1 the president proposes a $J_N$ yielding $\overline{E}$. In stage 2 each lobby $l_s$ offers $c_{ls} = 0 \forall v \in V$, the president offers $c_{ps} = \max[0, -S_s(E)]$ for all voting profiles in which senator $s$ is pivotal and votes for the proposal, and offers $c_{ps} = 0$ for all other voting profiles. In stage 3 all senators vote for the proposal. No senator has an incentive to deviate. Suppose senator $s$ deviates to $v_s = 0$. Since 99 senators continue to vote $v_s = 1$, the outcome will still be $\overline{E}$. But, since $s$ is receiving no contributions either when voting for or against the nominee (since she is not pivotal), she would not be better off by deviating.

No local lobby has an incentive to deviate. Suppose lobby $l_s$ deviated by offering a positive amount to senator $s$ under certain voting profiles $v^* \in V^* \subset V$. If in any $v^*$ the remaining 99 senators do not all vote $v_s = 1$, this schedule is off the equilibrium path. (Each lobby can only influence the vote of her own senator, and deviations are holding everything else constant.) It does not affect the equilibrium outcome or payments; hence there is no gain in this deviation. If in any $v^*$ the remaining 99 senators all vote $v_s = 1$, then the outcome will still be $\overline{E}$ and lobby $l_s$ would be paying positive contributions, denoted as $c_{ls}^*$. Lobby $l_s$ would gain $S_s(\overline{E}) + 0 - S_s(\overline{E}) - c_{ls}^* < 0$; hence it would be worse off. Therefore local lobbies have no deviation.

The president has no profitable deviation in stage 2, since he is obtaining his preferred outcome for free. Likewise, the president has no profitable deviation in stage 1. If he nominates a $J_N$ yielding outcome $E < \overline{E}$, given the continuation game, this will be approved at no cost and the utility change for the president would be $P(E) - P(\overline{E}) < 0$. Since $\overline{E}$ is the upper bound, there is no profitable deviation for the president at all.

**Uniqueness.**

Suppose there were an equilibrium with either positive payments by the president
or outcome \( r \). This candidate equilibrium would be characterized by a voting profile \( \tilde{v} \), a contribution schedule for each local lobby \( \tilde{c}_{ls} \), and a contribution schedule for the president \( \tilde{c}_{ps} \). We show there is a pivot strategy that the president can play, with the result of having his proposal \( E \) approved at a cost of \( 61\varepsilon \), with \( \varepsilon > 0 \) arbitrarily small. If we show such a strategy exists, then no equilibrium with (a) positive payments by the president or (b) \( E \) rejected could exist. That is, for any candidate equilibrium satisfying property (a) or (b), the president could deviate and play the pivot strategy. This deviation would reduce his payments by choosing \( \varepsilon \) to be smaller than previous payments and would sustain \( E \) as the outcome, where \( P(E) - 61\varepsilon > P(r) = 0 \).

We are left to prove such a pivot strategy exists. Consider a generic senator \( s \). Define \( c_{ls}^{0,v_{-s}} \) to be the contribution schedule offered to senator \( s \) by lobby \( l_s \) whenever \( v_s = 0 \) and the remaining 99 senators vote according to voting profile \( v_{-s} \). (The profile \( v_{-s} \) has 99 elements, each an element of the set \( \{0, 1\} \). The set of profiles \( (0, v_{-s}) \in V \) has \( 2^{99} \) elements.) The president can offer the following schedule to senator \( s \):

<table>
<thead>
<tr>
<th>( v_{-s} )</th>
<th>( v_s )</th>
<th>Contributions ( c_{ps} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) If ( v_{-s} ) has strictly less than 59 voting 1</td>
<td>1</td>
<td>( c_{ls}^{0,v_{-s}} + \varepsilon )</td>
</tr>
<tr>
<td>(b) If ( v_{-s} ) has exactly 59 voting 1</td>
<td>1</td>
<td>( c_{ls}^{0,v_{-s}} + \varepsilon + \max[0, -S_s(E)] )</td>
</tr>
<tr>
<td>(c) If ( v_{-s} ) has strictly more than 59 voting 1</td>
<td>1</td>
<td>( c_{ls}^{0,v_{-s}} + \varepsilon )</td>
</tr>
<tr>
<td>(d) For any other ( v_{-s} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Senator \( s \) has a dominant strategy, \( v_s = 1 \). In (a), the outcome is \( r \) regardless of the vote of senator \( s \), and his net utility from choosing action \( v_s = 1 \) rather than \( v_s = 0 \) is at least \( S_s(r) + c_{ls}^{0,v_{-s}} + \varepsilon - S_s(r) - c_{ls}^{0,v_{-s}} = \varepsilon > 0 \). It might be more if \( c_{ls}^{0,v_{-s}} > 0 \). In (b) senator \( s \) is pivotal. If she votes 0, her utility is \( S_s(r)(= 0) + c_{ls}^{0,v_{-s}} \). If she votes 1, her utility is at least \( S_s(E) + c_{ls}^{0,v_{-s}} + \varepsilon + \max[0, -S_s(E)] = \begin{cases} S_s(E) + c_{ls}^{0,v_{-s}} + \varepsilon & \text{if } S_s(E) \geq 0 \\ c_{ls}^{0,v_{-s}} + \varepsilon & \text{if } S_s(E) < 0 \end{cases} \). Her net utility from voting 1 as opposed to voting 0 will be at least \( \varepsilon > 0 \). In (c) the outcome is \( E \) regardless of the vote of senator \( s \). Her net utility from voting 1 as opposed to 0 is at least \( c_{ls}^{0,v_{-s}} + \varepsilon - c_{ls}^{0,v_{-s}} = \varepsilon > 0 \). Therefore, under any possible voting profile of the other senators, senator \( s \) has a higher payoff if she chooses action \( v_s = 1 \). Suppose the president plays the pivot strategy with 61 senators. For 61 senators it will dominant to vote \( v_s = 1 \); therefore the president will only have to offer each one of them \( c_{ls}^{0,v_{-s}} + \varepsilon \), with \( v_{-s} \) having strictly more than 59 voting in favor. However, when strictly more than 59 senators other than \( s \) vote in favor, a majority of senators is already voting 1, hence the vote of senator \( s \) is not relevant – senator \( s \) is not pivotal. By the refinement, \( c_{ls}^{0,v_{-s}} = 0 \) for any such voting profile. Therefore, the president would be playing the pivot strategy with 61 senators and obtain his preferred outcome for \( 61\varepsilon \).

**Proof of claim 2**

We now return to 2 and establish it as a special case of the previous result.

**Existence of equilibria**

The following is an equilibrium. In stage 1 the president proposes a nominee yielding
In stage 2 the president offers $c_{ps} = \max[0, -S_s(E)]$ for all voting profiles in which senator $s$ is pivotal and votes for the proposal, and $c_{ps} = 0$ for all other voting profiles. In stage 3 all senators vote in favor of the nominee. This is an equilibrium by the existence argument in claim 3.

Uniqueness
Suppose there were an equilibrium with either positive payments by the president or outcome $r$. Call this the candidate equilibrium. But then the president could deviate from the candidate equilibrium and make the following offer to senators in $\{S_{40}, ..., S_{100}\}$: offer $c_{ps} = \max[0, -S_s(E)] + \varepsilon$ for each voting profile in which $s$ votes for the proposal and is pivotal, offer $c_{ps} = \varepsilon$ for each voting profile in which $s$ votes for the proposal and is not pivotal, and offer zero for all other voting profiles. To senators in $\{S_1, ..., S_{39}\}$ the president offers $c_{ps} = 0, \forall v$. Senators in $\{S_{40}, ..., S_{100}\}$ have a dominant strategy, $v_s = 1$. The argument is as follows. If in the candidate equilibrium they were not pivotal and voted against the proposal, they would obtain $\varepsilon$ more by deviating. If in the candidate equilibrium they were pivotal and voted against the proposal, they would again obtain $\varepsilon$ more by deviating. By playing this deviation, the president induces at least 61 senators to vote for the proposal. But then, no one is pivotal, and the president has to pay only $61\varepsilon$. If in the candidate equilibrium the outcome were $r$, now the outcome will be $E$ and $\varepsilon$ can be chosen such that the deviation is profitable: $\varepsilon < \frac{P(E)}{61}$. If in the candidate equilibrium the outcome were $E$ but the president was paying total positive contributions $x$, the president can deviate and reduce his contributions by picking $\varepsilon < \frac{x}{61}$.

Proof of claim 4
We use many of the arguments proved in claim 3; therefore we avoid repetition of the same arguments, giving a more synthetic proof.

Existence of equilibria
The following set of strategies is an equilibrium. For any voting profile in stage 3, each senator casts the vote that gives a higher utility. If the utility is the same, she is indifferent. In stage 2, the following strategies are an equilibrium. The president offers $c_{ps} = I(J_N - S_s(E))$ to senators in $\{S_{40}, ..., S_{CP}\}$ for all voting profiles in which they vote $v_s = 1$ and are not pivotal; $I(J_N - S_s(E)) + \max[0, -L_s(E)] + \max[0, -S_s(E)]$ to senators in $\{S_{40}, ..., S_{CP}\}$ for all voting profiles in which they vote $v_s = 1$ and are pivotal; $\max[0, -L_s(E)] + \max[0, -S_s(E)]$ to senators in $\{S_{CP}, ..., S_{100}\}$ for all voting profiles in which they vote $v_s = 1$ and are pivotal; and zero for all other voting profiles and all other senators (for any voting profile). The lobbies play the following strategies. Lobbies with $L_s(E) \geq 0$ offer $c_{ls} = \min[L_s(E), \max[0, -S_s(E)]]$ for all voting profiles in which their senator is pivotal and votes $v_s = 1$, zero for all other voting profiles. Lobbies with $L_s(E) < 0$ offer $\min[-L_s(E), \max[0, S_s(E)]]$ for all voting profiles in which their senator is pivotal and votes $v_s = 0$, and zero for all other voting profiles. In stage 1, the president proposes the policy that maximizes his utility, net of contributions paid.

Equilibrium actions: senators in $\{S_{40}, ..., S_{100}\}$ vote $v_s = 1$ and senators in $\{S_1, ..., S_{39}\}$ vote $v_s = 0$; the president pays $I(J_N - S_s(E))$ to senators $\{S_{40}, ..., S_{CP}\}$ and zero to all others; and each local lobby pays zero.

No one has an incentive to deviate. In stage 3, no senator will deviate: senators in
\{S_{40}, \ldots, S_{100}\} are made exactly indifferent between \(v_s = 1\) and \(v_s = 0\), while senators in \(\{S_1, \ldots, S_{39}\}\) are not pivotal and hence cannot affect the outcome. They receive zero contributions for either vote, and they do not incur an ideological cost when voting in accord with their constituency.

In stage 2, no lobby has an incentive to deviate. Since each lobby can influence only its senator and no senator is pivotal, no lobby can affect the outcome. Therefore, since no lobby is paying positive contributions, no lobby has a deviation. The president has no incentive to deviate. If he deviates to contribute less to one of the pivotal senators than in the equilibrium voting profile, the senator will switch vote, only 60 senators will vote for the proposal and the president will have to spend more in contributions by assumption H1.

**Uniqueness**

The president chooses \(J_N = \arg \max P(E(J_N)) - \sum_{40}^{CP} I(J_N - S_s(E)) > 0\).\(^{31}\) (If no \(J_N\) yields a positive result, the president proposes the reversion policy point.) We argue that, given a proposal \(E\) that respects maximization in stage 1, there is no equilibrium in which either the president pays \(\sum_{40}^{CP} I(J_N - S_s(E)) + x\), \(x > 0\) or the proposal is rejected.

Suppose there were one, called the *candidate equilibrium*. Consider a generic senator \(s\). The candidate equilibrium includes an offer \(\tilde{c}_l\) for any possible voting profile. Define \(\tilde{c}_l^{0,v_s}\) to be the contribution schedule offered to senator \(s\) by lobby \(l_s\) whenever \(v_s = 0\) and the remaining 99 senators vote according to voting profile \(v_{-s}\). The president can offer the following schedule to senator \(s\), a pivot strategy creating a deviation from the *candidate equilibrium*:

<table>
<thead>
<tr>
<th>To senators</th>
<th>(v_{-s})</th>
<th>(v_s)</th>
<th>Contributions (c_{ps})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ({S_{40}, \ldots, CP})</td>
<td>(\sum_{i \neq s} v_i \leq 59)</td>
<td>1</td>
<td>(\tilde{c}<em>l^{0,v</em>{-s}} + \varepsilon + I(J_n - S_s(E)))</td>
</tr>
<tr>
<td>(b) ({CP + 1, \ldots, S_{100}})</td>
<td>(\sum_{i \neq s} v_i &lt; 59)</td>
<td>1</td>
<td>(\tilde{c}<em>l^{0,v</em>{-s}} + \varepsilon)</td>
</tr>
<tr>
<td>(c) ({S_{40}, \ldots, S_{100}})</td>
<td>(\sum_{i \neq s} v_i = 59)</td>
<td>1</td>
<td>(\tilde{c}<em>l^{0,v</em>{-s}} + \varepsilon + \max[0, -S_s(E)] + I(J_n - S_s(E)))</td>
</tr>
<tr>
<td>(d) ({CP + 1, \ldots, S_{100}})</td>
<td>(\sum_{i \neq s} v_i = 59)</td>
<td>1</td>
<td>(\tilde{c}<em>l^{0,v</em>{-s}} + \varepsilon + \max[0, -S_s(E)])</td>
</tr>
<tr>
<td>(e) ({S_1, \ldots, S_{39}})</td>
<td>(\forall v_{-s})</td>
<td>1 or 0</td>
<td>0</td>
</tr>
<tr>
<td>(f) ({S_{40}, \ldots, S_{100}})</td>
<td>(\forall v_{-s})</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

with \(\varepsilon < \frac{2}{61}\).

By the same argument as in the proof of claim 3, senators \(S_s \in \{S_{40}, \ldots, S_{100}\}\) have a dominant strategy, \(v_s = 1\). Since no senator is pivotal in this deviation, \(\tilde{c}_l^{0,v_s} = 0\) in equilibrium by the refinement. Therefore, the president has a profitable deviation from the *candidate equilibrium*. Thus, he can obtain his preferred outcome for \(61\varepsilon + \sum_{40}^{CP} I(J_N - S_s(E))\).

**Proof of claim 5**

We first establish three additional claims, used in the proof.

**Claim 7** In equilibrium,

\(^{31}\)Note that CP is a function of \(J_N\).
1. neither the president nor a lobby makes a positive payment if the preferred outcome is not chosen; and

2. neither a lobby nor the president makes a payment exceeding the net benefit it (he) gets from the outcome it (he) likes most if the preferred outcome is chosen.

These results are apparent from a consideration of the motivation of any agent to deviate. No agent has such a motivation in the circumstances given in this claim.

Claim 8  In equilibrium, a non-pivotal senator receives no contributions.

Proof. Suppose not. Any lobby \( l_s \) (or the president) contributing to the non-pivotal senator could eliminate the contribution without affecting the outcome. She is not pivotal, and other senators observe only the contributions offered to them; hence their votes are not affected. The contributing agent is better off deviating. ■

Claim 9  Given a nomination \( J_N \) yielding outcome \( E \) \((r \leq E \leq \bar{E})\), there always exists a continuation equilibrium in which the proposal is approved by at least sixty-one senators and no contributions are offered.

Proof. Given \( J_N \) and resulting outcome \( E \), suppose \( v_s = 1 \) for sixty-one senators, each lobby offers \( c_{ls} = (0,0) \), and the president offers \( c_{ps} = (0,0) \) \( \forall s \). No senator has an incentive to deviate since she neither receives a positive contribution no matter what vote she casts, nor is she pivotal. Hence her vote cannot influence her personal outcome-dependent utility. No local lobby has an incentive to deviate, since its corresponding senator is not pivotal. The president is obtaining his preferred outcome for free and, since \( P(E) \geq P(r) \), he has no deviation. ■

We now prove claim 5.

Existence of the continuation game equilibrium

Order senators according to \( LS_s(E) \). Given a reversion policy \( r \) and a generic nomination yielding \( E \) s.t. \( r \leq E \leq \bar{E} \), and \( P(E) \geq \sum_{s=41}^{100} \max[0,-S_s(E)-L_s(E)] \), there exists a continuation equilibrium in which each player plays the following strategies:\textsuperscript{32}

The president offers \( c_{ps} = \{ \max[0,-S_s(E)-L_s(E)], 0 \} \) to senators \( S_s \in \{S_{41},...,S_{100}\} \) and \( c_{ps} = \{0,0\} \) to senators \( S_s \in \{S_1,...,S_{40}\} \).

Each lobby with \( L_s(E) > 0 \) and \( l_s \in \{l_{41},...,l_{100}\} \) offers \( c_{ls} = \{ \min[L_s(E), \max[0,-S_s(E)]], 0 \} \).

Each lobby with \( L_s(E) < 0 \) and \( l_s \in \{l_{41},...,l_{100}\} \) offers \( c_{ls} = \{0,-L_s(E)\} \).

Each lobby with \( l_s \in \{l_1,...,l_{40}\} \) offers \( c_{ls} = \{0,0\} \).

Each senator plays the following strategy: whenever she is pivotal, she votes \( v_s = 1 \) \((v_s = 0)\) if \( S_s(E) \) plus the total contributions offered to vote for \( J_N \) is strictly higher (lower) than \( S_s(r) \) plus the total contributions offered to vote against \( J_N \). If the sums are equal, the senator is indifferent, and she can vote either way. Whenever she is not pivotal, she votes \( v_s = 1 \) \((v_s = 0)\) if the total contributions offered to vote for \( J_N \) are

\textsuperscript{32}Recall that \( c_{ps} \) and \( c_{ls} \) are ordered pairs with the first element the contribution if \( v_s = 1 \) and the second element the contribution if \( v_s = 0 \).
strictly higher (lower) than the total contributions offered to vote against \(J_N\). If the contributions are equal, she is indifferent.

In equilibrium, the following actions are played. Senators \(S_s \in \{S_1, \ldots, S_{40}\}\) play \(v_s = 0\), senators \(S_s \in \{S_{41}, \ldots, S_{100}\}\) play \(v_s = 1\). The president pays \(\max[0, -S_s(E) - L_s(E)]\) to senators \(S_s \in \{S_{41}, \ldots, S_{100}\}\) and pays zero to all others. Lobbies \(l_s \in \{l_1, \ldots, l_{40}\}\) pay zero contributions and lobbies \(l_s \in \{l_{41}, \ldots, l_{100}\}\) pay \(\min[L_s(E), \max[0, -S_s(E)]]\) if \(L_s(E) \geq 0\) and zero if \(L_s(E) < 0\). To see that this is an equilibrium, consider each stage of the game.

**Stage 3.** In stage three, no senator has an incentive to deviate:

- Senators \(S_s \in \{S_1, \ldots, S_{40}\}\) are not pivotal and thus cannot affect the outcome; therefore, they have a personal outcome dependent utility of \(S_s(E)\) regardless of their vote. They are offered zero contributions for any vote; hence they have no deviation from \(v_s = 0\).

- Each senator \(S_s \in \{S_{41}, \ldots, S_{100}\}\) is pivotal, but she has no incentive to change her vote since she is exactly compensated for any outcome-dependent utility loss and lobbyist contribution (if any).

**Stage 2.** In stage two, the president has no incentive to deviate. He has a positive benefit of \(P(E) - \sum_{s=41}^{100} \max[0, -S_s(E) - L_s(E)]\); therefore he prefers the equilibrium to paying nothing and obtaining \(r\). Furthermore, he cannot reduce his contributions:

- Senators \(S_s \in \{S_1, \ldots, S_{40}\}\) are not pivotal, therefore, by claim 8, they are paid no contributions for equilibrium action \(v_s = 0\). Furthermore, the president has no incentive to offer positive contributions for action \(v_s = 1\). If the senator deviates, and supports the president’s nominee, the outcome is unchanged and the president wastes the contribution paid.

- Senators \(S_s \in \{S_{41}, \ldots, S_{100}\}\) are all pivotal. The president is paying \(\max[0, -S_s(E) - L_s(E)]\). If \(\max[0, -S_s(E) - L_s(E)] \leq 0\), then the president is obtaining the vote of the senator for free, therefore he has no deviation. If \(\max[0, -S_s(E) - L_s(E)] > 0\), positive contributions are paid. Suppose the president deviated to offer \(\{\max[0, -S_s(E) - L_s(E)] - \varepsilon, 0\}\). There are two cases: (a) \(L_s(E) \geq 0\) and \(S_s(E) < 0\) (therefore \(-S_s(E) > L_s(E)\)) and (b) \(L_s(E) < 0\). (a) Contributions offered by the lobby are \(\{\min[L_s(E), \max[0, -S_s(E)]], 0\}\} = \{L_s(E), 0\}\}. Therefore the senator has a utility of \(S_s(E) - S_s(E) - L_s(E) + L_s(E) - \varepsilon = -\varepsilon\) if she votes \(v_s = 1\) and a utility of 0 if she votes \(v_s = 0\). The senator would deviate to vote \(v_s = 0\) and the nominee would be rejected. (b) Contributions offered by the local lobby are \(\{0, -L_s(E)\}\}. The benefit of the senator from voting \(v_s = 1\) would be \(S_s(E) - S_s(E) - L_s(E) - \varepsilon = -L_s(E) - \varepsilon\). The benefits from voting \(v_s = 0\) would be \(-L_s(E) > 0\). Therefore, the senator would deviate to \(v_s = 0\) and the nominee would be rejected.
The president would never deviate to contribute more, since these contributions would be wasted. Furthermore, the president cannot recruit a cheaper set of senators, since he is already recruiting the cheapest set by our ranking of senators.

No lobby has an incentive to deviate. Each lobby $l_s \in \{l_1, \ldots, l_100\}$ is paying zero contributions and cannot affect the outcome, because its senator is not pivotal. Therefore, it has no deviation – any positive contributions could only reduce its welfare.

Each lobby $l_s \in \{l_{41}, \ldots, l_{100}\}$ has no incentive to deviate either. There are three cases: (a) $L_s(E) > 0$ and $-S_s(E) > L_s(E)$, therefore $c_{ls} = \{\min[L_s(E), \max[0, -S_s(E)]], 0\} = \{L_s(E), 0\}$; (b) $L_s(E) > 0$ and $-S_s(E) \leq L_s(E)$, therefore $c_{ls} = \{0, S_s(E)\}$; and (c) $L_s(E) < 0$, therefore $c_{ls} = \{0, -L_s(E)\}$.

(a) The lobby does not deviate to offer more by claim 7. The lobby would be worse off by deviating to offer less, say $L_s(E) - \varepsilon$. If it deviated, the senator would have a benefit of $S_s(E) - S_s(E) - L_s(E) + L_s(E) - \varepsilon = -\varepsilon < 0$ from voting $v_s = 1$ and a benefit of 0 from voting $v_s = 0$. Therefore, the senator would deviate and the lobby would not be better off.

(b) There are two subcases: $\max[0, -S_s(E)] = 0$ and $\max[0, -S_s(E)] > 0$. If $\max[0, -S_s(E)] = 0$, then no contributions are offered by the lobby and the preferred outcome is obtained; therefore no deviation is profitable. If $\max[0, -S_s(E)] = -S_s(E)$, then $S_s(E) < 0$ and $S_s(E) + L_s(E) > 0$; therefore the president’s contributions are $\{\max[0, -S_s(E) - L_s(E)], 0\} = \{0, 0\}$. If the lobby deviates to offer $\{-S_s(E) - \varepsilon, 0\}$, then the senator deviates and the outcome changes: the payoff from voting $v_s = 1$ is $S_s(E) - S_s(E) - \varepsilon < 0$, while the payoff from voting $v_s = 0$ is zero. Thus the lobby does not have a profitable deviation.

(c) The lobby does not obtain its preferred outcome. Given the president’s offer, the lobby would have to offer strictly more than $-L_s(E)$ to induce its senator to vote $v_s = 0$. But then, even though the senator would deviate, the lobby would be worse off. Therefore, the lobby has no deviation.

Uniqueness of the continuation game outcome

No equilibrium with outcome $r$ can arise if $P(E) > \sum_{s=41}^{100} \max[0, -S_s(E) - L_s(E)]$. Suppose there were a candidate equilibrium in which the outcome were $r$. But then the president could offer $\max[0, -S_s(E) - L_s(E)] + \varepsilon$ to $S_s \in \{S_{41}, \ldots, S_{100}\}$, picking $\varepsilon$ such that $P(E) \geq \sum_{s=41}^{100} \max[0, -S_s(E) - L_s(E)] + 60\varepsilon$. But then, $S_s \in \{S_{41}, \ldots, S_{100}\}$ would vote for the proposed policy. The benefits from voting for the proposed policy would be at least $\max[ -S_s(E) - L_s(E), 0] + \varepsilon + S_s(E)$. There are two cases: (a) $L_s(E) \geq 0$ and (b) $L_s(E) < 0$

(a) If $L_s(E) \geq 0$, then, by the assumption that the president can extract the full surplus from the local lobbies, a contribution of $\min[L_s(E), \max[0, -S_s(E)]]$ is added to vote for the proposal.
If the senator is pivotal, the benefits from voting for the proposal are $\max[-S_s(E) - L_s(E), 0] + \varepsilon + S_s(E) + \min[L_s(E), \max[0, -S_s(E)]] > 0$, while the benefits from a vote in favor of $r$ are zero. If the senator is not pivotal, the gain from voting $E$ as opposed to $r$ is $\max[-S_s(E) - L_s(E), 0] + \varepsilon + \min[L_s(E), \max[0, -S_s(E)]] > 0$. Hence, it is a dominant strategy for the senator to vote $v_s = 1$. (Note that, if the president had not extracted the full surplus from the local lobbies in the candidate equilibrium, the candidate equilibrium itself would not have been an equilibrium.)

(b) If $L_s(E) < 0$, then, by claim 7, the most that the senator could be offered to vote $v_s = 0$ would be $-L_s(E)$. Given the offer of the president, if the senator is pivotal and votes for the proposal, her benefit is $\max[-S_s(E) - L_s(E), 0] + \varepsilon + S_s(E)$ which is strictly higher than the benefit from voting $v_s = 0$, $-L_s(E) + 0$; therefore the senator would vote for the proposal. If the senator is not pivotal and votes for the proposal, his benefit is $\max[-S_s(E) - L_s(E), 0] + \varepsilon$, which is still strictly higher than $-L_s(E) + 0$. Therefore, it is a dominant strategy for the senator to vote for the proposal.

Therefore, at least 60 senators would be voting for the proposal and the proposal would result in equilibrium. The president would be better off since $P(E) > \sum_{s=41}^{100} \max[0, -U_{ss}(E) - L_s(E)]$. Hence, $r$ can never result in equilibrium because the president has a strategy that makes him better off and results in the proposal being approved.

Given the continuation equilibrium described above, in stage 1 the president chooses the policy that maximizes his utility net of contributions, $\arg \max_J P(E) - \sum_{s=41}^{100} \max[0, -S_s(E) - L_s(E)]$ (recalled that $E$ is a function of $J$).