SnapVX: A Network-Based Convex Optimization Solver

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Abstract

SnapVX is a high-performance Python solver for convex optimization problems defined on networks. For these problems, it provides a fast and scalable solution with guaranteed global convergence. SnapVX combines the capabilities of two open source software packages: Snap.py and CVXPY. Snap.py is a large scale graph processing library, and CVXPY provides a general modeling framework for small-scale subproblems. SnapVX offers a customizable yet easy-to-use interface with "out-of-the-box" functionality. Based on the Alternating Direction Method of Multipliers (ADMM), it is able to efficiently store, analyze, and solve large optimization problems from a variety of different applications. Documentation, examples, and more can be found on the SnapVX website at http://snap.stanford.edu/snapvx

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1. Introduction

Convex optimization has become a widely used approach of modeling and solving problems in many different fields, as it offers well-established solution methods for finding the global optimum. Many optimization software packages exist (Mosek 2010, Byrd et al. 2006, Diamond et al. 2014, Sturm 1999), but they typically rely on algorithms that are difficult to parallelize. However, new classes of problems are appearing which are too large for these common, yet centralized, solvers. Currently, solving these problems requires developing problem-specific solvers, which can be very fast but require significant optimization expertise to build. Furthermore, they are limited in scope, as they must be fine-tuned to only one particular type of problem and thus are hard to generalize.

Many large convex optimization examples, however, follow a common form. They can be split up into a series of subproblems using a network (graph) structure. Nodes are subproblems, representing anything from timestamps in a time-series dataset to users in a social network. The edges then define the coupling, or relationships between the different nodes, and the combination yields the original convex optimization problem. This representation
can refer to examples based on actual networks, such as social or transportation systems, or questions classically modeled in other ways, such as control theory or time-series analysis.

In this paper, we look at the general form of an optimization problem defined on a network. We present SnapVX, a fast, scalable, and parallelizable solver for this class of problems, which combines the graph capabilities of Snap.py (Leskovec and Sosić 2014) with the general modeling framework from CVXPY (Diamond et al., 2014). We show how SnapVX works, present syntax and supported features, scale it to large problems, and describe how it can be used to solve convex optimization problems from a variety of different fields. The full version of our solver can be downloaded at the project website, http://snap.stanford.edu/snapvx

Problem Setup. Consider an optimization problem on an undirected graph \( G = (V, E) \), with vertex set \( V \) and edge set \( E \), of the form:

\[
\text{minimize} \quad \sum_{i \in V} f_i(x_i) + \sum_{(j,k) \in E} g_{jk}(x_j, x_k).
\]  

Variable \( x_i \) is associated with node \( i \), for \( i = 1, \ldots, |V| \) (the size of \( x_i \) can vary at each node). In problem (1), \( f_i \) is the cost function at node \( i \), and \( g_{jk} \) is the cost associated with edge \( (j,k) \). Extended (infinite) values of \( f_i \) and \( g_{jk} \) represent constraints. Note that nodes and edges can also have local, private optimization variables (which can also vary in size). These remain confined to a single node or edge, though, whereas the \( x_i \)'s are shared between different parts of the network. We consider only convex objectives and constraints for \( f_i \) and \( g_{jk} \), so the entire problem is a convex optimization problem.

2. Implementation

To solve problem (1), SnapVX uses a splitting algorithm based on the Alternating Direction Method of Multipliers (ADMM) (Boyd et al., 2011). With ADMM, each individual component of the graph solves its own subproblem, iteratively passing small messages over the network and eventually converging to an optimal solution. See http://snap.stanford.edu/snapvx/snapVX_math.pdf for more details on the underlying math. By splitting up the problem, SnapVX is able to solve much larger convex optimization examples than solvers using standard (centralized) methods. ADMM also keeps the optimality benefits that general solvers enjoy: not only are we guaranteed to obtain an optimal solution, but we also are given a certificate of optimality in the form of primal and dual residuals; see (Parikh and Boyd, 2014).

SnapVX stores the network using a Snap.py graph structure. This allows fast traversal and easy manipulation, as well as efficient storage. On top of the standard graph, each node/edge is given convex objectives and constraints using CVXPY syntax. To find a global solution, SnapVX automatically splits up the problem and solves each subproblem using CVXPY, iteratively handling the ADMM message passing behind the scenes. Though wrapped in a Python layer, CVXPY uses ECOS and CVXOPT (two high-performance numerical optimization packages) as its primary underlying solvers (Domahidi et al., 2013; Andersen et al., 2014), so the Python overhead is not significant and allows for easier interpretability and improved user interface. To parallelize the solver over multiple cores, a worker pool coordinates updates for the separate subproblems using Python’s multipro-
cessing library. As such, SnapVX is built to run on a single machine, parallelizing across multiple cores while also allowing “out-of-the-box” functionality even on standard laptops.

3. Syntax and Supported Features

We now look at the syntax for a basic example. Complete documentation and more examples are available on the SnapVX website. Consider two nodes with an edge between them. We solve for a problem where each node has an unknown variable \( x_i \in \mathbb{R}^1 \). The first node’s objective is to minimize \( x_1^2 \) subject to \( x_1 \leq 0 \), the second’s is to minimize \(|(x_2 + 3)|\), and the edge objective penalizes the square norm difference between the two variables, \( \|x_1 - x_2\|_2^2 \).

```python
from snapvx import *
gvx = TGraphVX()  # Create a new graph
x1 = Variable(1, name='x1')  # Create a variable for node 1
gvx.AddNode(1, Objective=square(x1), Constraints=[x1 <= 0])  # Add new node
x2 = Variable(1, name='x2')  # Repeat for node 2
gvx.AddNode(2, abs(x2 + 3), [])
gvx.AddEdge(1, 2, Objective=square(norm(x1 - x2)), Constraints=[[]])  # Add edge between the two nodes

gvx.Solve()  # Solve the problem
gvx.PrintSolution()  # Print the solution on a node–by–node basis
```

As SnapVX is meant to be a general-purpose solver, it has many customizable features to help easily and efficiently solve a wide range of convex optimization problems.

**Bulk loading.** Often, the objectives and constraints at each node or edge will share a common form. For example, all the nodes could be trying to minimize \( \|x - a_i\|_2 \) for different values of \( a_i \). Or, the edges could represent Laplacian regularization \( \|x_j - x_k\|_2^2 \) between different nodes. Rather than manually inputting each of these nodes/edges, SnapVX allows the user to “bulk load” this data. The functions `AddNodeObjectives` and `AddEdgeObjectives` allow the user to specify a set of nodes (or edges) and an external file with separate data, for example the \( a_i \)'s at each node. Then, the user defines the general form of the objectives and constraints, and SnapVX fills in the details. This functionality makes practical and significantly speeds up the loading of very large datasets.

**ADMM \( \rho \)-update.** The convergence time of ADMM is heavily dependent on the value of the penalty parameter \( \rho \) \cite{nishihara2015}, as it affects the tradeoff between primal and dual convergence, both of which need to be obtained for the overall problem to be “solved”. Aside from allowing the user to choose their preferred value of \( \rho \) (it defaults to \( \rho = 1 \)), SnapVX users are also able to define a function to update \( \rho \) after each step based on the primal and dual residual values, an approach with much active research \cite{he2000, fougner2015}.

**Parallelizability.** When running on multi-core machines, SnapVX automatically determines the number of processors on the current machine and splits the problem accordingly with Python’s multiprocessing library to balance the workload and speed up the solver.

**Additional features.** SnapVX allows for many different options when solving convex optimization problems. These include options such as turning ADMM on/off, verbose mode (to list intermediate steps at each iteration), and defining convergence thresholds.
4. Scalability

One of the biggest benefits of SnapVX is that it allows us to solve large problems very efficiently. Convergence time depends on the problem complexity, but we empirically observe that SnapVX scales approximately linearly with problem size. In Figure 1 we compare SnapVX and a centralized solver for a problem on a 3-regular graph. Each node solves for an unknown variable in $\mathbb{R}^{9000}$, and has an objective involving huber penalties (Huber, 1964), defined as

$$\text{huber}(x) = \begin{cases} x^2, & \text{if } |x| \leq 1 \\ 2x - 1, & |x| > 1. \end{cases}$$

At node $i$, the objective is $\sum_{j=1}^{9000} \text{huber}(x_{i,j} - a_j)$. The $a$'s at each node are drawn independently from a normal distribution with zero mean and identity covariance. Every edge has a network lasso penalty ([Hallac et al., 2015], $\|x_i - x_j\|_2$). By varying the number of nodes, we can span a wide range of problem sizes. The time required for SnapVX to converge, on a 40-core CPU where the entire problem can fit into memory, is shown in Table 1. Figure 1 displays a comparison to a centralized method (CVXPY, using ECOS) in log-log scale.

5. Potential Applications

SnapVX allows users without significant optimization expertise to apply ADMM to various applications. Common examples from many different fields can be formulated in a SnapVX-friendly manner. Several of these are listed on the project website. Examples include financial modeling, image processing, event detection, time series analysis, and housing price prediction. We provide an easy-to-use solver that is able to scale to large problems and apply to a wide variety of examples in many different fields. With an active userbase and rising interest from a range of scientific and engineering fields, we hope that SnapVX can become a useful tool for convex optimization problems, both in research and industry.
References


