Rethinking Modulation and Detection for High Doppler Channels

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Abstract

We present two modulation and detection techniques that are designed to allow for efficient equalization for channels that exhibit an arbitrary Doppler spread but no delay spread. These techniques are based on principles similar to techniques designed for time-invariant delay spread channels (e.g., Orthogonal Frequency Division Multiplexing or OFDM) and have the same computational complexity. Through numerical simulations, we show that effective equalization is possible for channels that exhibit a high Doppler spread and even a modest delay spread, whereas equalized OFDM exhibits a strictly worse performance in these environments. Our results indicate that, in rapidly time-varying channels, such as those found in high-mobility or mmWave deployments, new modulation coupled with appropriate channel estimation and equalization techniques may significantly outperform modulation and detection schemes that are designed for static or slowly time-varying multipath channels.

Index Terms — 5G Mobile Communication, Multipath Channels, Time-varying Channels, mmWave

I. INTRODUCTION

In many sub-6 GHz wireless systems such as LTE and WiFi, the delay spread of the point-to-point wireless channel is typically much smaller than the coherence time [2], [3]. A long channel coherence time allows the wireless channel to be treated as a time-invariant channel with frequency-selective channel gains. In this regime, OFDM, or general multicarrier modulation schemes, with rate and power adaptation, are well known to be information-theoretically optimal in terms of spectral efficiency [4], [5]. In these channels, a long coherence time enables accurate
channel estimation with negligible overhead. Obtaining an accurate estimate of the channel becomes difficult as the coherence time decreases relative to the block length of the waveform, which can lead to significant performance degradation when using multicarrier modulation.

In general, the wireless channel is both time dispersive, introducing a delay spread, and frequency dispersive, introducing a Doppler spread [6]. The capacity for a general time-varying channel with imperfect knowledge of and/or adaptation to the channel state is unknown. Channels with non-zero Doppler spread are no longer time invariant. Moreover, as the coherence time (roughly the inverse of the Doppler spread) shrinks relative to the block length, frequency-domain equalization methods, used with modulation and equalization techniques such as OFDM or Single-Carrier modulation with Frequency-Domain Equalization (SC-FDE), will no longer be effective as channel estimations will become inaccurate. Examples of wireless channels with significant Doppler spreads include mmWave systems and systems with high mobility [7]. Significant time-variations can also arise in non-terrestrial settings such as underwater systems [8], or in satellite to earth communication links [9]. Additionally, impairments such as phase noise in mmWave systems may manifest in ways similar to high Doppler spread in rapidly time-varying channels [10]. The time duration over which the channel can be assumed to be time-invariant in such systems is much shorter than in typical sub-6 GHz systems; these time-variations will affect the performance of algorithms that depend on accurate channel state information.

One approach to adapt modulation and detection techniques such as OFDM or SC-FDE to deal with the effects of time-variation is to limit symbol or block duration. Indeed, it is typical for OFDM deployments in time-varying channels to limit the symbol time, or, equivalently, increase the subcarrier spacing, so that the product of the overall symbol duration and the Doppler spread is small. In this regime, the time-frequency dispersive channel behaves approximately as a static delay spread channel at the expense of an increased cyclic prefix overhead. A detailed performance analysis of such schemes is presented in [11]. This class of schemes will be further discussed in Section IV.

A different approach to simultaneously deal with dispersion in the time and frequency domains is via general time-frequency signaling [6]. In [12], the authors describe a general framework for a time-frequency modulation scheme for time-frequency dispersive channels. The time-frequency representation in [12] uses the Short-time Fourier basis (SFT), and proposes ways to deal with the loss of orthogonality between the basis functions induced by the time-frequency dispersive channel. Another work, [13], introduces Orthogonal Time-Frequency-Space signaling (OTFS).
OTFS utilizes the delay-Doppler representation of the signal and the channel. OTFS has been demonstrated to have advantages over OFDM in specific high-Doppler channels. However, while [13] presents an overall framework for designing waveforms, the performance of OTFS in a time and frequency dispersive channel will depend on a large number of tunable parameters.

In this work, we first consider a specific class of time-varying channels, which are the time-frequency duals of time-invariant, frequency-selective channels. Specifically these channels have zero delay spread and a finite Doppler spread. We then describe two related modulation and detection schemes that are well suited to compensate for the impairments caused by these channels. We show that, provided a proper choice of waveform parameters and channel estimation algorithms, we may perfectly and efficiently compensate for the effects of this class of time-varying channels via equalization. Additionally, through numerical simulations, we show that these modulation and detection schemes perform well in channels that exhibit low to moderate delay spread and high Doppler spread. We describe realistic terrestrial environments where such channels exist.

The modulation and detection schemes we present are the time-frequency duals of OFDM with Frequency-Domain Equalization\(^1\) (OFDM-FDE) and SC-FDE. These proposed techniques, which we term Single-Carrier modulation with Time-Domain Equalization, or SC-TDE, and Frequency-Domain Multiplexing with a Frequency-Domain Cyclic Prefix, or FDM-FDCP, compensate for the effect of an arbitrary Doppler spread by performing linear equalization in the time domain. This class of modulation and detection techniques is not degraded by arbitrary Doppler spreads because it directly estimates and equalizes the Doppler profile of the wireless channel rather than estimating and equalizing the delay-spread profile as is the case with OFDM-FDE. Due to the fact that our techniques are the time-frequency duals of OFDM-FDE and SC-FDE, they inherit most of the computational and implementation benefits found in these systems. The relationships between our techniques and OFDM-FDE and SC-FDE are shown in Figure 1.

In addition to introducing this new class of modulation and detection schemes, we present several numerical simulations involving general time-varying, frequency selective channels. We show several regimes where frequency-domain equalization schemes such as OFDM-FDE are strictly sub-optimal compared to our techniques in terms of spectral efficiency and symbol error rates.

\(^1\)We refer to this modulation and detection technique as OFDM-FDE to emphasize that we are only considering OFDM with linear frequency-domain equalization as opposed to non-linear FDE or additional time-domain equalization.
Conversely, in delay spread channels with no or little Doppler spread, our techniques perform worse than OFDM-FDE or SC-FDE. Our work suggests that rethinking common assumptions about joint waveform and equalizer design may result in significantly improved performance over current methods in many Doppler-spread channels of interest. Moreover, these proposed techniques have comparable computational complexities to existing methods.

The rest of the paper is organized as follows. In Section II, we describe notation and assumptions used throughout the paper. In Section III, we give a complete description of both the FDM-FDCP and SC-TDE modulation and detection techniques. In Section IV we present numerical results pertaining to various performance metrics and summarize the advantages and disadvantages of FDM-FDCP and SC-TDE compared to OFDM-FDE and SC-FDE. Our conclusions are presented in Section V.
II. System Model and Notation

The complex baseband representation of the general time-varying impulse response function associated with a wireless channel is given by

\[ h(t, \tau) = \sum_{i=0}^{K-1} \alpha_i e^{j2\pi f_i t} \delta(\tau - \tau_i), \]  

where \( K \) is the number of multipath components and \( \alpha_i, f_i, \tau_i \) are the complex gain, Doppler frequency and the delay associated with the \( i^{th} \) multipath component, respectively. This is an example of a channel introducing both time shifts and frequency shifts; an overview of this class of channels can be found in [6]. The channel is linear time-invariant only when \( f_i = 0 \) for all \( i \), and the gains and delays associated with each individual component do not change in time. Since we study only the effects of time-dispersion and frequency-dispersion, we assume henceforth that the delays, Doppler shifts and gains of the multipath components are constant over the transmit block duration.

Throughout this work, when we refer to a technique such as OFDM-FDE or FDM-FDCP, we are jointly considering modulation and detection, including the combination of channel estimation and equalization as part of the detection process. Implicitly, this requires us to also make a set of assumptions about the nature of the channel (i.e. whether it is highly dispersive in time or frequency) in order to effectively measure and equalize the channel. We precisely define how we perform channel estimation and equalization for our proposed techniques in Section III.

We assume that the time-varying impulse response function \( h(t, \tau) \) is unknown at the transmitter and the receiver. The receiver performs channel estimation based on pilot symbols transmitted using the modulation technique under consideration. For reasons further discussed in Section III-C, this means that different modulation and detection techniques will obtain different estimates for the channel with the same time-varying impulse response function even in the absence of additive noise. As a result, when used in differing classes of time-varying channel impulse response functions, for example with low or high Doppler or delay spreads, different techniques may have a substantially different SER performance, even without any additive noise (i.e., with infinite SNR). We refer to channel estimates obtained in the limit of infinite SNR as perfect channel estimates.

We note that for modulation and detection methods such as OFDM-FDE or SC-FDE, techniques exist that allow one to compensate for the effects of arbitrary Doppler spread. For example, one may perform additional equalization in the frequency-domain, as described in [14]. However,
such schemes require a complexity that is quadratic in the block length of the system, i.e. $O(N^2)$, and thus, in general are not well suited for real-time, high-throughput communications. In order to obtain a fair basis of comparison for all techniques presented in this paper, and also to provide constructions that are suitable for deployment in practical wireless systems, we only allow equalization that has an almost linear complexity, limiting the overall cost of modulation and demodulation to the cost of the FFT operation, namely $O(N \log N)$.

We use the boldface notation $\mathbf{x}$ to denote a length $N$ discrete sequence, with $x[n]$ referring to the $n^{th}$ element of the sequence. Unless otherwise specified, we use lowercase $x$ to represent the time-domain sequence and uppercase $X$ to denote its frequency domain representation. The variable $N$ refers to the block length of our waveform construction. Sequences of length $N$ are isomorphic to $N$-dimensional column vectors with complex entries in a natural way; hence we use $x$ to refer to both a column vector and a finite length sequence of dimension $N$. The operator $*$ represents linear convolution, the operator $\circ$ represents circular convolution, and the operator $\odot$ represents the Hadamard product (element-wise scalar multiplication). $\mathcal{F}$ and $\mathcal{F}^{-1}$ represent the DFT and inverse DFT operations respectively. The notation $\frac{1}{x}$ refers to a sequence $y$ whose $n^{th}$ element is given by $y[n] = 1/x[n]$, $x^2 = x \odot x$, and $x/y = x \odot \frac{1}{y}$. Additive white Gaussian noise is denoted as $w(t)$ or $w[n]$.

III. MODULATION AND DETECTION FOR FREQUENCY DISPERSIVE CHANNELS

Techniques such as OFDM-FDE or SC-FDE, which rely on linear frequency-domain equalization, are designed for time-invariant delay spread channels. Under time-invariance, the two-dimensional time-varying impulse response function reduces to a one-dimensional impulse response function, $h(\tau)$. When the channel is not entirely time-invariant, $h(\tau)$ will not fully capture the effect of the two-dimensional channel. Even if the channel estimate is obtained perfectly, i.e. in the absence of AWGN, the resulting equalization using this channel estimate may exhibit a residual error floor. We refer to this effect as model mismatch. In Section IV, we measure the effect of this model mismatch through the error vector magnitude (EVM) of the demodulated QAM symbols, measured in the absence of AWGN.

In order to study the potential gains that can be realized by changing our assumptions about the general behavior of the wireless channel, we begin by considering the time-frequency dual of time-invariant delay spread channels. Specifically, we consider channels that have an arbitrary
Doppler spread and zero delay spread. This assumption allows us to reduce the two-dimensional time-varying channel impulse response function to a different, one-dimensional function, namely
\[ h(t, \tau) = h(t) = \sum_{i=0}^{K-1} \alpha_i e^{j2\pi f_i t}. \] (2)

For comparison, the corresponding one-dimensional channel response function for a time-invariant, non-zero delay spread, for which OFDM-like schemes are commonly used in many modern standards, is given by
\[ h(t, \tau) = h(\tau) = \sum_{i=0}^{K-1} \alpha_i \delta(\tau - \tau_i). \] (3)

Notice that (2) is a time-frequency dual of (3). In discrete time, the effect of the channel (2) is given by:
\[ y[n] = \sum_{i=0}^{K-1} \alpha_i e^{j2\pi f_i T_s} x[n], \] (4)

where \( T_s \) is the sampling period. Assuming that the \( f_i \) are integer multiples of \( \frac{1}{NT_s} \), where \( NT_s \) is the overall duration of the waveform, we can take the Fourier transform on both sides to yield
\[ Y[k] = \sum_{i=0}^{K-1} \alpha_i X[k - k_i], \] (5)

where \( k \) indexes the discrete frequency axis and
\[ H[k] = \sum_{i=0}^{K-1} \alpha_i \delta[k - k_i]. \]

In the above equations, the Fourier transform \( S[k] \) of a finite sequence \( s[n] \) of length \( N \) is defined as \( S[k] \Deltaq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s[n] e^{-j2\pi k n/N} \). \( k_i \Deltaq f_i T_s \). Thus, the effect of the channel given by (2) on the symbols \( S[k] \) in the frequency domain is the same as the effect of the channel given by (3) on time domain symbols. This implies that techniques used to correct for the delay spread in time invariant channels can be used to correct for the Doppler spread in time-varying channels. Exploiting this duality is the primary design idea behind the modulation and detection techniques that we present in this paper for high-Doppler spread channels.

We note that, in practice, the assumption used in (5), that \( f_i \) is discrete-valued, is not realistic. If our waveform is not properly constructed, this discrepancy can lead to large side-lobes in the frequency domain. That is, for a channel with \( K \) multipath components, these sidelobes can cause a single symbol to interfere with more than \( K \) symbols. We note that a similar effect, often
termed “tap leakage”, may also occur in OFDM-FDE- or SC-FDE-based systems when \( \tau_i \) is not discrete-valued. This effect is discussed in [15]. In practice, this effect is mitigated by considering appropriate windowing or pulse shaping functions in the time and frequency domains. For SC-TDE and FDM-FDCP we may employ windowing functions which are approximately the duals of those commonly applied to OFDM-FDE and SC-FDE; this is described further in Section III-D.

SC-TDE is a single-carrier transmission scheme and carries QAM symbols in the time-domain; we denote the baseband sequence of QAM symbols as \( r[n] \). Conversely, FDM-FDCP is a multicarrier modulation scheme and contains QAM symbols in the frequency domain; the baseband QAM symbols of FDM-FDCP are denoted as \( R[n] \). We note that when referring to SC-TDE signals, we use \( R[n] \) to denote \( F \{ r[n] \} \), and similarly, when referring to FDM-FDCP symbols, the sequence \( r[n] \) denotes \( F^{-1} \{ R[n] \} \).

A. Equalization and the frequency-domain cyclic prefix

In the channel given by (5), Doppler spread causes inter-carrier interference between frequency domain symbols that, if not corrected for, impairs performance and affects SER even without any additive noise. As previously mentioned, one possible method to compensate for this interference is through equalization between frequency bins [14]. More commonly, OFDM is adapted to time-varying channels by increasing the width of the subcarrier bin by reducing the overall symbol duration. However, while this approach will reduce the ICI, the presence of Doppler will still lead to an error in channel estimation and may result in an increased symbol error rate. This effect is explored numerically in Section IV.

In the remainder of this section, we show how to compensate for the effect of Doppler spread through the use of a frequency-domain cyclic prefix. We first observe that in (5), the effect of the channel is equivalent to linear convolution in the frequency domain. This linear convolution may be converted to circular convolution by adding a cyclic prefix to the data symbols \( X \) in the frequency domain. Similar to the time-domain cyclic prefix found in OFDM-FDE or SC-FDE, the width of the cyclic prefix in the frequency domain depends on the Doppler spread of the channel. Specifically, if the Doppler spread in the discrete frequency domain is within the range \([-f_{\text{max}}, f_{\text{max}}]\), then the number of frequency domain cyclic prefix elements (FDCP) that need to be appended to \( X \) is \( 2f_{\text{max}} + 1 \).
Denote $L \triangleq 2f_{\text{max}} + 1$, and assume that there are $N - L$ data symbols such that $L \ll N$. Let $\tilde{R} = \{\tilde{R}[n]\}_{n=0}^{N-1}$ be defined such that
\[
\tilde{R}[n] = \begin{cases} 
R[n - L] & n \geq L, \\
R[N - L + n] & \text{otherwise}.
\end{cases}
\] (6)
This operation is illustrated in Figure 2. Denote $\tilde{r} = \mathcal{F}^{-1}\{\tilde{R}\}$, of length $N$, as the time-domain sequence to be transmitted over the channel after pulse shaping. At the receiver, the stream of $N$ received symbols can be passed through a serial to parallel converter, and the resulting $L$ symbols can be removed from the frequency domain through the use of an FFT. The remaining $N - L$ symbols form the sequence $Z$. Ignoring additive noise, we have the following relation:
\[
Z = H \ast R.
\] (7)
In the discrete frequency domain, the action of the channel can now be described as circular convolution, or equivalently as element-wise multiplication in the discrete time domain. Equalization can thus be achieved by performing element-wise division by $h$ in the time domain, i.e.
\[
\hat{r} = \mathcal{F}^{-1}\{Z\} / \mathcal{F}^{-1}\{H\} = z / h.
\]
Fig. 3. Zero-padded FDM-FDCP: Rather than including a cyclic-prefix in the frequency domain, we can simply add guard bands in the frequency domain, analogous to zero-padded OFDM-FDE. The receiver performs equalization by adding the frequency-domain tails on both sides, represented here by shaded regions outside of the block labeled “FDM-FDCP Symbols”, onto the original symbols and then performing element-wise division in the time domain.

B. Zero-Padding

In OFDM-FDE, the cyclic-prefix can be replaced by a guard interval in the time-domain equal to the length of the delay spread [16]. At the receiver, the resulting tail of each OFDM-FDE block falling into this guard interval is then added back to the beginning of the OFDM-FDE block, thus emulating the effect of the cyclic prefix. This technique is known as zero-padded OFDM-FDE.

One may take a similar approach to emulate the frequency-domain cyclic prefix in FDM-FDCP. In this case, at the transmitter, we must simply leave empty spectrum on both sides of the signal. The receiver can then copy the tails in the frequency domain back into the FDM-FDCP block as shown in Figure 3. This approach has only a minimal impact on the complexity at the receiver.

A practical method of achieving zero-padding in a system transmitting FDM-FDCP is to simply place guard bands in the frequency domain. In wireless systems, guard bands are already used to meet spectral mask requirements, and to help simplify filter design. We note that unlike in the case of OFDM-FDE, the use of zero-padding over the full cyclic prefix results in a slightly reduced peak-to-average power ratio (PAPR). This is discussed further in Section III-E.
C. Channel estimation

The measurement of the time-varying channel impulse response function, $h(t, \tau)$, is subject to the uncertainty principle [17] arising from the fact that time and frequency are Fourier duals. Given finite resources in time and bandwidth, there are fundamental limits to how accurately one can measure the delay and Doppler components of a given time-varying channel impulse response function. Any error associated with a measurement of the channel impulse response function will have an impact on the overall system performance of a modulation and detection scheme.

In OFDM systems, the assumption of time-invariance of the wireless channel helps with channel estimation. In the absence of Doppler spread, the channel impulse response is of the form (3) and is only a function of $\tau$. A non-zero delay spread gives rise to frequency selectivity; the frequency response, and hence the delay spread, can be measured to any arbitrary precision by placing pilot symbols in the frequency domain.

Equivalently, one may measure the entire frequency response of the channel by sending a single impulse in the time domain, which would correspond to sending an OFDM-FDE block with a constant data symbol in each frequency bin. If the channel were truly time invariant, this estimate would then be valid for all future channel uses. In practice, OFDM-FDE pilots are placed in the frequency domain and the channel gains are interpolated between pilots. However, a variety of strategies exist to perform casual channel estimation, for example see [15] or [18]. For channels that are rapidly time-varying channels, these estimation strategies break down.

For FDM-FDCP and SC-TDE, channel estimation can be performed by assuming that a pilot sequence, $\tilde{r} = \mathcal{F}^{-1}\{R\}$ is known at the receiver. An estimate $\hat{h}$ of the channel response $h \triangleq \mathcal{F}^{-1}\{H\}$ can be calculated by

$$\hat{h} = \mathcal{F}^{-1}\{Z\} / \mathcal{F}^{-1}\{R\}.$$  

(8)

Similar to the case for OFDM-FDE in the limit of zero Doppler spread, for FDM-FDCP with no delay spread, a full estimation may be performed by sending a signal tone in the frequency domain, or transmitting a block with a constant single occupying every time slot. Equivalently, one may simply insert a pilot in the frequency domain and send no signal in an appropriate width surrounding the pilot, as shown in Figure 4. In the absence of delay spread, this channel estimate will remain valid over all blocks. For SC-FDE, one may simply place pilots in the
Fig. 4. A causal channel estimation strategy for FDM-FDCP. A pilot tone may be placed in the frequency domain, surrounded by a guard interval at least as large as twice the Doppler spread. The receiver may then recover an estimate of the Doppler profile, and hence the channel impulse response function, by simply measuring the resulting the received spectral components surrounding the pilot symbol.

time domain and interpolate between these pilot symbols, in a manner analogous to performing estimation for OFDM-FDE.

Using FDM-FDCP, if the channel has a non-zero delay spread, the channel will become frequency selective. This implies that the channel estimate may differ depending on where the pilot is placed in the frequency domain. The can be contrasted to the use of OFDM-FDE in a high-Doppler environment where the channel will vary over the length of the block, making the channel estimate inaccurate. In OFDM-FDE, the time-duration of the block length is limited by the coherence time of the channel, which is inversely proportional to the Doppler spread. In contrast, FDM-FDCP is limited in bandwidth by the coherence bandwidth of the channel, which is inversely proportional to the delay spread of the channel. The effectiveness of our modulation and detection techniques at equalizing channels with non-zero delay spread will be investigated further through numerical simulations in Section IV.

D. Modulation and detection

Having described the operations of channel estimation and equalization, we now fully describe how to modulate and detect FDM-FDCP and SC-TDE. Overall block diagrams for both systems are presented in Figure 5. At the transmitter, in both systems, QAM symbols are converted from serial to parallel blocks of length \( N - L - P \), where \( P \) is the number of pilot symbols, including any guard intervals associated with the pilot if applicable. Pilot tones are then inserted along
Fig. 5. On the top, a system diagram of FDM-FDCP. Pilot symbols and a frequency-domain cyclic prefix are added to the data symbols, which are then modulated through an inverse FFT. Pulse shaping is performed in the time domain analogous to OFDM-FDE. SC-TDE is shown on the bottom, which differs from FDM-FDCP through an additional Fourier transform. Transmit pulse shapes are windowed in the frequency domain in analogy to SC-FDE. For both systems, the FFT and IFFT are the most expensive operations from the perspective of computational complexity and require $O(N \log N)$ operations.

side the data symbols as described in Section III-C. The frequency domain cyclic prefix is then appended to the waveform, and the resulting signal is then passed through an appropriate pulse shaping or windowing function before being transmitted over the air.

For this work, we consider only a single pulse shape and attempt to use equivalent pulse-shaping methods to compare each modulation and detection technique. A more complete description on the effect of pulse shaping on our techniques is beyond the scope of this work. However, we note that due to the similarity of our techniques to existing constructions, existing
work on pulse shaping for OFDM-FDE or SC-FDE may be applied, see for example [19] or [20]. For each modulation technique, pulse shaping is applied at the transmitter in the domain that is dual to the data symbols. Specifically, we rely on the standard root raised cosine response, $G(s)$, given by

$$G(s) = \begin{cases} \sqrt{T} & 0 \leq |s| \leq \frac{1-\beta}{2T} \\ \sqrt{T} \left( 1 + \cos \left( \frac{\pi T}{\beta} \left( |s| - \frac{1-\beta}{2T} \right) \right) \right) & \frac{1-\beta}{2T} \leq |s| \leq \frac{1+\beta}{2T} \\ 0 & |s| > \frac{1+\beta}{2T}. \end{cases} \tag{9}$$

For FDM-FDCP, we multiply the time-domain signal by $G(t)$ with $\beta = 0.1$ and $T = T_s$. This yields orthogonal sub-carriers that consist of Nyquist-like pulses with a rapidly decaying tail. For SC-TDE, we multiply the frequency-domain signal by $G(f)$ with the same parameters so that the resulting time-domain signal is a Nyquist pulse with 10% excess bandwidth.

Since channel estimation is performed by transmitting pilots through the appropriate pulse-shaping function, we may define the effective channel response as the composition of the pulse-shaping operator and the channel operator. That is, assuming our channels have zero delay spread\(^2\), $h_{eff}[n] = h[n] \odot G(t)$ for FDM-FDCP and $h_{eff}[n] = h[n] \otimes g(t)$, for SC-TDE, where $g(t) = F^{-1} \{G(f)\}$. Then we can follow exactly the same steps as in (8) for channel estimation and data demodulation. If $y[n]$ is the signal received after discretization at the receiver, and assuming the receiver has recovered an estimate of the channel, $\hat{h}_{eff}[n]$ we may estimate $\hat{r}[n]$ as

$$\hat{r}[n] = y[n] / \hat{h}_{eff}[n]. \tag{10}$$

Finally, an estimate of the data symbols may be recovered by removing the frequency-domain cyclic prefix and pilot symbols in their appropriate domains. We note that computing the Fourier transform and inverse-Fourier transform of a block of symbols is the most computationally expensive operation and requires $O(N \log N)$ operations.

**E. Peak-to-Average Power Ratio Considerations**

Multicarrier modulation schemes such as OFDM often have a large peak-to-average power ratios (PAPR). This may lead to difficulties in the implementation of such schemes due to

\(^2\)For the general channel with non-zero delay and Doppler spread, the convolution and multiplication operators are replaced with more general compositions for linear operators.
non-linearities present in transmitter power amplifiers. We note that due to the similarity of our waveforms to existing constructions, it should be possible to apply many PAPR reduction techniques developed for OFDM to our waveforms, for example see [21].

In general, the peak-to-average power ratios of the transmitted FDM-FDCP and SC-TDE waveforms are very similar to those of OFDM-FDE and SC-FDE respectively. We expect that the overhead of the frequency-domain cyclic prefix will cause a slight increase in PAPR compared to schemes that use only a time-domain cyclic prefix. That is, we expect FDM-FDCP will have a slightly higher PAPR than OFDM-FDE. The SC-TDE waveform will have a lower PAPR due to the fact that the data symbols are transmitted in the time domain. However, the application of the frequency-domain cyclic prefix will increase the PAPR beyond that of the SC-FDE waveform. As the analytic study of PAPR can often be difficult, a more complete discussion of PAPR is beyond the scope of this work.

IV. NUMERICAL RESULTS

We now present a series of simulations that show the potential of both FDM-FDCP and SC-FDE to mitigate the effects of Doppler spread. The channel model used in these simulations is described in detail in Section IV-A. In order to highlight the effect of model mismatch, we assume that a perfect, non-casual channel estimate is available at the receiver. As described in Section III, this estimate is not the two-dimensional scattering function, but rather $h_{\text{eff}}[n]$ which captures the effect of using a pulse shaping filter.

In order to quantify the effects of model mismatch, we rely on the error-vector magnitude (EVM) metric, which is defined as the ratio of the amplitude of the error vector to the root mean squared amplitude of the received symbol, or

$$\text{EVM} = \frac{\sqrt{P_{\text{error}}}}{\sqrt{P_{\text{mean}}}}.$$  \hspace{1cm} (11)

It is also convenient to express decoding error in terms of an irreducible-error floor. If we assume that the error-vector is approximately Gaussian, then using MQAM modulation, we can approximate the irreducible-error floor in terms of SER using the standard expression for the SER performance of MQAM in AWGN (e.g. see [2, Chapter 6]), namely

$$P_{s,\text{floor}} = 4Q\left(\frac{3}{(M-1)(\text{EVM})}\right).$$  \hspace{1cm} (12)

We note that our simulations show little difference in performance between single and multicarrier modulation formats (i.e. SC-FDE vs. OFDM-FDE or SC-TDE vs. FDM-FDCP).
is because we do not assume CSI knowledge at the transmitter and therefore there is no rate or power adaptation across subcarriers or time-domain symbols. For SC-FDE, if CSI is known at the transmitter, the variable-rate and variable-power QAM modulation scheme described by [22] may be employed. Further performance differences between single and multicarrier schemes may be observed in channels with deeper fading characteristics or when using error-correcting codes.

A. Channel Models

The channel models used in these simulations are based on the 3GPP channel models for 0.5 to 100 GHz found in TR 38.901 [23]. To emphasize the effects of delay and Doppler spread on the modulation format, we choose a non-line-of-sight link between a single transmitter and a single receiver. Specifically, we consider that each channel is drawn according to the two-dimensional channel scattering function

$$h(t, \tau) = \sum_{i=1}^{n} \alpha_i e^{2\pi f_s t} \delta(\tau - \tau_i).$$

(13)

For a specified delay spread, the power and delay of each received component is fixed according to the Tapped-Delay Line Model-A (TDL-A); the normalized values of $|\alpha_i|$ and $\tau_i$ are given in Table I. For each simulation, each received component is assigned a phase uniformly at random.

<table>
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<th>Normalized Power Delay (dB)</th>
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<td>0.7618</td>
<td>-15.9</td>
</tr>
<tr>
<td>1.5375</td>
<td>-15.9</td>
</tr>
<tr>
<td>1.8978</td>
<td>-6.6</td>
</tr>
</tbody>
</table>

| 2.2242                      | -16.7                       |
| 2.1718                      | -12.4                       |
| 2.4942                      | -15.2                       |
| 2.5119                      | -10.8                       |
| 3.0582                      | -11.3                       |
| 4.0810                      | -12.7                       |
| 4.4579                      | -16.2                       |
| 4.7966                      | -18.3                       |
| 5.0066                      | -16.6                       |
| 5.3043                      | -19.9                       |
| 9.6586                      | -29.7                       |
Further, each \( f_i \) was drawn according to a Jakes’ spectrum [24]; that is, each \( f_i \) is a random variable that is drawn independently according to the PDF

\[
p(f) = \begin{cases} 
\frac{1}{\pi f_D \sqrt{1-(f/f_D)^2}}, & |f| < f_D \\
0, & \text{otherwise}
\end{cases}
\] (14)

All channel parameters are assumed to be constant over each transmission block length.

The total channel model can now be fully characterized by specifying only the desired delay and Doppler spread. We note that a specified Doppler spread gives the maximum deviation of the Doppler relative to the carrier center frequency. In contrast, as shown in Table I, received components will have a maximum delay that is nearly ten times the delay specified by the delay spread; however, over 90% of the signal power will be captured within the time interval indicated by the delay spread.

The range of delay and Doppler spreads used in our simulations correspond to realistic wireless environments. In particular, for delay spread we consider a minimum of 10 ns, which in [23] is described as a “very short” delay spread, and is typical of an indoor or office environment. We also consider delay spreads as high as 100 ns which [23] describes as a “normal” delay spread and would be typical in an urban canyon deployment, for example. In Table II, we list typical deployment scenarios corresponding to differing delay spread for 6 GHz deployments. We note that higher-frequency deployments such as 20 or 60 GHz would experience delay spreads slightly lower than indicated by Table II.

The Doppler spread values considered range from 50 Hz to 5000 Hz. We note that Doppler spreads as high as 500 Hz would be typical for a 60 GHz deployment in an environment with
low mobility (walking speeds), and would arise in 6 GHz deployments with vehicles moving at surface-road speeds. Table II lists scatterer speeds relative to either transmitter or receiver that result in 50 or 500 Hz Doppler shifts for various center frequencies. Several representative channel responses obtained using this model are presented in Figure 6. These responses are measured through the FDM-FDCP estimation routine described in Section III-C.

B. Decoding Performance under AWGN

To demonstrate the ability of our modulation and detection techniques to compensate for arbitrary Doppler spread, we simulate the decoding performance under AWGN for channels with various Doppler spreads. The first set of simulations are given in Fig. 7 where, for comparison, we also provide a simulation of OFDM-FDE. For both techniques, we fix the blocklength to be $N = 2048$ with a sampling rate of 1.92 MHz. We simulate transmitting 16 QAM over all channels. In order to emphasize channel impairments caused by Doppler, we choose a small delay spread of 50 ns, typical of an indoor setting or a mmWave urban-canyon deployment. For both techniques, we assume that CSI is known perfectly at the receiver.

For small Doppler spreads, i.e. 200 Hz and below, FDM-FDCP is able to almost entirely equalize the channel. As the Doppler spread increases, the small amount of delay spread somewhat impairs our ability to fully equalize the channel. This effect is considered more carefully in
Fig. 7. On the left, decoding performance of 16 QAM using FDM-FDCP with a 1.92 MHz sampling rate and a block length of \( N = 2048 \) under AWGN. The channel model is based on the 3GPP TDL-A model with a normalized delay spread of 50 ns. On the right, OFDM-FDE in the same channel using the same set of parameters. For both techniques, we assume perfect CSI is available at the receiver.

Fig. 8. On the left, decoding performance of 16 QAM using FDM-FDCP with a 4.5 MHz sampling rate and a block length of \( N = 2048 \) under AWGN. The channel model is based on the 3GPP TDL-A model with a normalized delay spread of 20 ns. In this environment, increasing Doppler spread has little effect on FDM-FDCP whereas OFDM-FDE fails to equalize the channel at high Doppler.

Section IV-C. In contrast, OFDM-FDE is unable to equalize the channel except when the Doppler spread is below 100 Hz; however, even in this case OFDM-FDE has substantially worse SER performance than FDM-FDCP.
In Figure 8 we present an additional set of simulations where the sampling rate has been increased to 4.5 MHz and the delay spread has been decreased to 20 ns. Here, due to the reduced delay spread, we see that the ability of FDM-FDCP to compensate for arbitrary Doppler is only slightly effected by increasing the Doppler spread. In contrast, the shorter block length allows OFDM-FDE to more effectively equalize the channel. However, once the Doppler spread exceeds 300 Hz, FDM-FDCP outperforms OFDM-FDE. These results suggest that FDM-FDCP offers an attractive method of transmission in high-Doppler spread channels that are bandlimited either due to constraints imposed by resource allocation or hardware.

C. Channel Parameter Sweeps

In this section we characterize the effectiveness of the considered modulation and detection techniques at equalizing channels that are impaired predominantly by a large delay spread or a large Doppler spread. We begin by fixing the sampling rate and block length associated with all four modulation and detection techniques and simulate the techniques across a variety of channel conditions. Specifically, we fix the block length to be $N = 1024$ and the sampling rate to be 4.5 MHz.

In Figure 9, we fix the delay spread to be 50 ns and sweep the Doppler spread from 50 Hz to 5000 Hz. We see that OFDM-FDE and SC-FDE are able to effectively equalize the channel as long as the Doppler period remains small compared to the block duration. However, for large Doppler spreads, equalization becomes uneffective. Additionally in Figure 9, we see that FDM-FDCP and SC-TDE are able to effectively equalize the channel even as the Doppler spread grows. We notice a slight degradation in performance for high Doppler spread channels. This is a result of the non-zero delay spread present in the channel; we demonstrate a similar degradation for OFDM-FDE in the next set of simulations.

In Figure 10, the Doppler spread is fixed to 500 Hz and the delay spread is swept from 10 ns to 1000 ns. We observe that FDM-FDCP and SC-TDE are able to effectively equalize these channels as long as the delay spread remains small compared to the symbol period (here $T_s = 22$ ns). In contrast, the OFDM-FDE and SC-FDE perform well over all delay spreads. The similarity between Figures 9 and 10 should not be surprising as the channel models and modulation and detection techniques can all be related through the principle of time-frequency duality.
Fig. 9. In this plot, the block length is fixed to $N = 1024$, the sample rate is fixed to 4.5 MHz, and the delay spread is fixed to 50 ns. As the Doppler spread increases, OFDM-FDE is no longer able to effectively equalize the channel. Notice that OFDM-FDE becomes ineffective when the duration of the block, here 0.23 ms, is roughly one-tenth of the coherence time of the channel.

Fig. 10. Here, the block length is again fixed to $N = 1024$, the sample rate is fixed to 4.5 MHz. The Doppler spread is fixed to 500 Hz. As the delay spread increases, our time-domain equalization process is no longer effective. Our equalization technique becomes ineffective roughly when the delay spread is close to the symbol period.

D. Waveform Parameter Sweeps

Modulation and detection techniques based on frequency domain equalization, such as OFDM-FDE and SC-FDE, can effectively compensate for arbitrary delay spread by assuming that the channel is time-invariant over each transmission block. The period of the largest Doppler component (approximately the coherence time of the channel) will limit how large $NT_s$ can be while still allowing for effective equalization. Roughly, these techniques will only effectively equalize the channel if the coherence time of the channel is about ten times larger than $NT_s$. In contrast, our modulation and detection techniques, which are based on time-domain equalization, can equalize arbitrary Doppler spread assuming that the delay spread is small in comparison to $T_s$. As a rule of thumb, we claim that our techniques will effectively equalize Doppler spread if the symbol period is roughly twice the delay spread.

Both of these claims are supported by the simulations shown in Figure 11, which simulates 16-QAM symbols over all four modulation and detection techniques discussed in this paper. In this set of simulations, we fix the delay spread to be 50 ns and the Doppler spread to be 500 Hz. The block length is fixed to $N = 1024$ and the sample rate is varied from 450 kHz to 45 MHz.
Fig. 11. Here the block length is fixed to $N = 1024$ and the sampling rate is swept from 450 kHz to 45 MHz. The channel has a delay spread of 100 ns and a Doppler spread of 500 Hz. Our techniques perform well as long as the symbol period is small, or roughly the same order, compared to the delay spread. In contrast, OFDM-FDE and SC-FDE perform well if the block length is much smaller than the coherence time of the channel.

We note that these values extend slightly beyond the range considered by the LTE standard; this is done to emphasize the performance of all techniques at the extremes of high and low sampling rates.

As described previously, one method typically used to adapt OFDM-FDE to channels with a short coherence length is to shorten the transmit block duration ($NT_s$). In Figure 12, we present the effects of changing the block length $N$ while holding the sampling rate constant at 4.5 MHz. These simulations use the same parameters as those simulations presented in Figure 11. As expected, the error floor for OFDM-FDE vanishes for small block lengths, whereas the error floor for FDM-FDCP and SC-TDE is not effected by varying the block length.

V. CONCLUSIONS

Many modulation and detection techniques used in current wireless systems, such as OFDM-FDE or SC-FDE, make use of the assumption of time-invariance, or approximate time-invariance, of the channel in order to efficiently equalize effects of inter-symbol interference. However, in practice, time variations in channel responses may occur at time scales much smaller that
the duration of the transmission block. These time variations may occur in wireless channels associated with mobile transceivers, high carrier frequencies, scheduling/resource allocation time scales, or large noise floors that necessitate time-averaging for noise suppression. As demonstrated in this paper, OFDM-FDE is no longer competitive for such time-varying channels. For such environments, we propose two new modulation and detection techniques, FDM-FDCP and SC-TDE, both of which use a frequency-domain cyclic prefix. This allows us to compensate for the presence of an arbitrary Doppler spread. These techniques can be related to existing constructions for time-invariant channels through the principle of time-frequency duality.

A complete evaluation of the relative benefits of different modulation and detection techniques in wireless channels would depend critically on the joint delay and Doppler spreads in the wireless channel. In this work, we show that that FDM-FDCP and SC-TDE can outperform OFDM-FDE and SC-FDE in situations where there is a significant Doppler spread and a low-to-moderate delay spread. In order to effectively mitigate the effects of Doppler spread, FDM-FDCP and SC-TDE require an overhead in bandwidth that is proportional to the Doppler spread of the channel. As described in this paper, the modulation and detection of these techniques can be implemented with a time complexity of $O(N \log N)$, making them competitive with both OFDM-FDE and SC-FDE in runtime and power consumption.

We do not explore a variety of design parameters such as choice of pulse shape, or channel estimation algorithms. We leave such comparisons as a topic of future work. We note that if CSI is available at the transmitter, it is possible to apply adaptive loading to the SC-TDE. It is an open question to explore the performance of such systems and their relation to the information-theoretic capacity of channels with high-Doppler and low-delay spread.

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**REFERENCES**


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