Energy Efficient Optimal Node-Source Localization using Mobile Beacon in Ad-Hoc Sensor Networks

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***Abstract*—In this paper, a single mobile beacon based method to localize nodes using principle of maximum power reception is proposed. Optimal positioning of the mobile beacon for minimum energy consumption is also discussed. In contrast to existing methods, the node localization is done with prior location of only three nodes. There is no need of synchronization, as there is only one mobile anchor and each node communicates only with the anchor node. Also, this method is not constrained by a fixed sensor geometry. The localization is done in a distributed fashion, at each sensor node. Experiments on node-source localization are conducted by deploying sensors in an ad-hoc manner in both outdoor and indoor environments. Localization results obtained herein indicate a reasonable performance improvement when compared to conventional methods.**

I. Introduction

The problem of localization is important in wireless sensor networks (WSN) applications like military surveillance, vehicular technology, agriculture monitoring. Various methods based on received signal strengths (RSS) [1] and other measurements like Direction of Arrival (DOA) [2], Time of Arrival (TOA) [3], Time Difference of Arrival (TDOA) [4], Angle of Arrival (AOA) [5] can be used for the localization. A distributed approach of localization has been shown with the help of simulations in [6]. In [7], the trajectory followed by the mobile anchor may not be energy-efficient. A guiding mechanism to minimize energy consumption for the mobile anchor is proposed in [8] . Although [8] uses only one anchor but we show that the energy consumption for this anchor is significantly higher. Energy efficient clustering algorithms are described in [9], [10]. In [11], a range-free energy efficient algorithm is presented and [12] describes a distributed target localization method in WSN. In this paper the terms beacon and anchor node are used synonymously. Source assumed in this paper refers to the mobile beacon at particular instant.

In this paper, maximum power reception based node-source localization is presented which initially requires location of three sensor nodes. The mobile anchor traverses and is positioned to an optimal location for further localization. The algorithm is called energy efficient because after the location of initial set of three nodes is calculated, the GPS of these nodes is switched off, as enabling GPS is energy consuming. This method performs optimal node-source localization at every instant, since the beacon moves incrementally to a position that requires minimal energy to communicate with its neighbouring nodes. Also, it does not require synchronization since each node computes its position using information transmitted from the anchor and does not require any information from the neighbouring nodes.

The following notations are used in this paper. Matrices or one dimensional column vectors are shown in bold faced letter (upper or lower case). $(.)^T$ and $(.)^*$ represent the transpose and optimal value respectively. $||.||$ is a 2-norm and used for showing the Euclidean distance between two one dimensional vectors. ζ_i represents the Gaussian noise. 2-space is denoted by $\mathbb{R}^2$.

The paper is organized as follows. Section II describes the energy efficient optimal node-source localization. In Section III, experimental performance evaluation is presented. A brief conclusion is presented in Section IV.

II. Energy Efficient Optimal Node-Source Localization

In this Section, the basic framework of node source localization is discussed first. The proposed node-source localization method using maximum power reception principle is then discussed. Optimal placement of the mobile anchor for incremental node-source localization is also described. An algorithmic description for the same is provided and finally significance of constrained multilateration on node localization is also illustrated.

A. Basic Framework for Node-Source Localization

The sensor network is modelled as an undirected graph, $G = (\chi, E, P)$, where χ and E are the set of sensor nodes and edges between each of pair of nodes respectively. $p_{i,j} \in P$ is the signal strength between anchor i and sensor nodes j. Also, $p_{i,j} = p_{j,i}$ is considered i.e., channel is invariant in both the forward and reverse directions.

Let $\chi + \{N+1\} = \{1, 2, \cdots, N, N+1\}$ is the set of nodes, out of which $\{1, 2, \cdots, N-3\}$ are the unknown nodes. $\{N-2, \cdots, N+1\}$ represents the three nodes whose locations are initially known and one anchor node . The problem of localization is to find $\upsilon = (x_i, y_i)\ \forall\ i = 1, 2 \cdots, N-3$ with the help of priori known coordinates $(x_{N-2}, y_{N-2}), \cdots, (x_N, y_N)$, (x^m, y^m) and $p_{i,j}$. Also, $m_{t_i} = (x^m_{t_i}, y^m_{t_i})$ and $m_t = (x^m_t, y^m_t)$ is used to denotes location of anchor at time t_i and temporary position of anchor. In the aforementioned framework, the distance between the anchor and the nodes needs to be computed and Friis transmission model [1] is used in this context.

In free space, Euclidean distance between the mobile anchor & node is calculated by Friis transmission model [1]

$$P_t - P_r + 10\log(G_tG_r) + 10n_e\log(\frac{\lambda}{4\pi}) = 10n_e\log(K) \quad (1)$$

where, P_r and P_t represents the received power (in dB) at sensor node and transmitted power of the mobile anchor respectively. The transmission and reception antenna gain are represented by G_t and G_r respectively. n_e is the path loss exponent. K is the distance between the mobile anchor node and unknown sensor node and λ is the carrier wavelength.

B. Node-Source Localization based on Maximum Power Reception Principle

The anchor node is moved in the direction of approximate maximum power, to find the most densely clustered nodes locally. Otherwise, the anchor could keep on traversing unnecessarily in sparsely deployed regions and hence the path might not be optimal and can consume more energy.

The method works with discrete time instants e.g., $t_1, t_2, \cdots, t_M$. Initially three nodes (S_a, S_b, S_c) as shown in

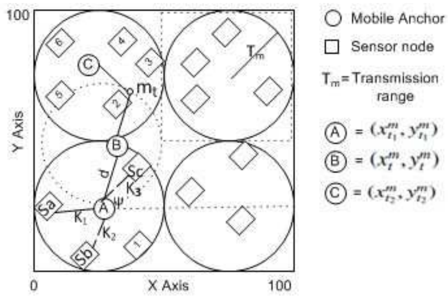

Fig. 1. Figure illustrating the placement of nodes and movement of anchor

Fig. 1 whose coordinates are $\{(x_{N-2}, y_{N-2}), \cdots, (x_N, y_N)\}$ are located using GPS or location aware devices. Subsequently, the location aware devices are disable to save energy.

The deployment is shown in Fig. 1. Note that square regions marked in Fig. 1 are considered for the purpose of localization. Additionally, the mobile anchor always traverses in straight line and assumes a direct line-of-sight communication between sender and receiver.

A mobile anchor is dropped at $(x_{t_1}^m, y_{t_1}^m)$ in the area of interest (AOI) of the network. This mobile anchor $(x_{t_1}^m, y_{t_1}^m)$ will transmit signals to the nodes (S_a, S_b, S_c) within its transmission region. The Euclidean distance K_1, K_2 and K_3 between $(x_{t_1}^m, y_{t_1}^m)$ and these nodes (S_a, S_b, S_c) is estimated by Equation 1. There maybe some error between the estimated distance and the actual distance as described by:

$$K_i = \|\mathbf{X}_i - \mathbf{X}_t^m\| + \zeta_i, \qquad \forall i = 1, 2, \cdots, N \quad (2)$$

where, $\mathbf{X}_i = \begin{bmatrix} x_i & y_i \end{bmatrix}^T$, $\mathbf{X}_t^m = \begin{bmatrix} x_t^m & y_t^m \end{bmatrix}^T$, ζ_i accumulates all the noise and interference existing in the environment.

$(x_{t_1}^m, y_{t_1}^m)$ is calculated by $(A^TA)^{-1}A^TB$ using $(x_{N-2}, y_{N-2}), \cdots, (x_N, y_N)$ and K_1, K_2 and K_3, where A and B are as defined in subsequent Section. The mobile anchor communicates to the remaining nodes (node 1) located in $(T_m)_{t_1}$ through the radio message of their location. These unknown nodes (node 1) also calculate the Euclidean distance to the mobile anchor. So, remaining nodes (node 1) located in $(T_m)_{t_1}$ are localized with help of location of the mobile anchor using the constrained optimization approach, which is described in subsequent Section. Hence, we follow a distributed approach for localization.

1) Initial Node-Source Localization under Centroid Ambiguity: When more than two hyperbolic curves intersect, the following approach is used to resolve the centroid [1] ambiguity. In general, Euclidean distance relation between known sensor node (x_i, y_i) and unknown node (x, y) can be expressed as a constrained optimization problem.[1]

$$\begin{aligned} &\text{Minimize } \|\mathbf{AZ} - \mathbf{B}\|^2 \\ &\text{Subject to} \\ &\qquad x_{t_i}^m - T_m \le x \le x_{t_i}^m + T_m \\ &\qquad y_{t_i}^m - T_m \le y \le y_{t_i}^m + T_m \end{aligned} \quad (3)$$

where, $x_{t_i}^m$, $y_{t_i}^m$, T_m is the azimuth, elevation at time t_i and the transmission range of mobile anchor respectively. Hereafter, we denote $a = x_{t_i}^m - T_m$, $b = x_{t_i}^m + T_m$, $c = y_{t_i}^m - T_m$ and $d = y_{t_i}^m + T_m$. Also,

$$\mathbf{A} = 2\begin{bmatrix} x_1 - x_n & y_1 - y_n \\ \vdots & \vdots \\ x_{(n-1)} - x_n & y_{(n-1)} - y_n \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x_1^2 - x_n^2 + y_1^2 - y_n^2 + Z_1 \\ \vdots \\ x_{(n-1)}^2 - x_n^2 + y_{(n-1)}^2 - y_n^2 + Z_{(n-1)} \end{bmatrix}$$

In matrix $\mathbf{B}$, $Z_i = K_n^2 - K_i^2$, for i=1,$\cdots$,(n-1) . $K_1, K_2, \cdots, K_n$ can be found out by Equation 1. A and B is calculated by the proposed algorithm.

C. Optimal Placement of the Mobile Anchor for Incremental Node-Source Localization

As described in the previous Section, initial location of the mobile anchor is used to determine the location of the neighbouring nodes. However, to localize the rest of the nodes covered in AOI, an optimal location of mobile anchor needs to be determined in the subsequent node-source localization process. The proposed method to determine such an optimal location requires the movement of mobile anchor to an intermediate position. Prior to determining optimal placement of the mobile anchor, two steps are required which are described herein.

[1]The solution is derived in the appendix.

1) Determining the Intermediate Location of the Mobile Anchor: The measurement of power in different directions is done at unknown nodes $(x^m_{t_1}, y^m_{t_1})$. Based on the magnitude of maximum power, the direction of movement of anchor, ψ, is chosen. The measurement of this angle ψ is done by drawing a line parallel to x axis. Now the anchor traverses to location (x^m_t, y^m_t) described by Equation 6 in the direction of approximate maximum power with certain angle ψ following a straight path. When traversing from $(x^m_{t_1}, y^m_{t_1})$to (x^m_t, y^m_t), the anchor will communicate to unknown nodes at-least three times in the vicinity of $(T_m)_{t_1}$.

$$||\mathbf{X}^m_{t_i} - \mathbf{Y}^m_t|| = (t_i - t_{i-1})V_{t_i,t_{i-1}} = d, \qquad \forall i, j = 1, 2, \cdots, M \quad (4)$$

$$\psi = \arctan(\frac{y_i - y_j}{x_i - x_j}), \qquad \forall i, j = 1, 2, \cdots, M \quad (5)$$

where, $\mathbf{X}^m_{t_i} = \begin{bmatrix} x^m_{t_i} & y^m_{t_i} \end{bmatrix}^T$, $\mathbf{Y}^m_t = \begin{bmatrix} x^m_t & y^m_t \end{bmatrix}^T$ and $V_{t_i,t_{i-1}}$ is the velocity of the mobile anchor between time t_i and t_{i-1}.

Let the solution of Equation 4 and 5 be represented by (x_p, y_p). In general,

$$\begin{aligned} x^m_t &= x^m_{t_i} + x_p \cos(\psi) \\ y^m_t &= y^m_{t_i} + y_p \sin(\psi) \end{aligned} \quad (6)$$

We get the position (x^m_t, y^m_t) using Equation 6.

2) Determining the Optimal Location of the Mobile Anchor: Optimal positioning of anchor is described in this Section such that it consumes minimal energy while communicating to the nodes at a particular time instant.

The anchor traverses in the direction of approximate maximum power received (cluster of nodes 2-4), following a straight path from $(x^m_{t_1}, y^m_{t_1})$ to m_t and arrives at temporary position (x^m_t, y^m_t). If the anchor is at m_t, surrounding few nodes 2-4 receive maximum power, but placing the anchor at optimal position $(x^m_{t_2}, y^m_{t_2})$ around nodes 2-6, which lie in the transmission range minimizes energy of the anchor, while communicating to the nodes. $(x^m_{t_2}, y^m_{t_2})$ is the optimal position which is to be calculated in this Section. A mobile anchor is being positioned in each time unit such that it consumes minimum energy while communicating to the sensor nodes. (x^m_t, y^m_t) is used as a initial point to iteratively solve for the optimal position [9] $(x^m_{t_i}, y^m_{t_i})$ of anchor.

$$\begin{aligned} &\text{Minimize } \sum_{k=1}^{N_{t_k}} (||\mathbf{X} - \mathbf{X}^m_{t_i}||) E_k \\ &\text{Subject to} \\ &\qquad ||\mathbf{X}^m_{t_i} - \mathbf{X}^m_{t_{i-1}}|| \geq T_m \\ &\qquad E_{j\in(H(m))} = \sum_{k=1}^{N_{t_k}} E_{kj} \\ &\qquad E_{kj} \geq 0 \end{aligned} \quad (7)$$

where, $\mathbf{X} = \begin{bmatrix} x_k & y_k \end{bmatrix}^\top$ and $\mathbf{X}^m_{t_i} = \begin{bmatrix} x^m_{t_i} & y^m_{t_i} \end{bmatrix}^\top$ is the position of sensor node and mobile anchor respectively. $H(m)$ is the set of total number of virtual anchor locations. E_k denotes energy transmitted by the anchor per unit of distance while communicating to the sensor nodes. N_{t_k} is used for the set of nodes within the communication range of anchor, m at time t_k excluding the nodes which have been localized previously.

At $(x^m_{t_2}, y^m_{t_2})$, the anchor node communicates and sends radio message to node (2-6) in its transmission range T_m and subsequently the distance is calculated with Equation 1. $(x^m_{t_2}, y^m_{t_2})$ and estimated location of nodes are used for further localization of remaining nodes. This process is repeated till all the nodes in the network are localized.

D. Algorithm for Optimal Node-Source Localization

The algorithmic description of the proposed method is listed in Algorithm 1.

Algorithm 1 Optimal node-source localization

1: **Initialization** : Take $G = (\chi, E, P)$ and $\chi_{kn} = \{\chi_k, \chi_{k+1}, \chi_{k+2}\}$, where, $\chi_{k+i} = (x_{k+i}, y_{k+i})$ for $i = 0, 1, 2$ is assumed known.
2: **Initial reduction of set of nodes** : $\chi' = \chi$ - χ_{kn}.
3: **Calculate initial position of anchor** : Find $m_{t_1} = (x^m_{t_1}, y^m_{t_1})$ using Equation 3.
4: **Identification of remaining unlocalized nodes** : Find all $\chi_i \in (T_m)_{t_1}$,transmission range of mobile anchor m at time t_1. If $\chi = \{\phi\}$ go to step 9.
5: **Calculate the Euclidean distance** : Find the Euclidean distance $d(m_{t_1}, \chi_i)$ between mobile anchor m_{t_1} and χ_i at time t_1 $\forall$ χ_i in step 4 using Equation 1.
6: **Apply the constrained multilateration method** : Apply Equation 3 using at-least 3 mobile anchor location $\forall$ $\chi_i \in (T_m)_{t_1}$ using χ_{kn} and $d(m_{t_1}, \chi_i)$ in step 5.
7: **Reduction of set of nodes** : $\chi'' = \chi'$ - $\{\forall$ $\chi_i \in (T_m)_{t_1}\}$.
8: **Traverse in direction of approximate maximum power** : Traverse a distance $d = V * (t_2 - t_1)$ at an angle ψ to m at time t_2.
9: **Find a temporary or intermediate position,** (x^m_t, y^m_t) **of mobile anchor** : Use Equation 6 to calculate this.
10: **Find new position,** $(x^m_{t_2}, y^m_{t_2})$ **of mobile anchor** : Use Equation 7 to calculate the new position.
11: **Calculate the Euclidean distance** : Again, find $d(m_{t_2}, \chi_j)$ such that $\chi_j \in \{(T_m)_{t_2}\}$ - $\{$*already estimated* χ_i *in step* 6$\}$
12: **Apply the constrained multilateration approach** : Estimate position of $\forall$ χ_j in step 11.
13: **Condition check** : Is $\chi = \{\phi\}$. If no, go to step 7.
14: **Termination** : Output υ.

E. Significance of Constrained Multilateration on Node Localization

As it can be seen from Algorithm 1 that a node requires atleast 3 mobile anchor location to localize itself. More than 3 location transmission from mobile anchor can also be used to effectively localize the node using the multilateration technique. As more number of transmission locations are used to localize the node, the location of node can be more accurate. In order to illustrate this, average localization error versus transmission from mobile anchor in Fig. 2 is investigated. This method is also compared with other methods [6], [7].

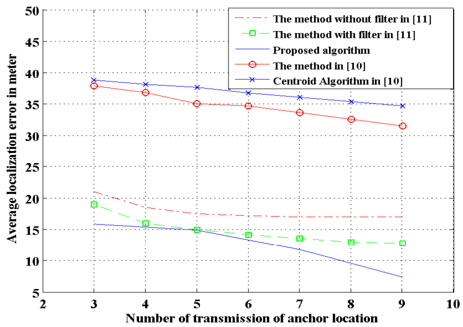

Fig. 2. Comparison of average localization error versus number of virtual anchor locations

It can be noted that the proposed algorithm performs reasonably better than other methods. Centroid algorithm [6] has highest average location error of around 37m for 5 anchor nodes. Results from [6] and [7] are reproduced in the Fig. 2. [6] has an average error of 35m for number of anchors varying between 3 and 9. With 20 anchors, [6] shows an average error of 21.6m while the Centroid algorithm used in [6] shows an error of 28m. It is also illustrated that the proposed method has lower average localization error with respect to [7] with and without filter.

III. Performance Evaluation

In this Section, localization error analysis is presented. Subsequently CRLB analysis is illustrated, and finally, experiments on node-source localization in a real deployment scenario are also conducted.

A. Localization Error Analysis

Localization error analysis using average error deviation (AED) is presented.

Fig. 3 shows the average error deviation plots of obtained location for four different methods. It can be seen from Fig. 3 that proposed algorithm shows lesser standard deviation of error compared with the work in [6] and [7]. Additionally,

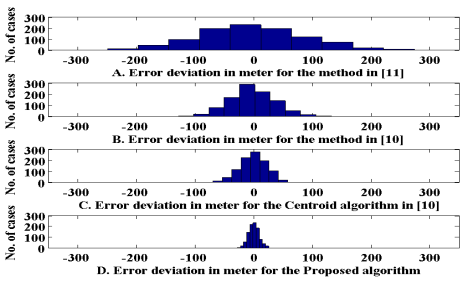

Fig. 3. Symmetrized average radial error deviation of four different methods

localization error is calculated for independent experiments conducted at different times. Fig. 4 shows the results obtained by performing the experiment 35 times and correspondingly plotting the instantaneous localization error. We can see from

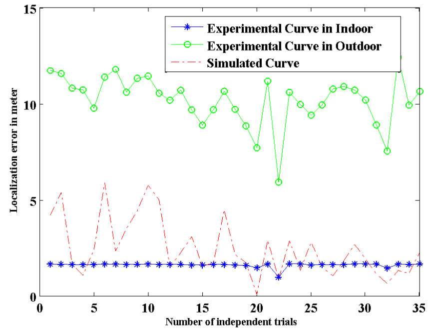

Fig. 4. Localization error versus number of independent trials

Fig. 4 that average localization error for outdoor environment is around 12m. For indoor average error is 2m and it remains consistent.

B. Cramer-Rao Lower Bound Analysis

The lower bound on error variance is analyzed using the Cramer-Rao lower bound estimators [13].

$$\sigma_k^2 \geq \frac{F_{Rxx} + F_{Ryy}}{F_{Rxx}F_{Ryy} - F_{Rxy}^2} \tag{8}$$

It can also be represented as:

$$\sigma_k^2 = \frac{1}{b} \frac{\sum_{i \in H(m)} d_{k,i}^{-2}}{\sum_{i \in H'(m)} \sum_{j=i+1}^{M} (\frac{d_{k \perp i,j} d_{i,j}}{d_{k,i}^2 d_{k,j}^2})^2} \tag{9}$$

where, F_{Rxx}, F_{Rxy}, F_{Ryy}, b, $d_{k \perp i,j}$ and $d_{i,j}$ are as described in [13]. $H(m)$ is the set of M nodes whose location is already estimated and $H'(m)$ represents the set of first $M-1$ localized nodes. Similarly, for each of the sensor node CRLB can be written as in Equation 8 & 9. Fig. 5 shows the CRLB analysis result on actual data. It can be noted from Fig. 5 that minimum variance achieved is 7.4m which means that error fluctuation is less.

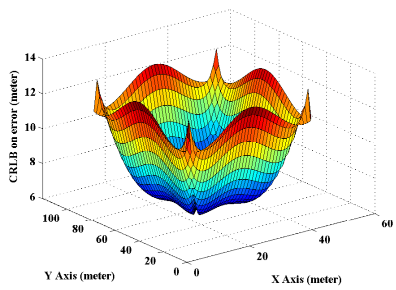

Fig. 5. CRLB plot on simulated data

C. Energy Efficiency of the Proposed Algorithm

In order to illustrate the energy efficiency of the proposed algorithm, total energy consumption of various algorithms are computed and shown in Table I. The proposed algorithm performs better than the $BB-AGM$ and $DB-AGM$ [8] in terms of total energy consumption. An energy of $0.02W$ is used for transmitting the beacon, while $1W$ energy is consumed by the anchor for a unit distance movement [8].

TABLE I
TOTAL ENERGY CONSUMED BY AN ANCHOR USING VARIOUS ALGORITHMS

Algorithms	**Total Energy (KW)**
Proposed Algorithm	143.5
DB-AGM [8]	282.5
BB-AGM [8]	394.4

D. Experiment on Node-Source Localization

In this Section, the performance of the proposed algorithm is evaluated by randomly deploying sensor nodes in a real deployed scenario.

1) Experiment on Localization in Outdoor Environment: The performance of proposed algorithm in an experimental deployment is shown in Fig. 6. We use National Instruments (NI) WSN- 3202, 3212 and gateway NI 9792. The nodes communicate through IEEE 802.15.4 protocol and have an outdoor range of 300m.

Fig. 6. Experimental deployment showing mobile anchor node, static sensor nodes, routers and gateway

The experiment is conducted during daytime on IIT Kanpur field of size $100m \times 100m$ in pre-winter. One node acts as a mobile anchor that is moved manually and other nodes are kept static in a random fashion. At each instant, static nodes continuously measure the link quality. Equation 1 is not a reliable estimate for location estimation from the link quality. This is because link quality does not depend only upon the power received. It actually represents the confidence with which the packets are received. Hence experiments were conducted to establish the dependency of the link quality on distance which varies with the wireless channel. If experimental conditions remain approximately same, the results are consistent.

Fig. 7, represents the CRLB plot on experimental data for the outdoor range. Experimental data is obtained by conducting sensor deployment on the field. Minimum variance of the localization error is $0.223m$ of a $1m \times 1m$, which is comparable to [13]. For $100m \times 100m$, it shows reasonably improved result.

We get Equation 10, similar to Equation 1 using curve fitting technique, which maps link quality to distance. The third and fourth term of Equation 1 is combined into a constant 102.9 in Equation 10. Non-linear least square method is opted here as it gives minimum error (RMSE= 8.106). Several trials of the experiment are conducted at different times to ensure reliable estimate of distance.

$$(L.Q.)_t - (L.Q.)_r + 102.9 = 37.45\, log(K) \qquad (10)$$

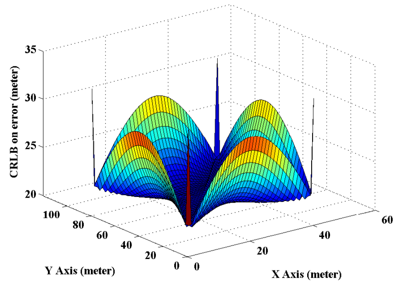

Fig. 7. CRLB plot on experimental data for outdoor

where, $(L.Q.)_t$ and $(L.Q.)_r$ are the transmitted and received link quality, and K is the distance between mobile anchor and sensor node in *meter*.

2) Experiment on Localization in Indoor Environment: The experiment is conducted indoor to evaluate the performance of proposed algorithm. Crossbow MTS310 sensor board,

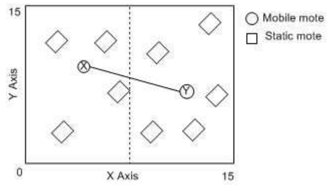

Fig. 8. Figure illustrating the placement of motes and movement of mobile mote in indoor

XM2110 IRIS board and MIB520 USB mote interface board acting as a gateway is deployed indoor of area $15m \times 15m$ as shown in Fig 8. The motes communicate through IEEE 802.15.4 protocol. One of the motes is acting as a mobile

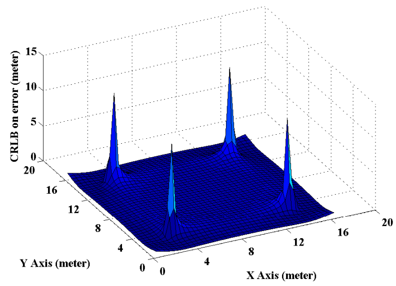

Fig. 9. CRLB plot on experimental data for indoor

anchor and other motes are kept randomly in the area of interest. Several trials of experiment is conducted to map a received signal strength (RSS) to a distance and establish the relation 11. Equation 11 is similar to Equation 1, where third and fourth term of Equation 1 is combined into a constant -49.74.

$$P_t - P_r - 49.74 = 37.45\, log(K) \qquad (11)$$

where, P_t and P_r are transmitted and received power respectively in dBm. K is the distance between mobile anchor and motes in *meter*. The non-linear least square method to map RSS into a distance is opted because it gives minimum error (RMSE= 2.593).

Fig. 9 shows the CRLB plot on experimental data collected through motes deployed indoor. The minimum variance obtained through this plot is very small which is consistent with the localization error plot as shown in Fig. 4.

IV. Conclusion

In this paper, an energy efficient method of node-source localization over randomly distributed wireless sensor nodes is proposed. The algorithm performs sensor node localization with the prior knowledge of location of only three nodes. The proposed method does not require synchronization since each node computes its position using information transmitted from the anchor and does not require any information from the neighbouring nodes. The additional novelty of the proposed method lies in the movement of the anchor in the direction of approximate maximum power. Additionally, the mobile anchor is placed such that minimal energy is required to communicate with the neighbouring nodes. The experimental results on an ad-hoc wireless sensor network deployment on the field are used to validate the high degree of reliability of sensor node localization. New methods that can use a mobile anchor that traverses a network where sensor nodes are also mobile is currently being explored on a real experimental set-up.

Acknowledgment

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References

[1] X. Shen, Z. Wang, P. Jiang, R. Lin, and Y. Sun, "Connectivity and rssi based localization scheme for wireless sensor networks," *Advances in Intelligent Computing*, pp. 578–587, 2005.

[2] K. Kucuk, A. Kavak, H. Yigit, and C. Ozdemir, "A novel localization technique for wireless sensor networks using adaptive antenna arrays," in *Radio and Wireless Symposium, IEEE*, Jan. 2008, pp. 483–486.

[3] Y.-T. Chan, H. Yau Chin Hang, and P.-C. Ching, "Exact and approximate maximum likelihood localization algorithms," *Vehicular Technology, IEEE Transactions on*, vol. 55, no. 1, pp. 10–16, Jan. 2006.

[4] K. Ho and W. Xu, "An accurate algebraic solution for moving source location using tdoa and fdoa measurements," *Signal Processing, IEEE Transactions on*, vol. 52, no. 9, pp. 2453–2463, 2004.

[5] R. Schmidt, "Multiple emitter location and signal parameter estimation," *Antennas and Propagation, IEEE Transactions on*, vol. 34, no. 3, pp. 276–280, 1986.

[6] H. Chen, P. Huang, M. Martins, H. C. So, and K. Sezaki, "Novel centroid localization algorithm for three-dimensional wireless sensor networks," *4th International Conference on Wireless Communications, Networking and Mobile Computing,WiCOM'08.*, pp. 1–4, 2008.

[7] J. Park, S.-m. Yoo, and C.-S. Pyo, "Localization using mobile anchor trajectory in wireless sensor networks," in *International Conference on Indoor Positioning and Indoor Navigation (IPIN) Guimaraes, Portugal*, Jan. 21-23 Sep. 2011, pp. 1–4.

[8] C.-T. Chang, C.-Y. Chang, and C.-Y. Lin, "Anchor-guiding mechanism for beacon-assisted localization in wireless sensor networks," *Sensors Journal, IEEE*, vol. 12, no. 5, pp. 1098–1111, 2012.

[9] W. Li, "Energy efficient clustering algorithm in wireless sensor networks based on geometric programming," *Electronic Commerce and Security, ISECS'09, Second International Symposium on*, vol. 2, pp. 525–527, 2009.

[10] S. Bandyopadhyay and E. J. Coyle, "An energy efficient hierarchical clustering algorithm for wireless sensor networks," in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications. IEEE Societies*, vol. 3. IEEE, 2003, pp. 1713–1723.

[11] S. Zhang, J. Cao, C. Li-Jun, and D. Chen, "Accurate and energy-efficient range-free localization for mobile sensor networks," *Mobile Computing, IEEE Transactions on*, vol. 9, no. 6, pp. 897–910, 2010.

[12] J. Lee, K. Cho, S. Lee, T. Kwon, and Y. Choi, "Distributed and energy-efficient target localization and tracking in wireless sensor networks," *Computer Communications*, vol. 29, no. 13, pp. 2494–2505, 2006.

[13] N. Patwari, A. O. Hero III, M. Perkins, N. S. Correal, and R. J. O'dea, "Relative location estimation in wireless sensor networks," *Signal Processing, IEEE Transactions on*, vol. 51, no. 8, pp. 2137–2148, 2003.

[14] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

Appendix

A. Solution to the constrained optimization problem in Equation 3

Let $\mathbf{Z}^* = [x^* \;\; y^*]^T$ and λ^* are the solution of primal and dual problem with strong duality holds. For this problem Karush-Kuhn-Tucker (KKT) condition [14] is satisfied. Taking the gradient of the Lagrange function and setting it equal to zero-

$$L(\mathbf{Z}, \lambda^*, \mu^*) = (\mathbf{Z}^*)^T \mathbf{A}^T \mathbf{A}\mathbf{Z}^* - 2(\mathbf{Z}^*)^T \mathbf{A}^T \mathbf{B} + \mathbf{B}^T \mathbf{B} +$$

$$\lambda^*_{u_1}(x_k - u_{ubound_1}) + \lambda^*_{l_1}(u_{lbound_1} - x_k) + \lambda^*_{u_2}(y_k - u_{ubound_2}) +$$

$$\lambda^*_{l_2}(u_{ubound_2} - y_k)$$

For the solution of x, $\left.\frac{\partial L(\mathbf{Z},\lambda^*,\mu^*)}{\partial x_i}\right|_{x_i = x_i^*} = 0$ gives

$$2x_i^* \sum_{j=1}^{n-1} \alpha_j^2 + 2y_i^* \sum_{j=1}^{n-1} \alpha_j \beta_j - 2\sum_{j=1}^{n-1} \alpha_j \gamma_j + \lambda_{u_1} - \lambda_{l_1} = 0$$

Case A: $\lambda_{u_1} = 0$, $\lambda_{l_1} > 0$ and $x_i^* = a$
$\Rightarrow 2a(\sum_{j=1}^{n-1} \alpha_j^2) + 2y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) - 2(\sum_{j=1}^{n-1} \alpha_j\gamma_j) - \lambda_{l_1} = 0$
$\Rightarrow -y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) \leq a(\sum_{j=1}^{n-1} \alpha_j^2) - \sum_{j=1}^{n-1} \alpha_j\gamma_j$

Case B: $\lambda_{l_1} = 0$, $\lambda_{u_1} > 0$ and $x_i^* = b$
$\Rightarrow 2b(\sum_{j=1}^{n-1} \alpha_j^2) + 2y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) - 2(\sum_{j=1}^{n-1} \alpha_j\gamma_j) - \lambda_{u_1} = 0$
$\Rightarrow y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) \leq -b(\sum_{j=1}^{n-1} \alpha_j^2) + \sum_{j=1}^{n-1} \alpha_j\gamma_j$

Case C: $\lambda_{u_1} = 0$, $\lambda_{l_1} = 0$ and $a \leq x_i^* \leq b$
$x_i^* = \frac{\sum_{j=1}^{n-1} \alpha_j\gamma_j - y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j)}{\sum_{j=1}^{n-1} \alpha_j^2}$

A solution to y can be proposed on similar lines and the optimal solution is :

$$x_i^* = \begin{cases} a, & \text{if } -y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) \leq a(\sum_{j=1}^{n-1} \alpha_j^2) - \sum_{j=1}^{n-1} \alpha_j\gamma_j \\ b, & \text{if } y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) \leq -b(\sum_{j=1}^{n-1} \alpha_j^2) + \sum_{j=1}^{n-1} \alpha_j\gamma_j \\ \frac{\sum_{j=1}^{n-1} \alpha_j\gamma_j - y_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j)}{\sum_{j=1}^{n-1} \alpha_j^2}, & \text{otherwise} \end{cases}$$

$$y_i^* = \begin{cases} c, & \text{if } -x_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) \leq c(\sum_{j=1}^{n-1} \beta_j^2) - \sum_{j=1}^{n-1} \beta_j\gamma_j \\ d, & \text{if } x_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j) \leq -d(\sum_{j=1}^{n-1} \beta_j^2) + \sum_{j=1}^{n-1} \beta_j\gamma_j \\ \frac{\sum_{j=1}^{n-1} \beta_j\gamma_j - x_i^*(\sum_{j=1}^{n-1} \alpha_j\beta_j)}{\sum_{j=1}^{n-1} \beta_j^2}, & \text{otherwise} \end{cases}$$

Where, $\alpha_i = x_i - x_n$, $\beta_i = y_i - y_n$, and $\gamma_i = x_i^2 - x_n^2 + y_i^2 - y_n^2 + Z_i$ $\forall i = 1, \ldots n-1$.