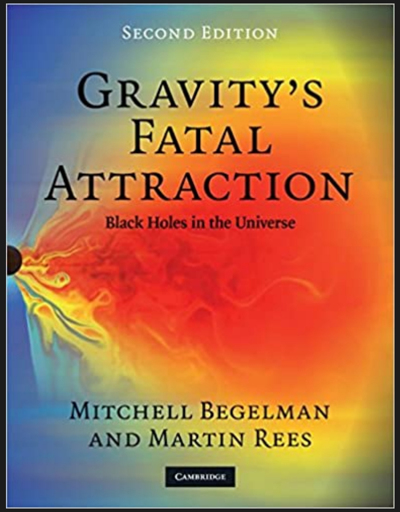
6: Introducing the black hole

Physics 17: Black Holes and Extreme Astrophysics

Goals

- Use Newtonian physics to discover the concept of black holes
- Find out what happens to objects and light close to black holes

Reading



Begelman & Rees

- Chapter 1: Introducing the black hole (p2-9, 13-18)

The story so far…

- Gravity is the mutual attraction between all massive objects in the Universe. $F = \frac{Gm_1 m_2}{r^2}$
- Total energy is always conserved (so is momentum, angular momentum)
- Light travels at a finite speed,
- Nothing (no information) can travel faster than the speed of light. Only massless particles can travel at the speed of light
- Space and time are unified into space time. Time dilation and length contraction for moving objects
- Energy and mass are equivalent

Q1:

We launch a rocket from the Earth straight upwards at a speed of 5km s^{-1}. What is its total energy?

How high does the rocket get before gravity starts to bring it back down?

Q2:

If we wanted to send the rocket to the outer parts of the Solar System, how fast would we need to launch it from the Earth for it to be able to escape the Earth's gravitational pull?

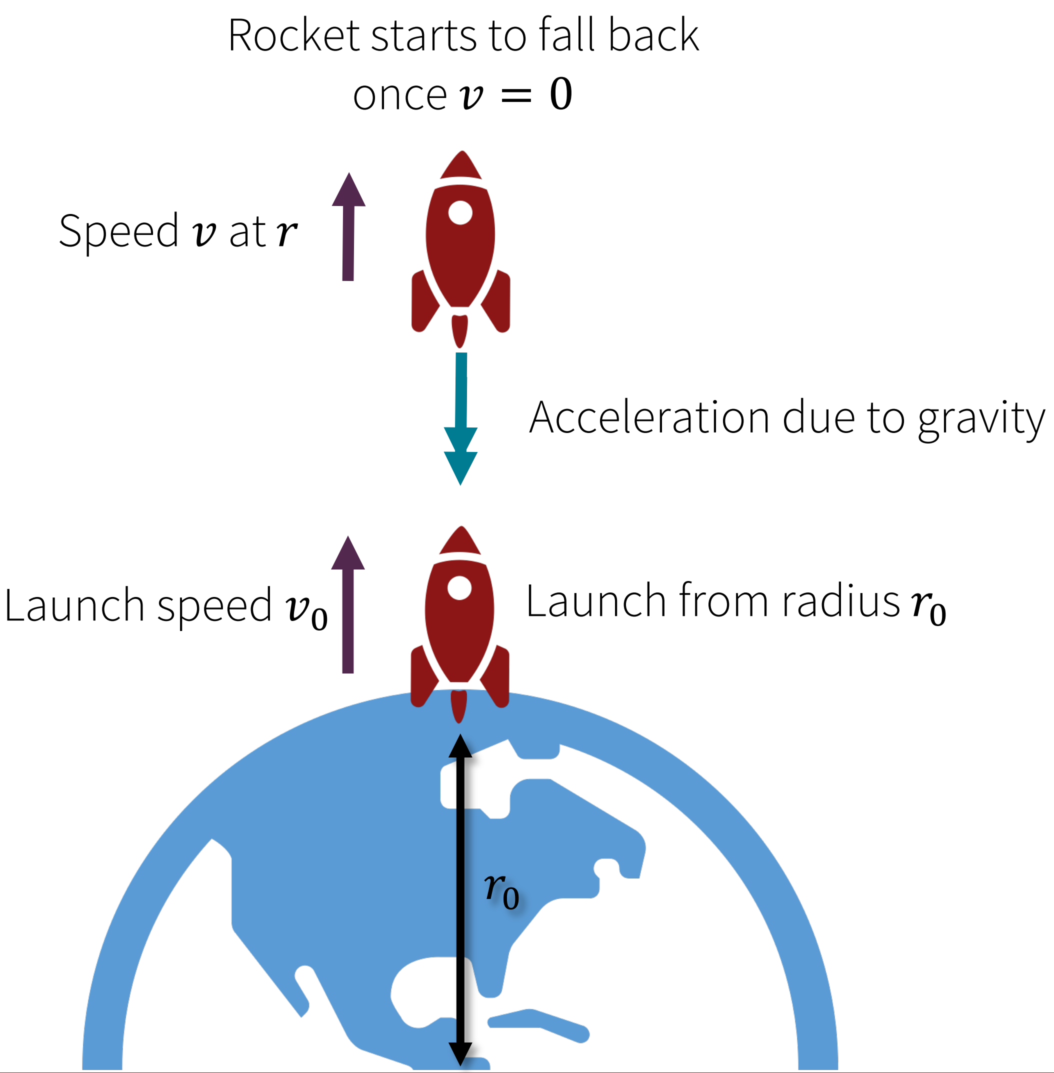
Q3:

Now suppose instead of launching a rocket from the Earth, we are launching our spaceship from the surface of the collapsed remnants of a star. The star has a similar mass to the Sun but is only 1km in radius.

How fast would we need to launch the spaceship from the surface in order to escape its gravity and get home?

What does this mean? How would this star appear from the outside?

Total energy is conserved



- The total energy of the rocket is the sum of its kinetic and gravitational potential energy

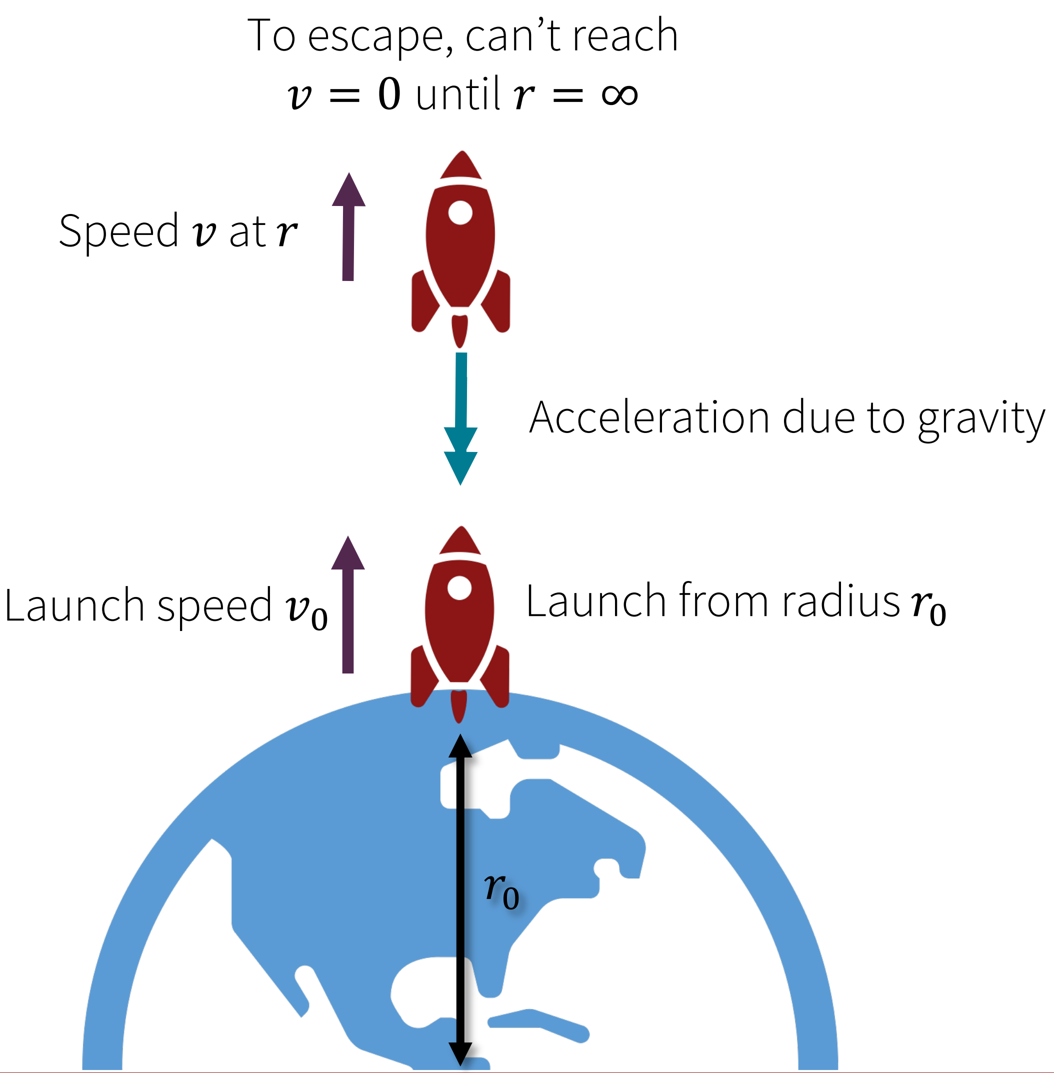
$$E = T + V$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

- At every point in its flight, the total energy, E, must be the same. If we launch at speed v_0 from the surface at radius

$$\frac{1}{2}v^2 - \frac{GM}{r} = \frac{1}{2}v_0^2 - \frac{GM}{r_0}$$

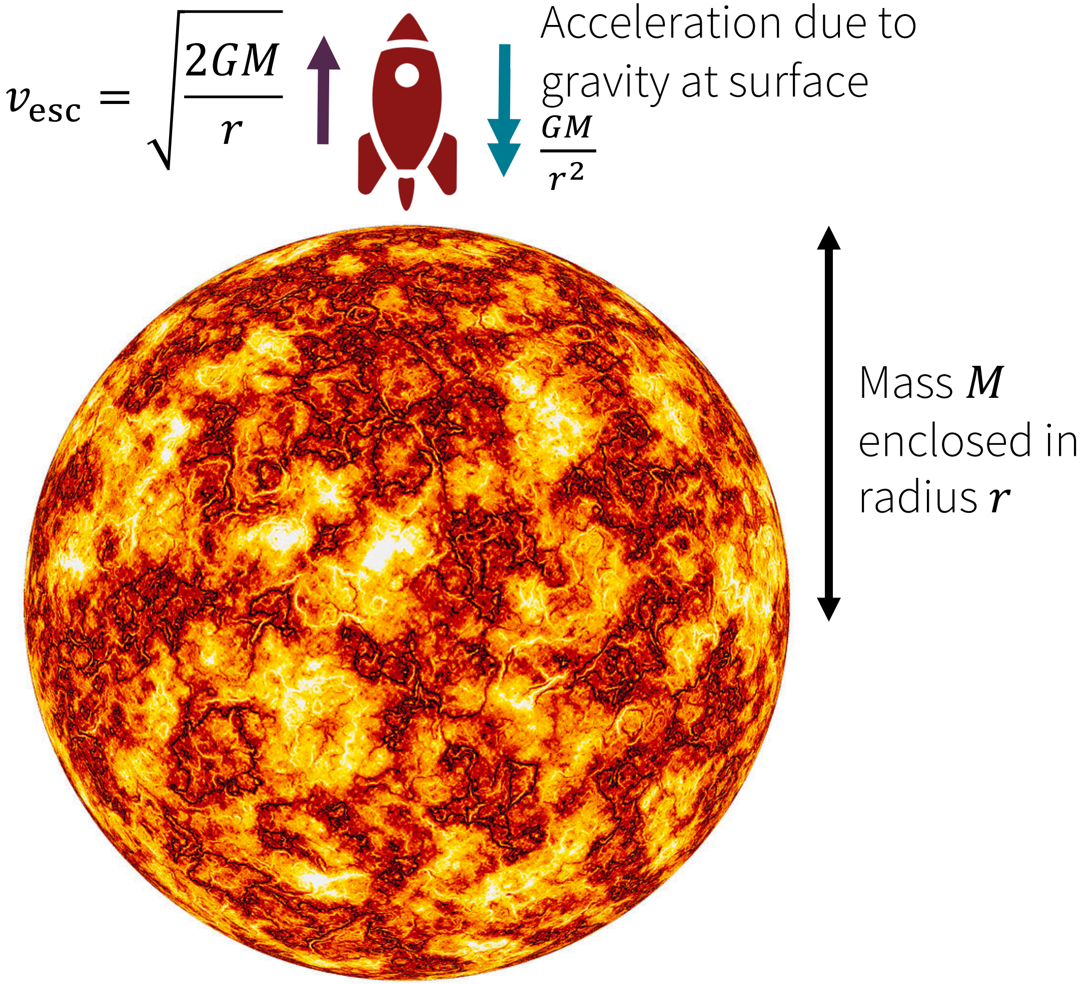
Escape Velocity



- If v reaches 0 at any point during the flight, the rocket will stop, then gravity will bring it back down
- In order to escape the Earth, we need v to not reach 0 until the rocket has travelled an infinite distance (as $r \to \infty, \frac{GM}{r} \to 0$), so we need total energy E = 0
- So to escape, we need to launch the rocket at

$$v_0 > \sqrt{\frac{2GM}{r_0}}$$ (the escape velocity)

Dark Stars



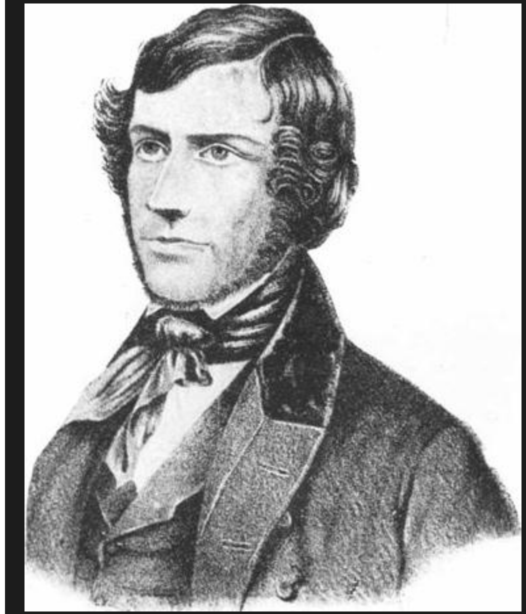
If $r = \frac{2GM}{c^2}$, $v_{\text{esc}} = c$

- From radii smaller than this, the escape velocity is greater than the speed of light
    - Not even light would be able to escape the surface of a star with $r < \frac{2GM}{c^2}$ (the Schwarzschild radius)
- Nothing can travel faster than the speed of light, so if light can't escape, nothing can

If some amount of matter, M, is compressed to radius $r < \frac{2GM}{c^2}$, nothing can escape the surface gravity, not even light
— we have created a black hole

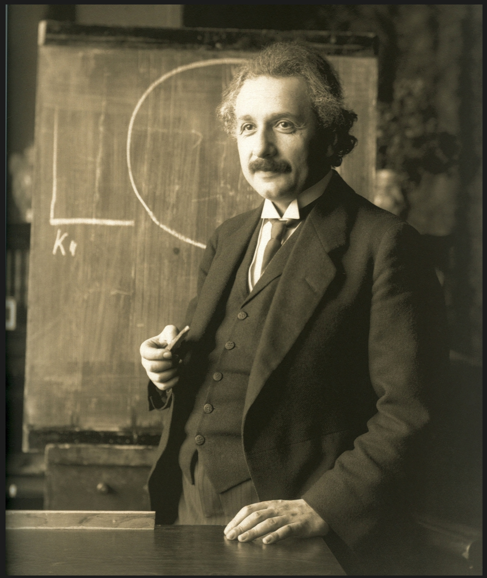
From Dark Stars to Black Holes

- This idea of 'dark stars' was first developed by John Michell (1783) and Pierre Simon Laplace (1796)
- This Newtonian picture implies for $r \gtrsim \frac{2GM}{c^2}$, light will escape at a reduced speed, and for $r < \frac{2GM}{c^2}$, light will travel up a small distance and then turn back to the surface (this is not true)
- BUT we're talking about <u>strong gravity</u> and motion <u>close to or at the speed of light</u> – Newtonian approximation breaks down
- Full theory of these gravitationally collapsed objects would need to wait for Einstein's theory of gravity, General Relativity (1916)
  - But Einstein believed they were too strange to be real!
- Term 'black hole' first coined by John Wheeler (1967)



John Michell

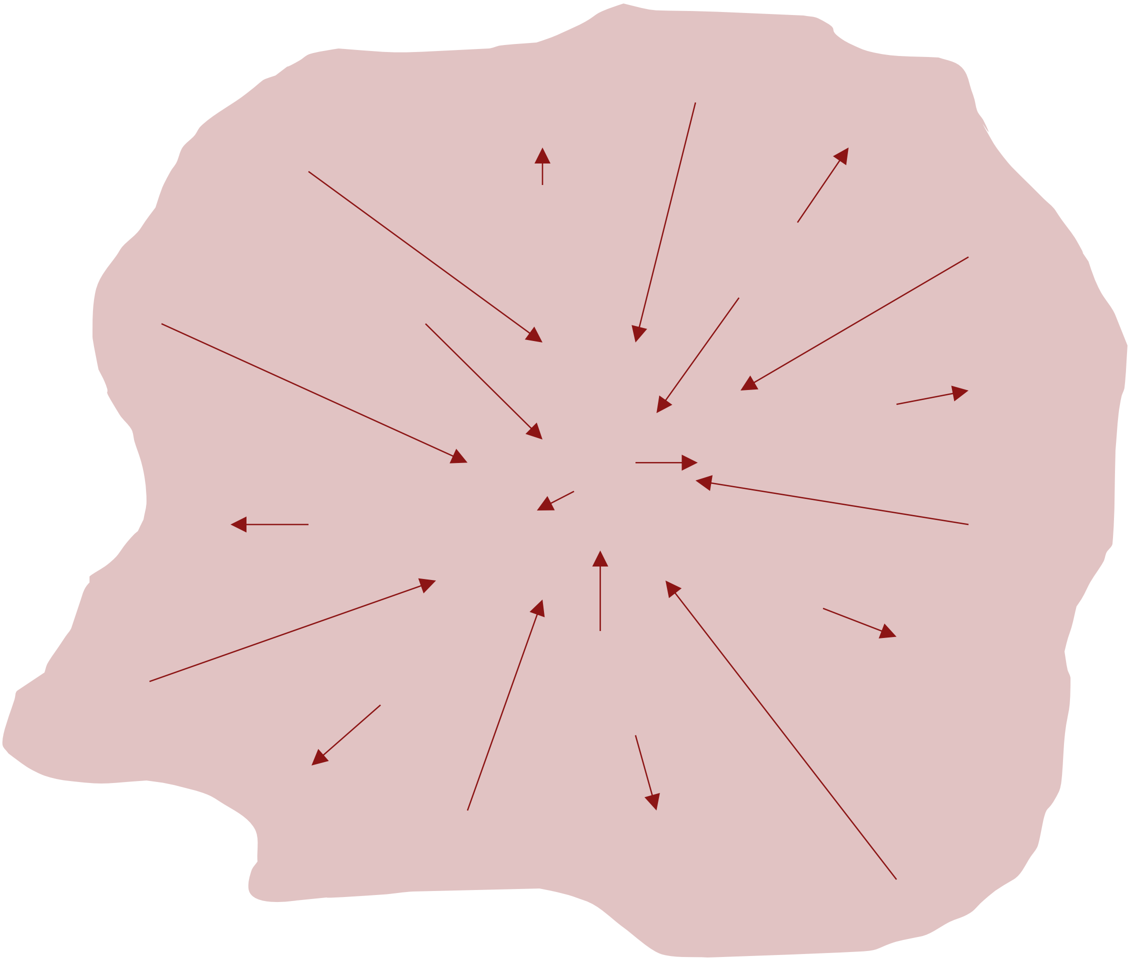
Pierre-Simon Laplace



Albert Einstein

John Wheeler

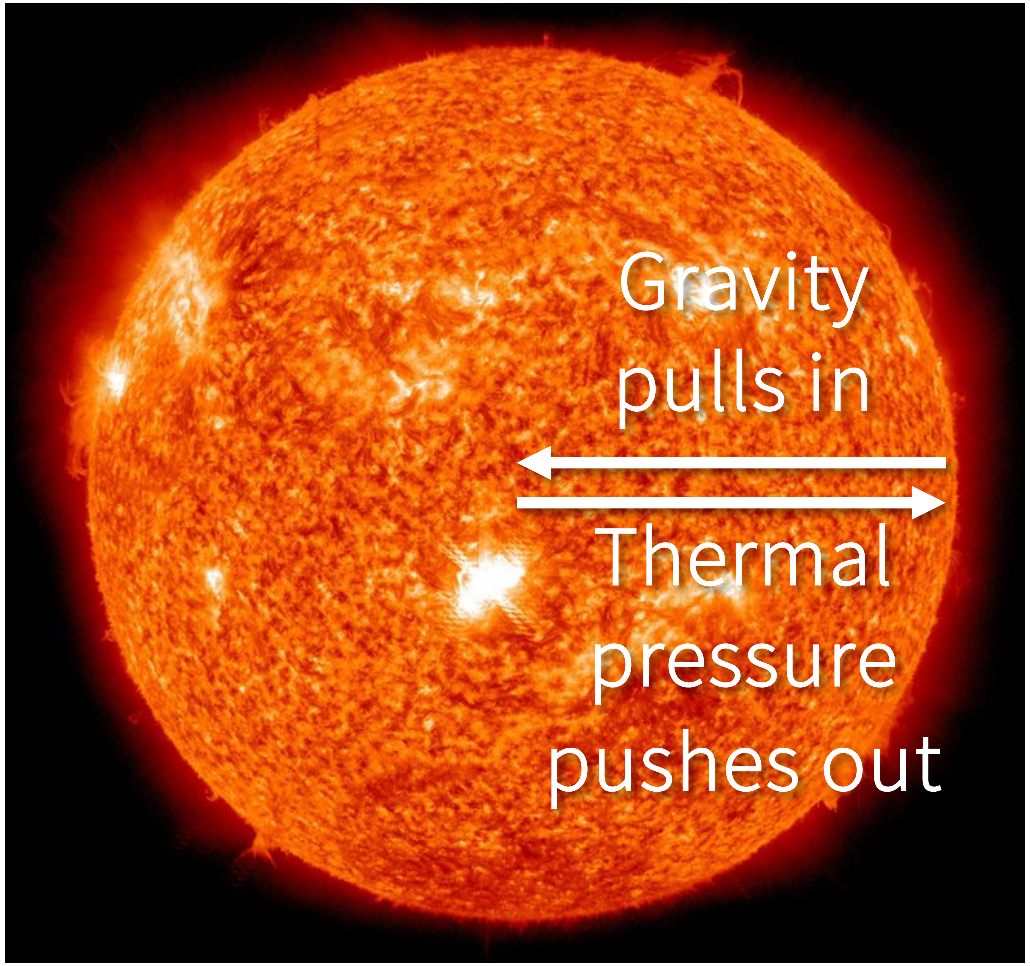
How can we make a black hole?



- Start with a cloud of gas in space
- Every part of the cloud is attracted to every other part of the cloud by the force of gravity
- Gravity pulls the cloud together and causes it to collapse to a smaller volume and the gas becomes denser
- Gravitational pull $F = \frac{Gm_1m_2}{r^2}$ gets stronger as the cloud collapses
  - Collapse gets faster & faster
  - 'Runaway collapse'

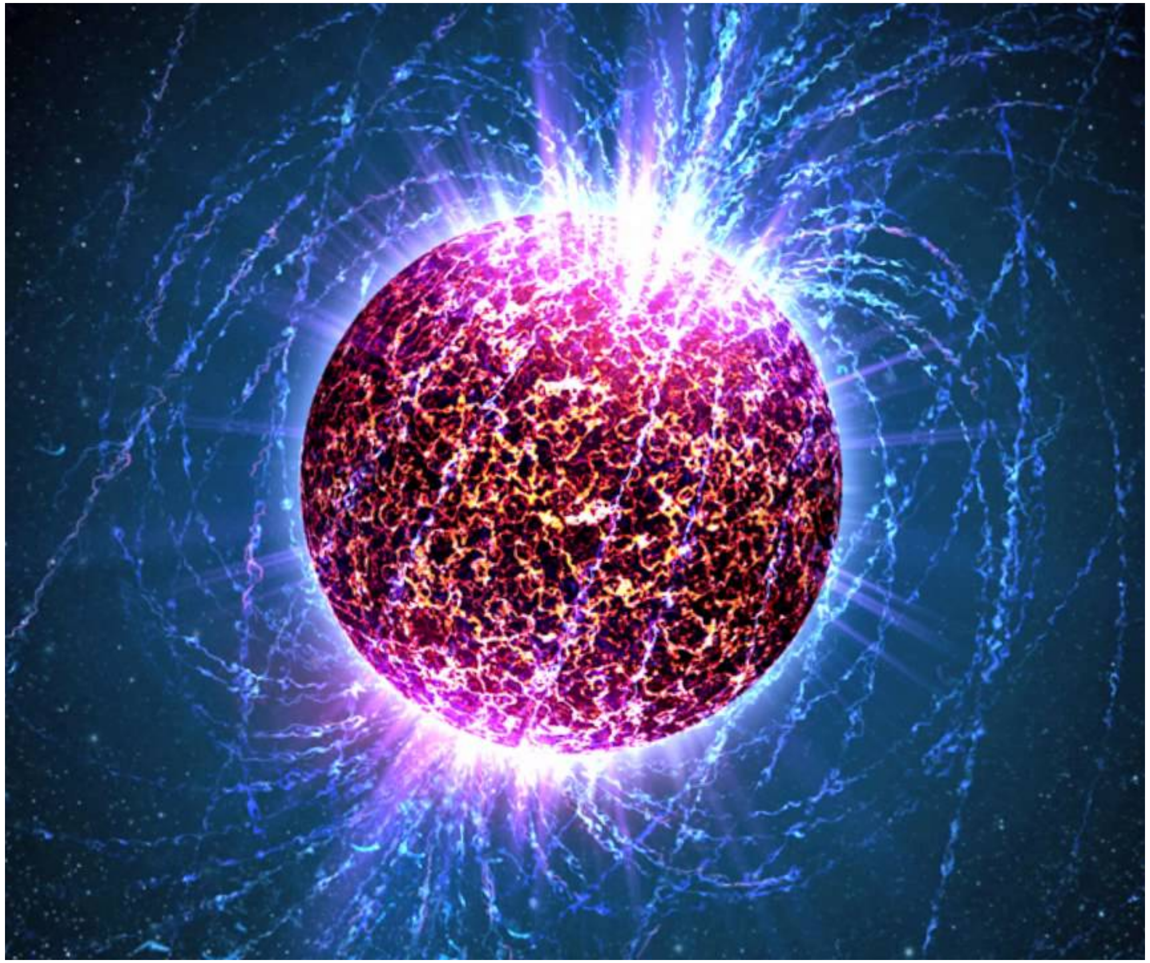
WILtY

How can we make a black hole?



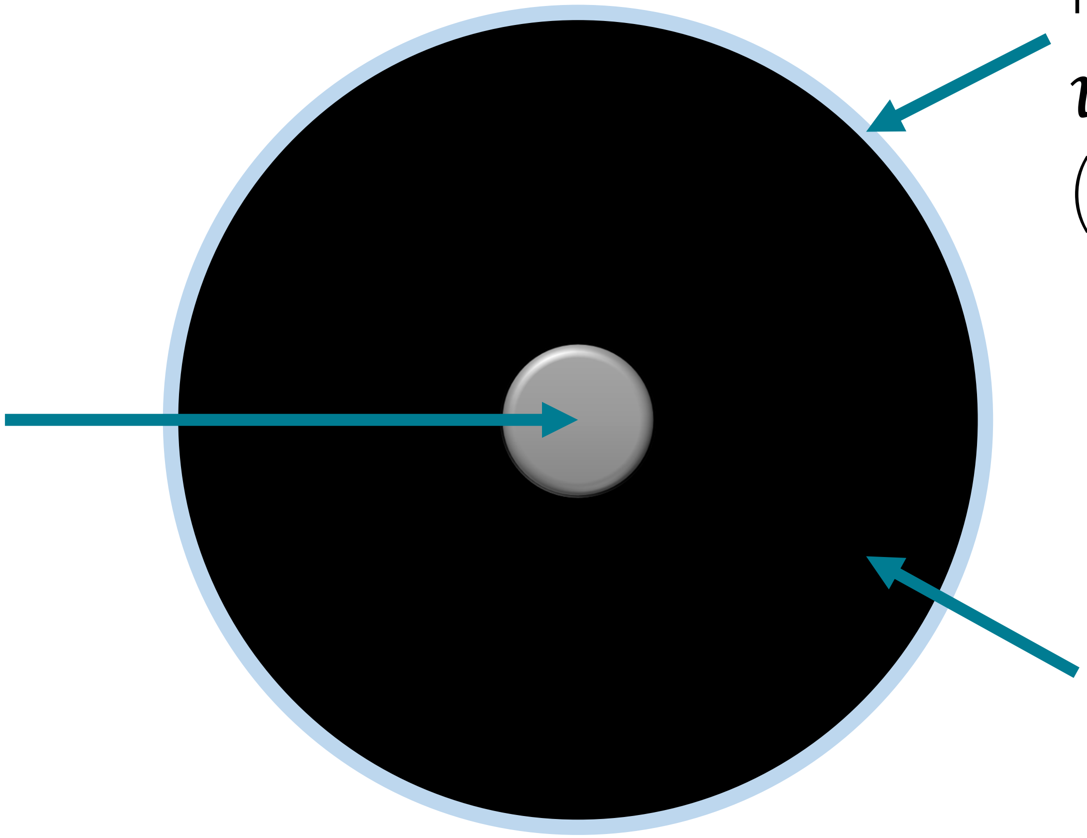
- As the gas gets compressed, it heats up
- Thermal pressure of the gas pushes out, balances the gravity and stops the collapse
- Pressure & density get high enough to force atomic nuclei together, starting nuclear fusion reactions in core, providing heat source to sustain thermal pressure
- Forms a star (coming up in Lecture 8)

How can we make a black hole?



- When star runs out of fuel for nuclear fusion reactions, nothing to keep the gas hot to maintain thermal pressure pushing against gravity
- Gravity pulls star together and it collapses further. Gravity gets stronger as it collapses.
- (a number of things can happen… coming up…)
- Various forces can stop collapse (from electrons, neutrons… coming up…)
- If there's enough mass collapsing, the gravity overwhelms other forces and nothing stops the collapse WILtY
- It gets small enough & dense enough that it forms a black hole

Anatomy of a black hole



Collapsed material (the *singularity*)

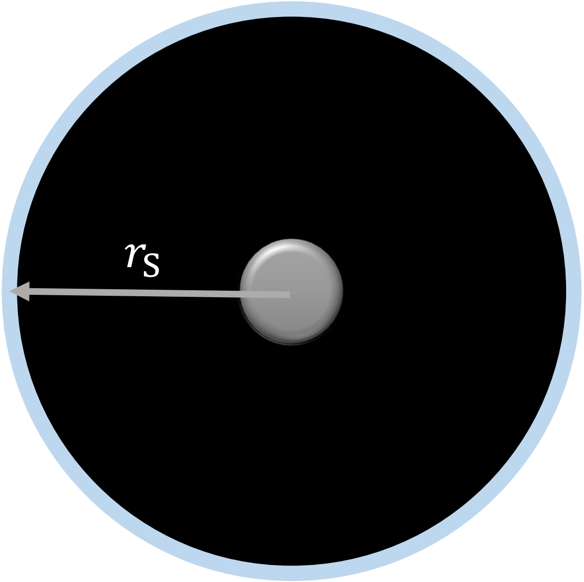
"Point of no return" at which $v_{\mathbf{esc}} = c$ (the *event horizon*)

$$r_H = \frac{2GM}{c^2}$$

Nothing (no object, no light, no information) can escape from inside the event horizon

QaD

How big is a black hole?



Radius of the event horizon of a (non-spinning) black hole of mass M is the Schwarzschild radius

$$r_S = \frac{2GM}{c^2}$$

The size of the event horizon (the Schwarzschild radius) depends on the mass of the black hole.

If the black hole is spherically symmetric, the gravitational force (and hence the radius of the event horizon) depend on the total amount of mass enclosed within the radius, hence it does not matter precisely how the matter is distributed inside the event horizon.

Mass	Schwarzschild Radius
Sun ($1\ M_\odot$)	1.5 km
Stellar mass black hole ($10\ M_\odot$)	15 km
Stellar mass black hole ($10^6\ M_\odot$)	1.5 x 10^6 km
2 protons with energy 7TeV	10^{-50} m

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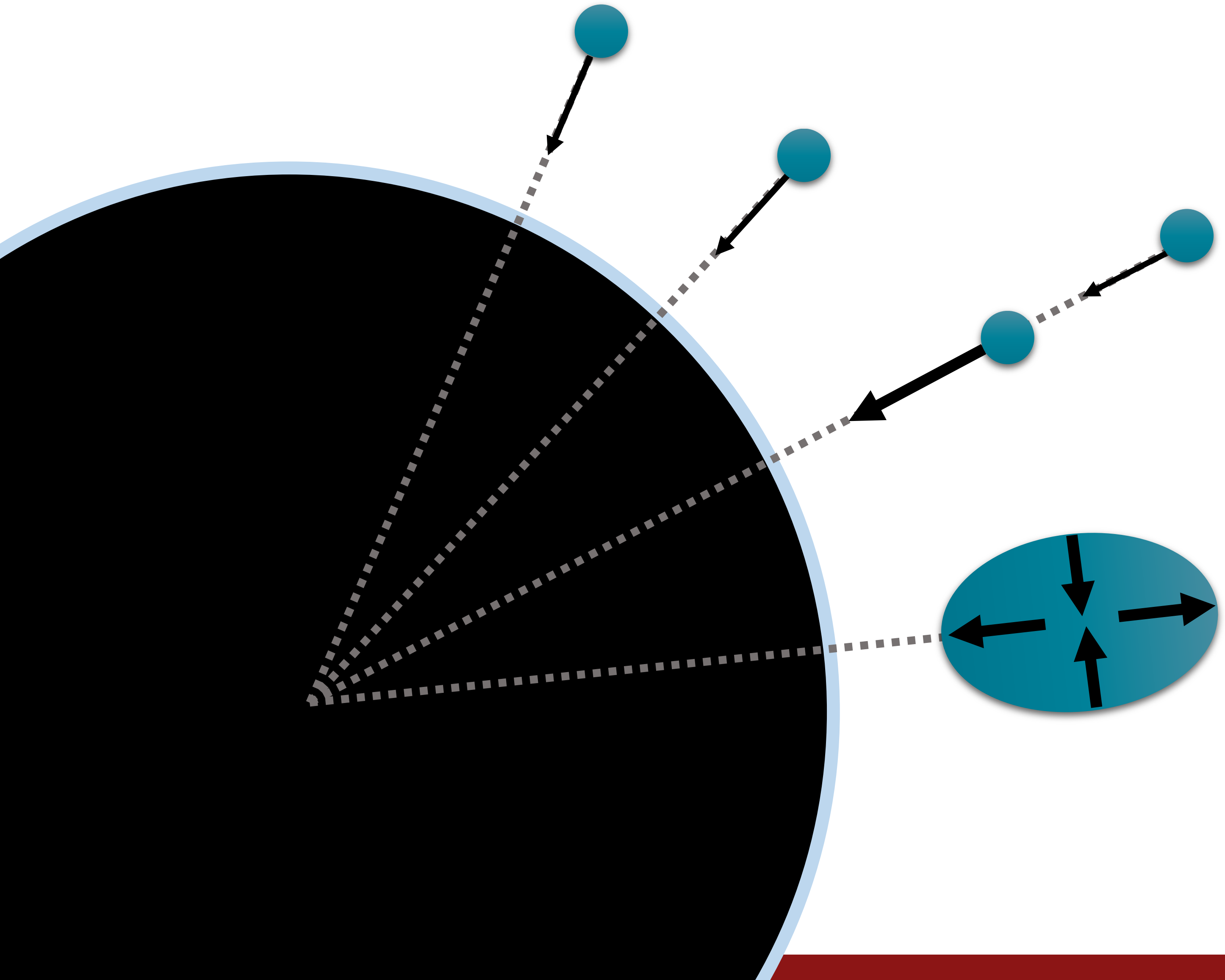
The pull of the black hole



Gravitational force pulls towards the center

- Objects get pulled in horizontally

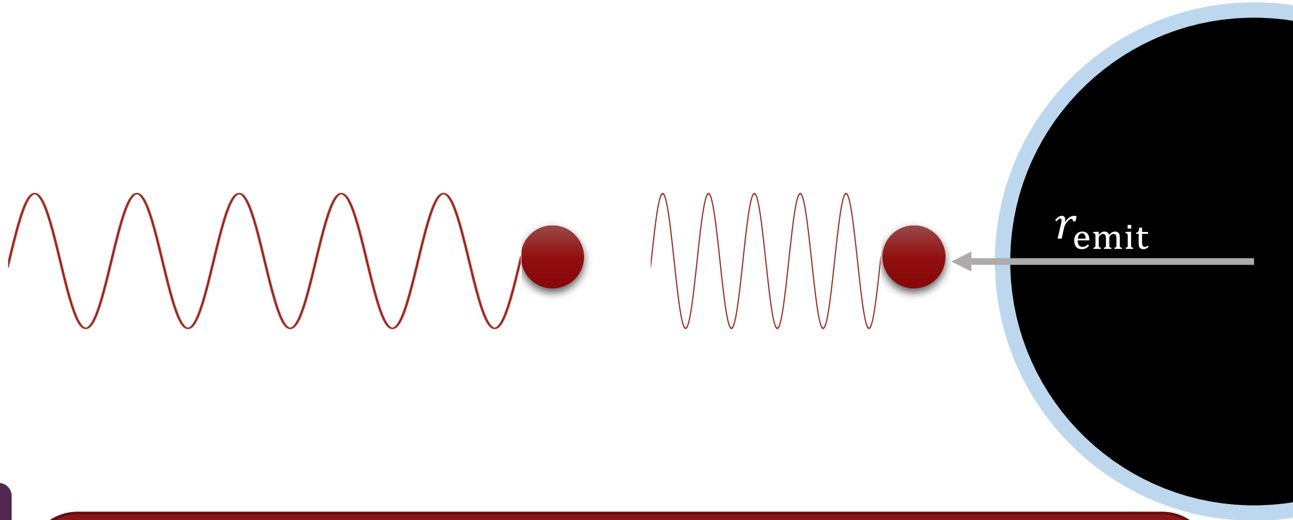
Further from the center, the gravitational force is weaker

- Parts of object closer to the center accelerate in more rapidly than those further out
- Objects get drawn out vertically as it falls in

Tidal force – the difference in gravitational pull on two sides of object $\sim {}^{1}/_{r^3}$

- Tidal force at event horizon is greater for smaller black holes

What happens to light emitted close to a black hole?



Think of light emitted (from outside the event horizon) as a stream of photons

Energy of each photon, $E = h\nu$

Using $E = mc^2$, we can identify an 'effective mass' $m = \frac{h\nu}{c^2}$ (note that a photon is actually massless) QaD

As the photon climbs away from the black hole, it climbs out of the gravitational potential, so loses energy

$$\Delta E = \frac{GMm}{r} - \frac{GMm}{r_{\mathrm{emit}}} = \frac{GMh\nu}{c^2}\left(\frac{1}{r} - \frac{1}{r_{\mathrm{emit}}}\right)$$

(we're a long way from the star so $r \to \infty$)

Gravitational Redshift

Photons lose energy and light is redshifted (shifted to longer wavelength/lower frequency) as it climbs out of a gravitational potential

$$\frac{\Delta E}{E} = \frac{\Delta \nu}{\nu} = -\frac{GM}{c^2 r_{\mathrm{emit}}}$$

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- Escape velocity
- A black hole is an object that has collapsed to within its Schwarzschild radius. Gravity at its surface becomes strong enough that the escape velocity exceeds the speed of light, so nothing can escape
- Event horizon – the point of no return from within which nothing can escape
- Size of the event horizon is the Schwarzschild radius
- Tidal forces
- Gravitational redshift