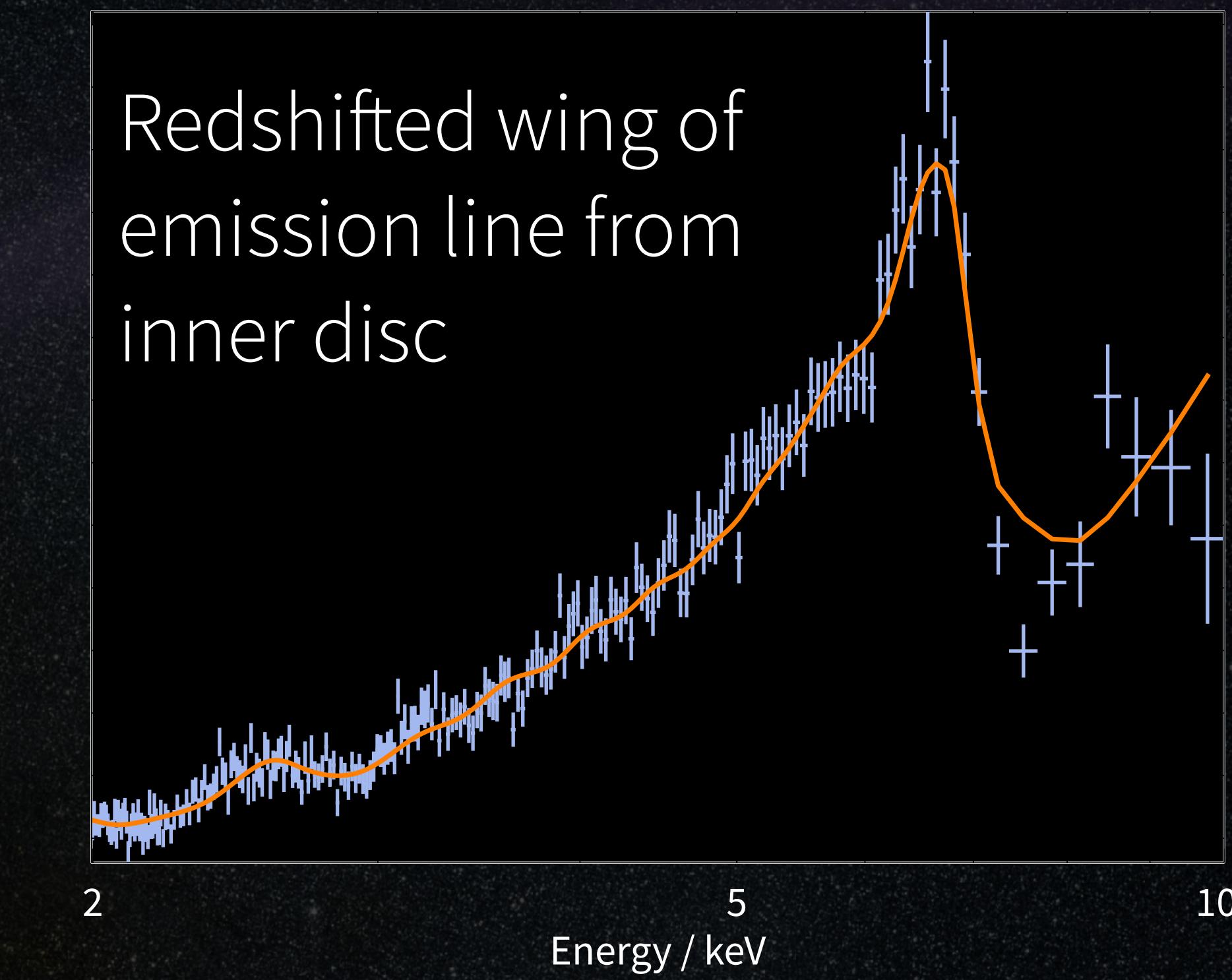
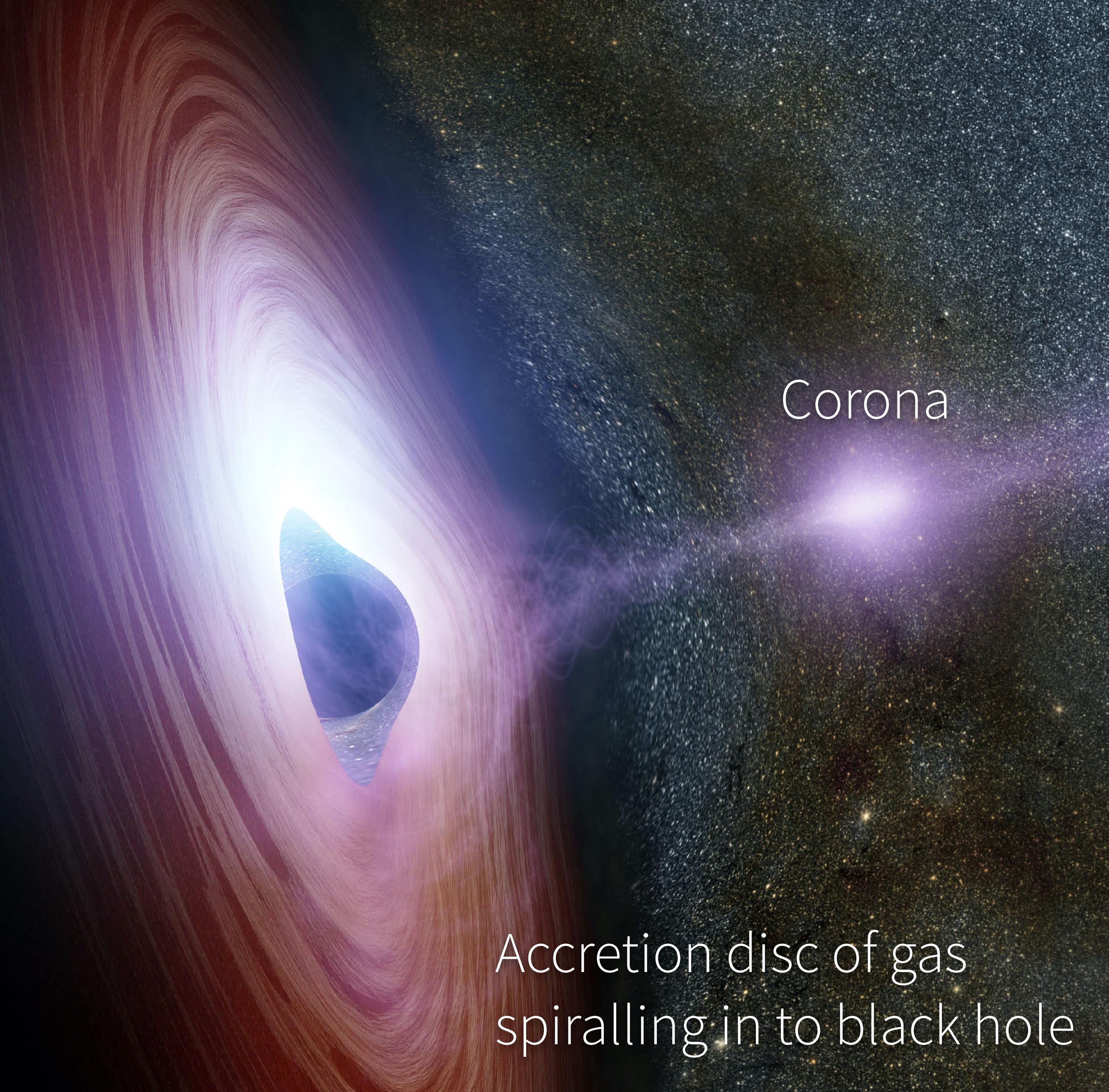


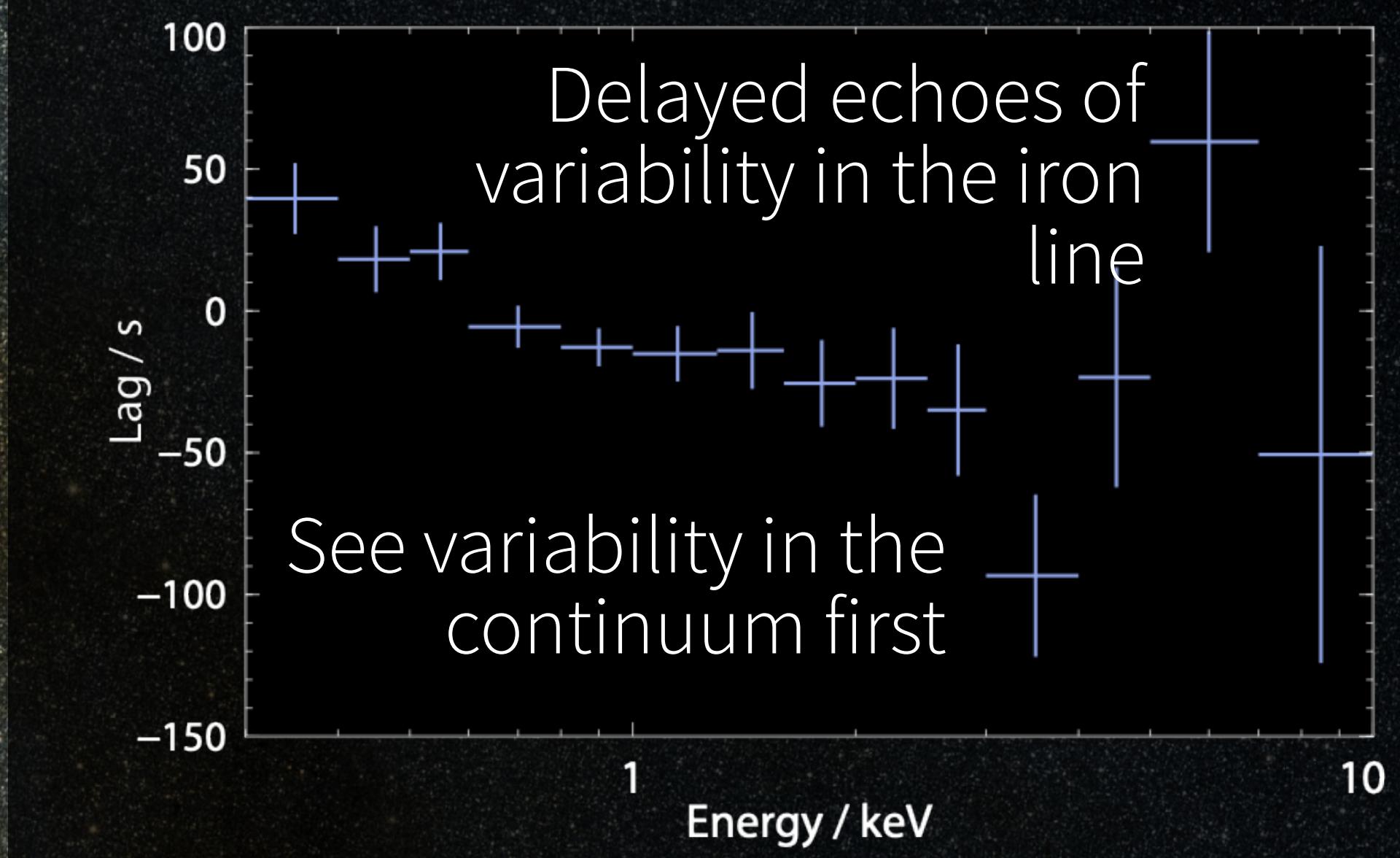
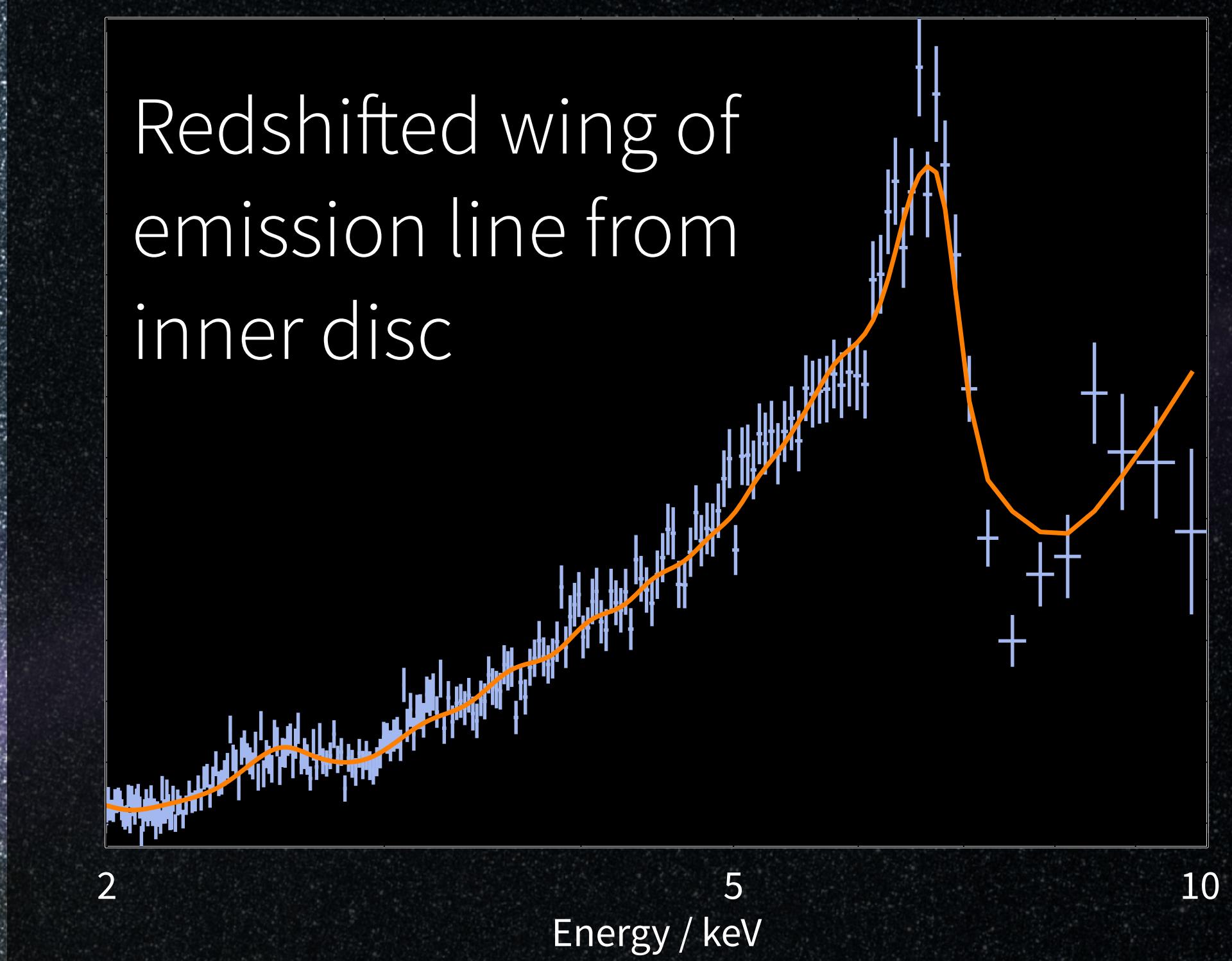
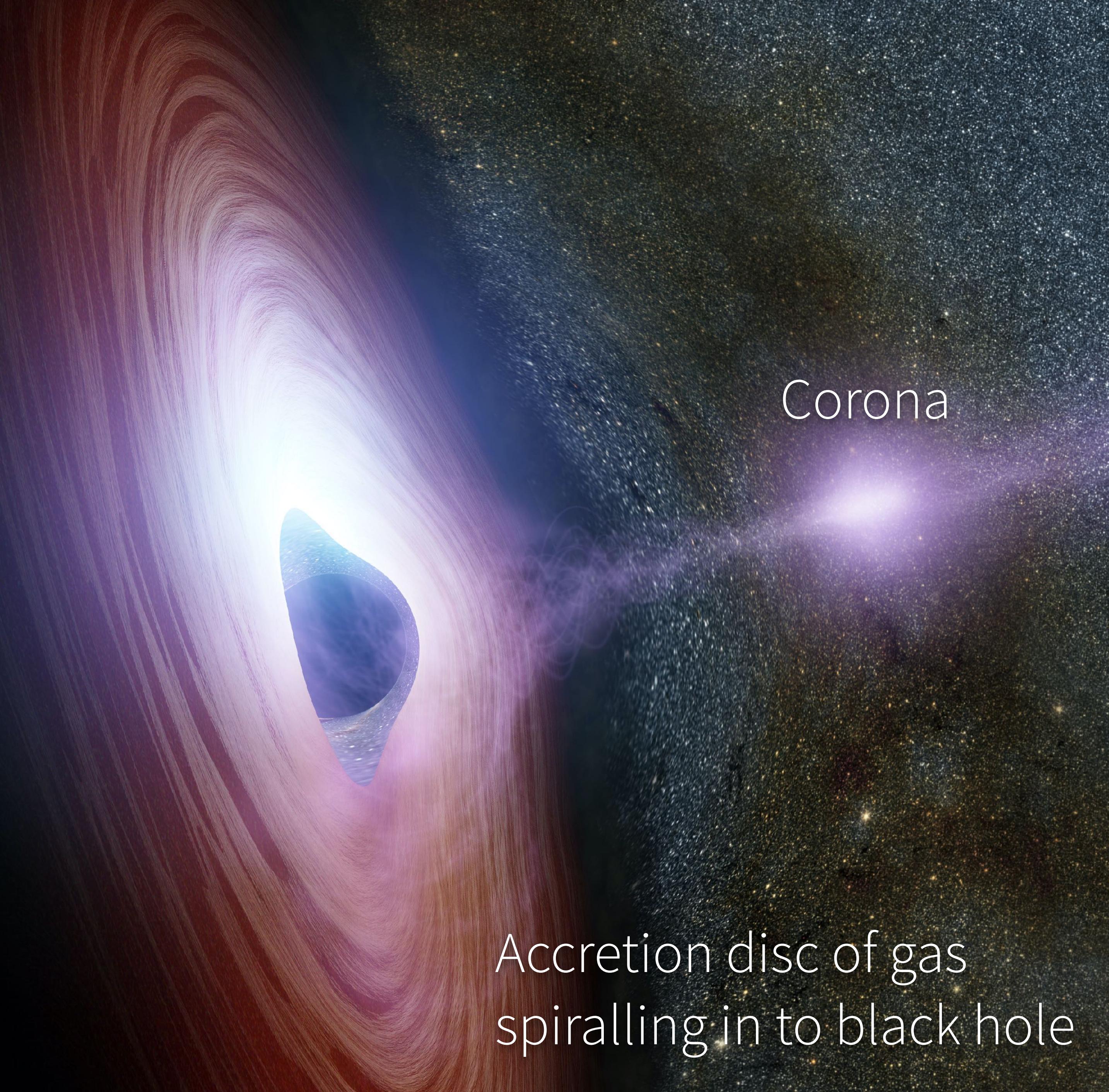
A statistical approach  
to measuring X-ray  
reverberation in  
gravitationally  
lensed quasars

Dan Wilkins

# Outline

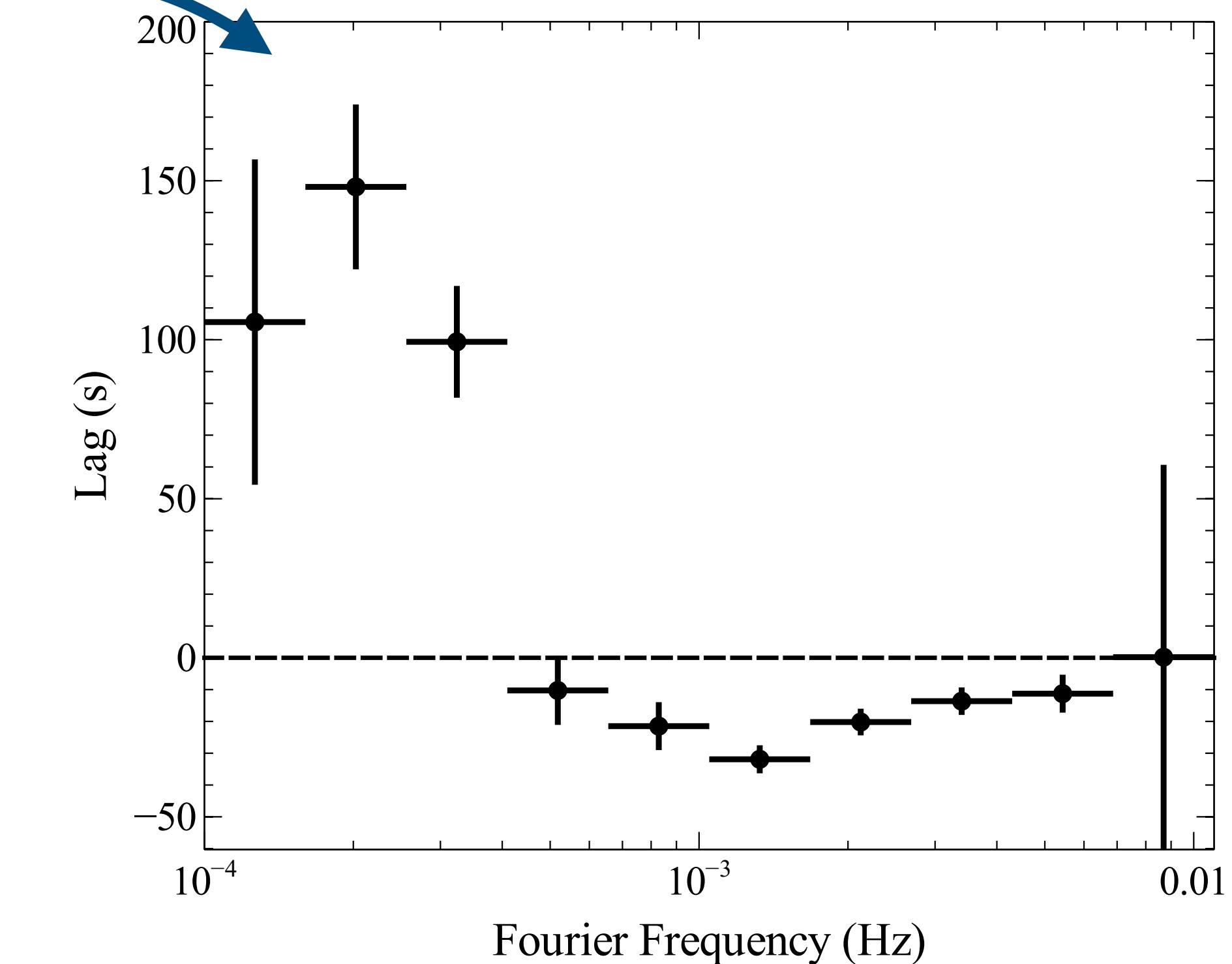
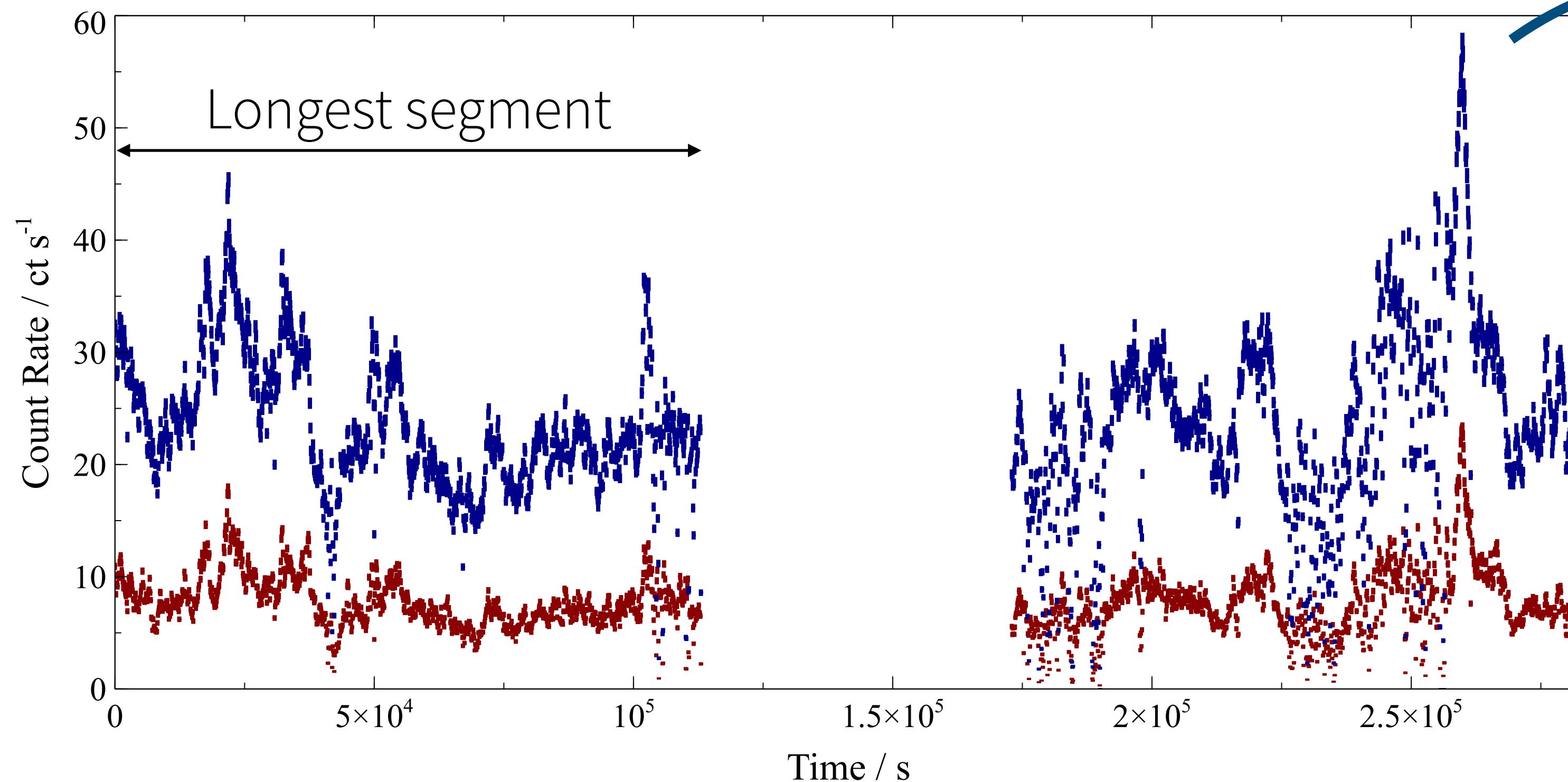
- X-ray reverberation mapping of the inner accretion flows around black holes
- The Gaussian process framework
- Measuring reverberation around high-mass black holes in radio galaxies
- X-ray reverberation in Chandra observations of gravitationally lensed quasars





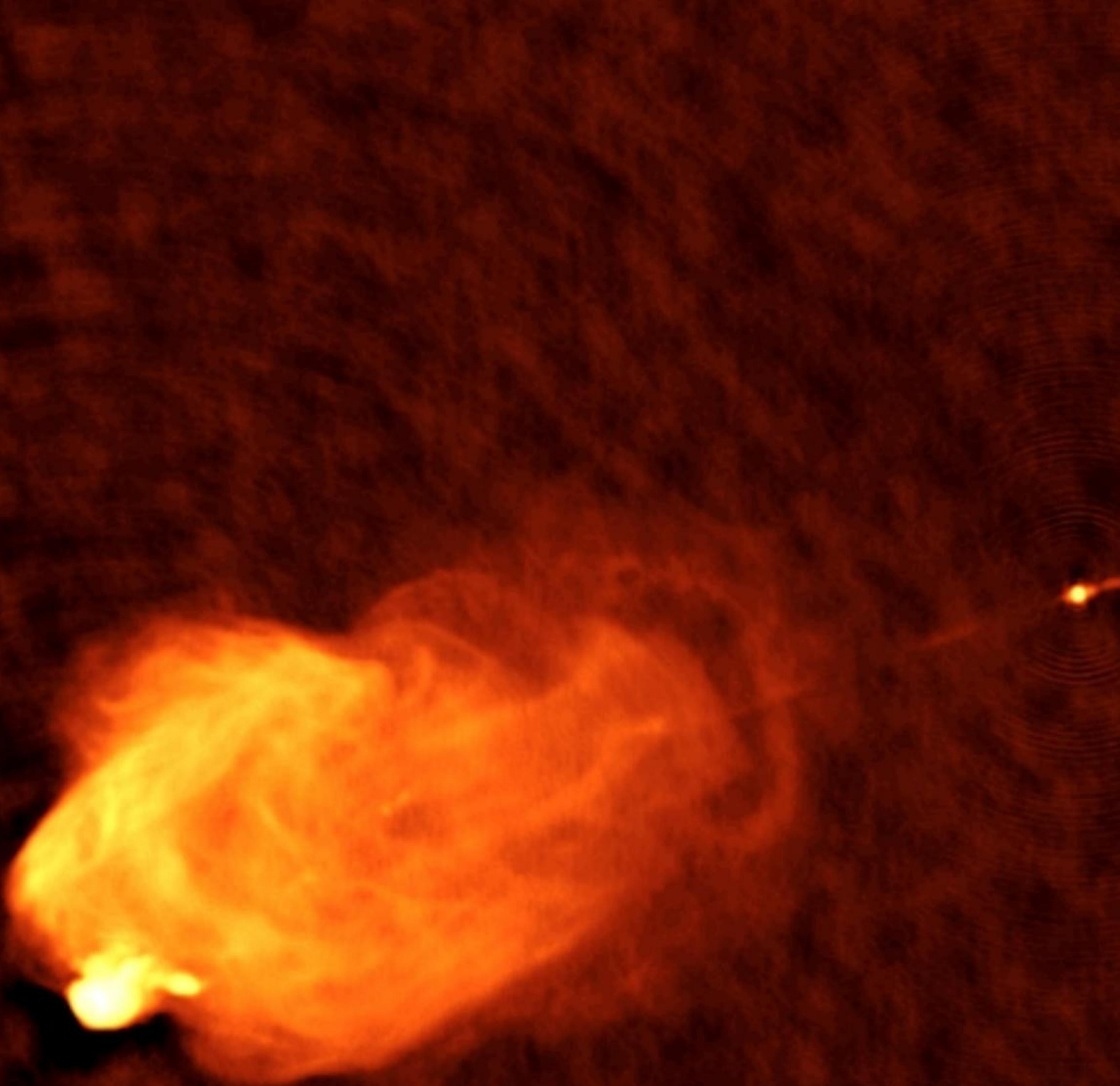
# Fourier Timing

Fourier transform  
Cross spectrum



- Requires high signal-to-noise light curves
- Longest measurable time lag limited by longest continuous segments

(see Uttley et al. 2014)



X-ray reverberation  
in radio galaxies

# Statistical frameworks for modelling time lags

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MNRAS **489**, 1957–1972 (2019)  
Advance Access publication 2019 August 16

doi:10.1093/mnras/stz2269



**Low-frequency X-ray timing with Gaussian processes and reverberation  
in the radio-loud AGN 3C 120**

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doi:10.1088/0004-637X/777/1/24

**CALCULATING TIME LAGS FROM UNEVENLY SAMPLED LIGHT CURVES**

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*Received 2013 July 1; accepted 2013 August 27; published 2013 October 10*

# Gaussian Processes (Wilkins 2019)

- Data vector containing the time series (N bins)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}$$

# Gaussian Processes (Wilkins 2019)

- Data vector containing the time series (N bins)
- Assume data vector is a random draw from an N-dimensional Gaussian – gives the likelihood function

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}$$

$$p(\mathbf{x}|\Sigma_{xx}) = \frac{1}{(2\pi)^N |\Sigma_{xx}|} \exp\left(-\frac{1}{2}\mathbf{x}^T \Sigma_{xx}^{-1} \mathbf{x}\right)$$

# Gaussian Processes (Wilkins 2019)

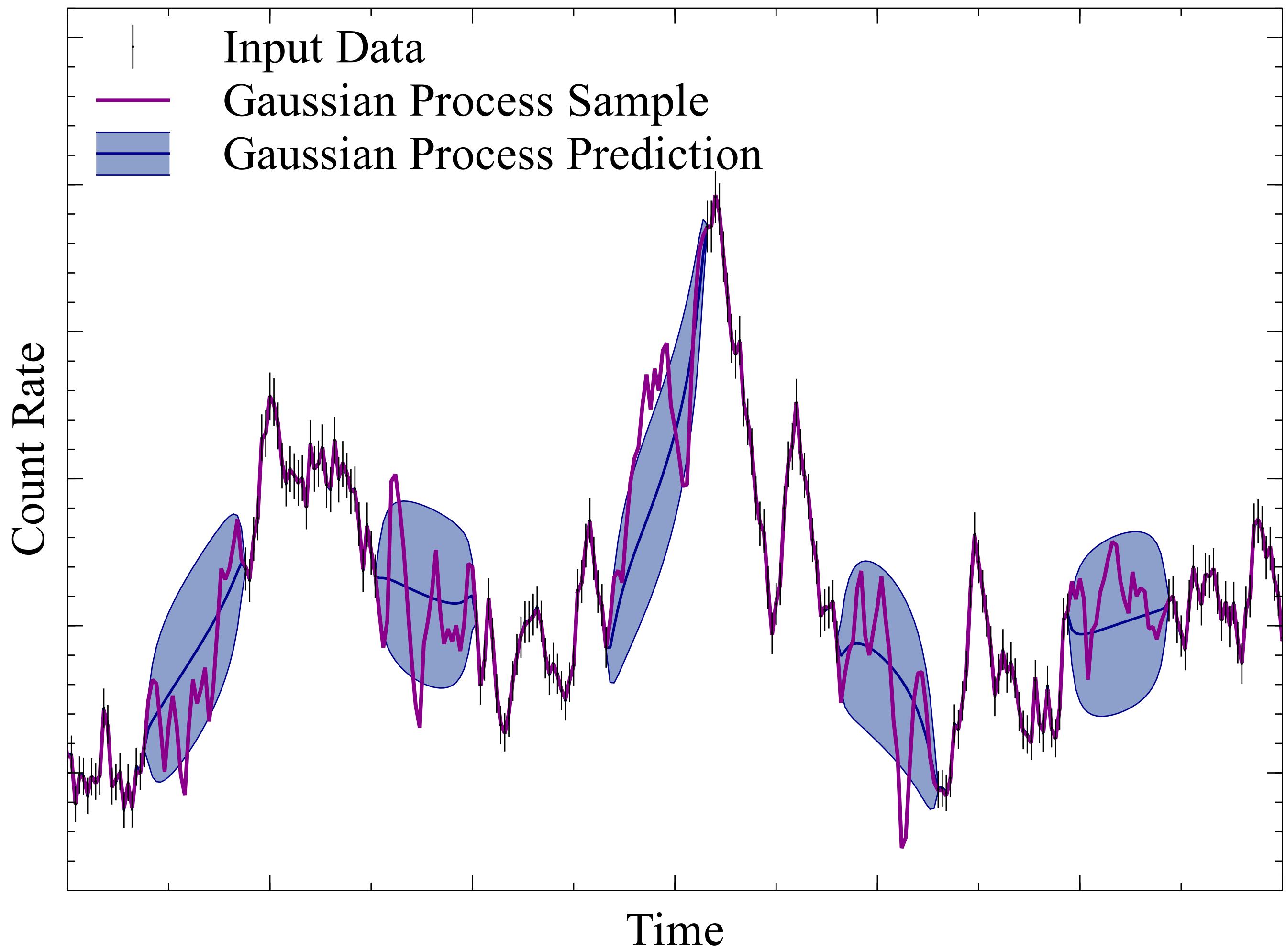
- Data vector containing the time series (N bins)
- Assume data vector is a random draw from an N-dimensional Gaussian – gives the likelihood function
- Covariance matrix encodes autocorrelation (i.e. covariances between pairs of time bins)
- Analytic form for  $\langle x_i x_j \rangle$  – the kernel function

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}$$

$$p(\mathbf{x}|\Sigma_{xx}) = \frac{1}{(2\pi)^N |\Sigma_{xx}|} \exp\left(-\frac{1}{2} \mathbf{x}^T \Sigma_{xx}^{-1} \mathbf{x}\right)$$

$$\Sigma_{xx} = \begin{pmatrix} \langle x_1 x_1 \rangle & \langle x_1 x_2 \rangle & \dots & \langle x_1 x_N \rangle \\ \langle x_2 x_1 \rangle & \langle x_2 x_2 \rangle & \dots & \langle x_2 x_N \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle x_N x_1 \rangle & \langle x_N x_2 \rangle & \dots & \langle x_N x_N \rangle \end{pmatrix}$$

# Filling in the gaps



Adopt simple analytic function  
for covariances

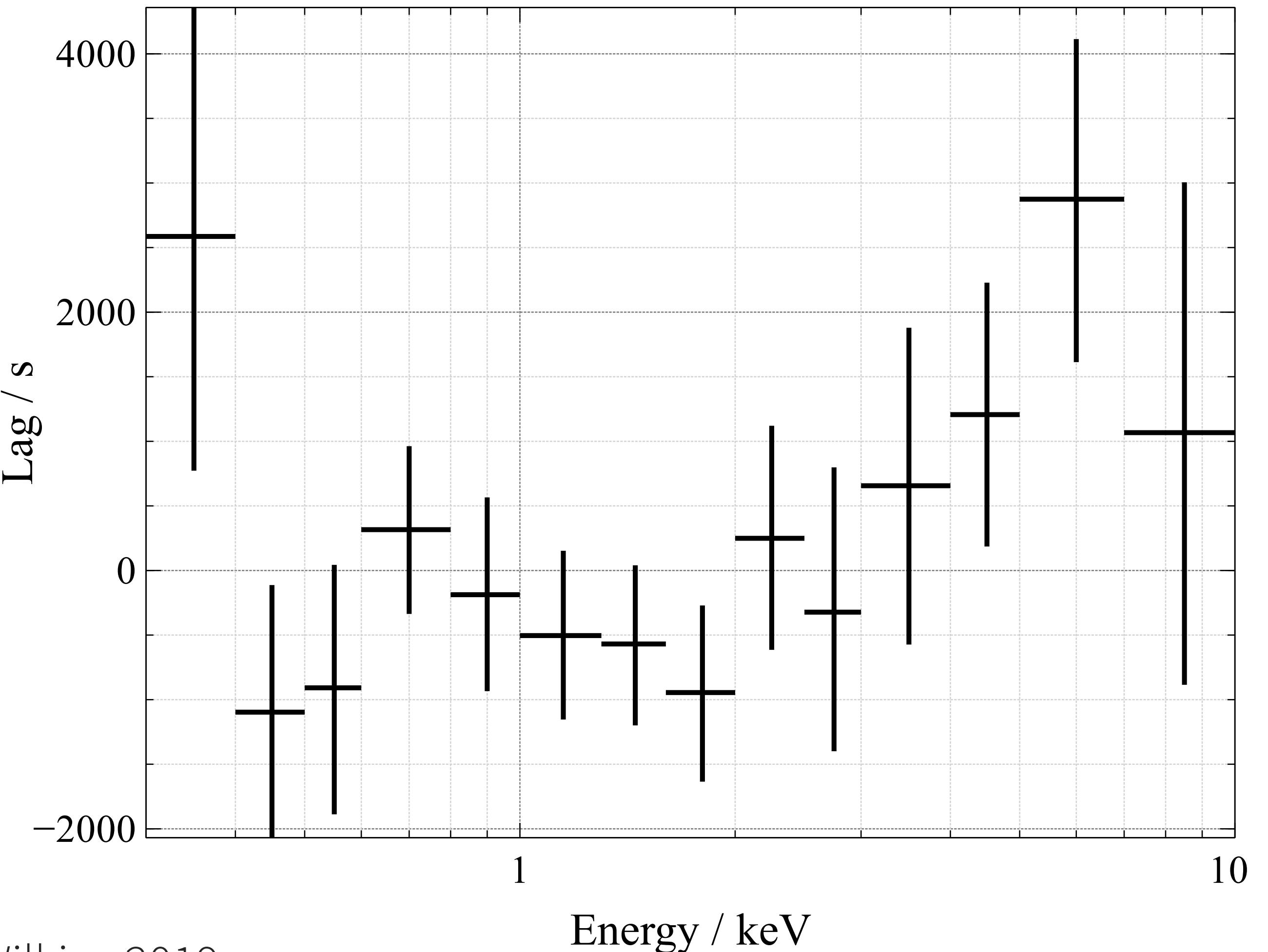
Fit hyper-parameters to observed  
time bins

Draw samples from  $(x_{\text{sample}} | x_{\text{obs}})$   
with these parameters to predict  
missing time bins

Average predictions over many  
samples

# X-ray Reverberation in 3C 120

Broad line radio galaxy



$M_{\text{BH}} = 5.7 \times 10^7 M_{\odot}$

Gaussian process fit to light curve  
in each energy bin

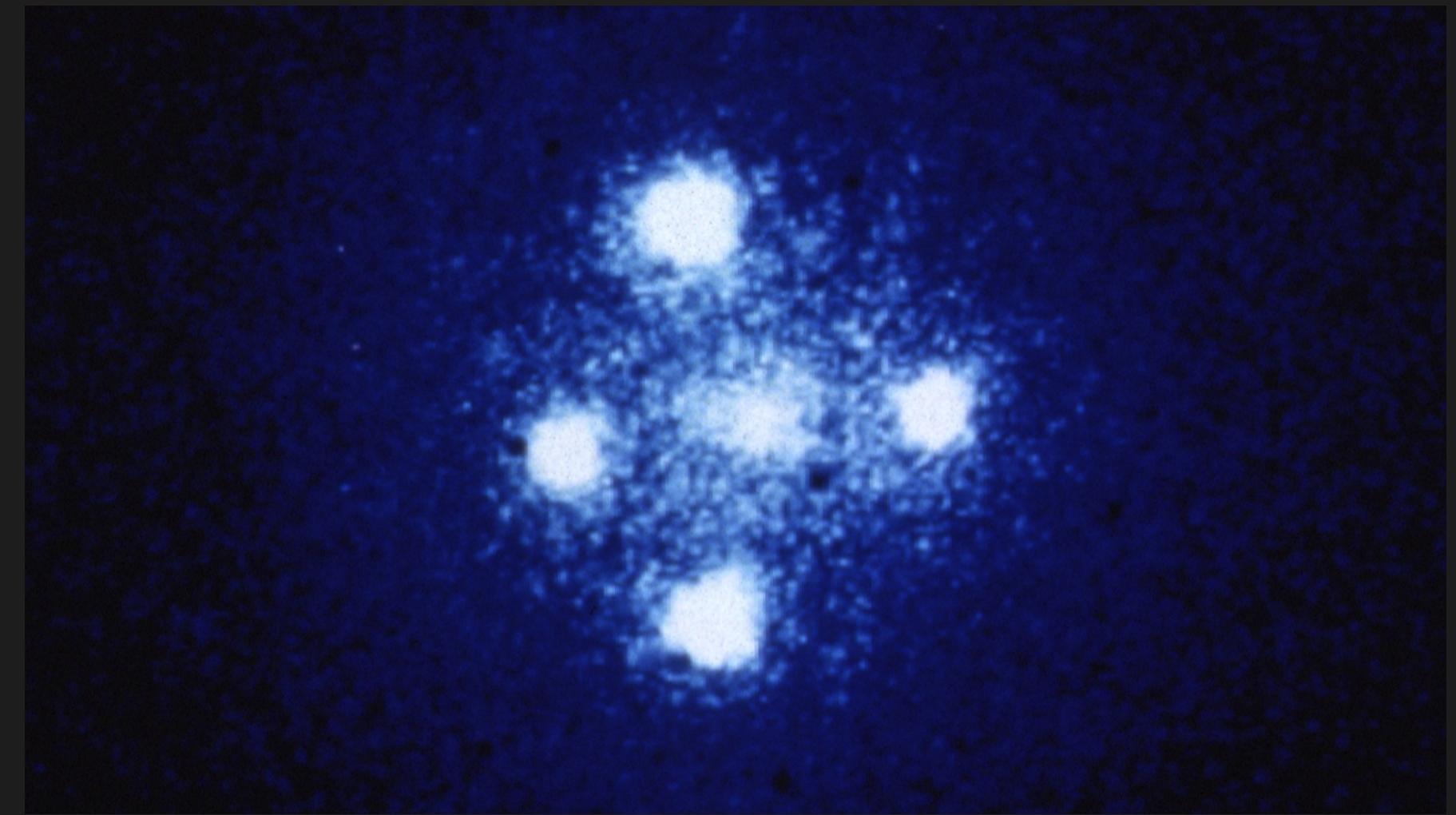
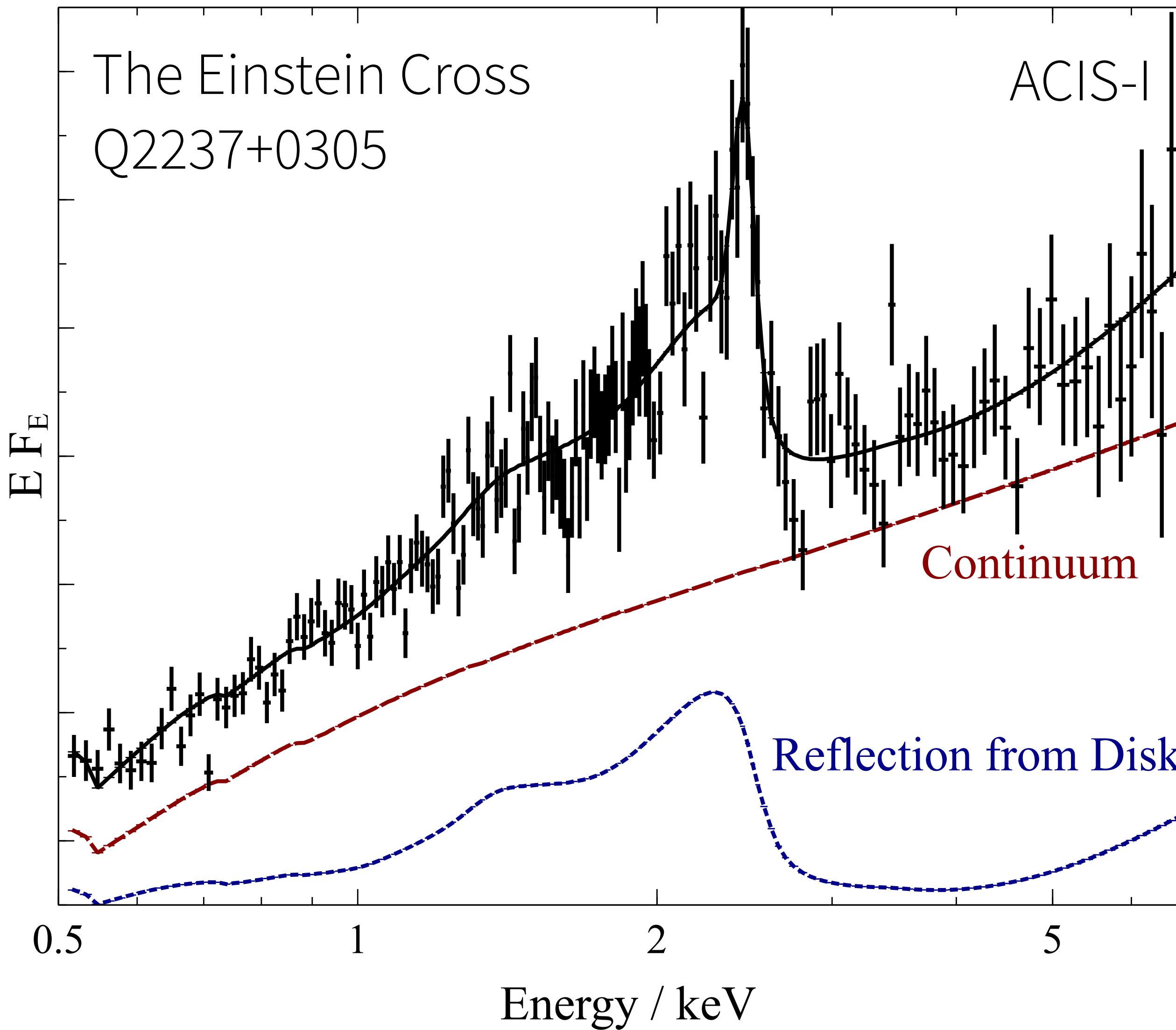
Predict missing time bins and  
calculate time lag using  
conventional Fourier method

Produces posterior distribution  
of time lag in each energy bin



X-ray reverberation  
in gravitationally  
lensed quasars

# Reverberation in lensed quasars



Chandra observations of lensed quasars show broad iron K lines, reflected from inner accretion disk — Reis et al. 2014, Reynolds et al. 2014, Walton et al. 2015

Lower count rates – direct lag measurement difficult

What range of time lags are consistent with the data?

# Fitting the lag between two energy bands (Zoghbi et al. 2013)

- Stacked data vector of the two time series,  $x$  and  $y$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

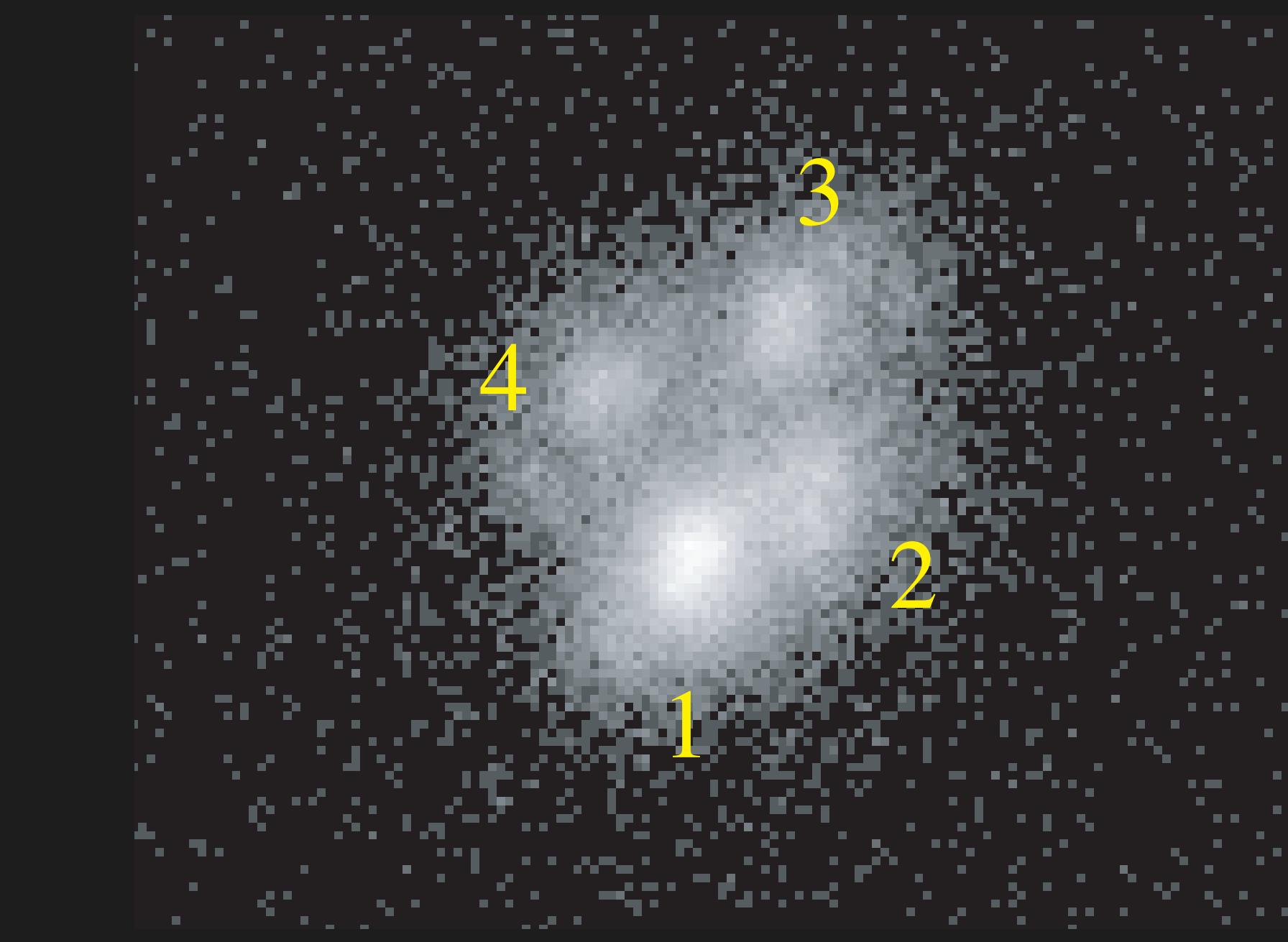
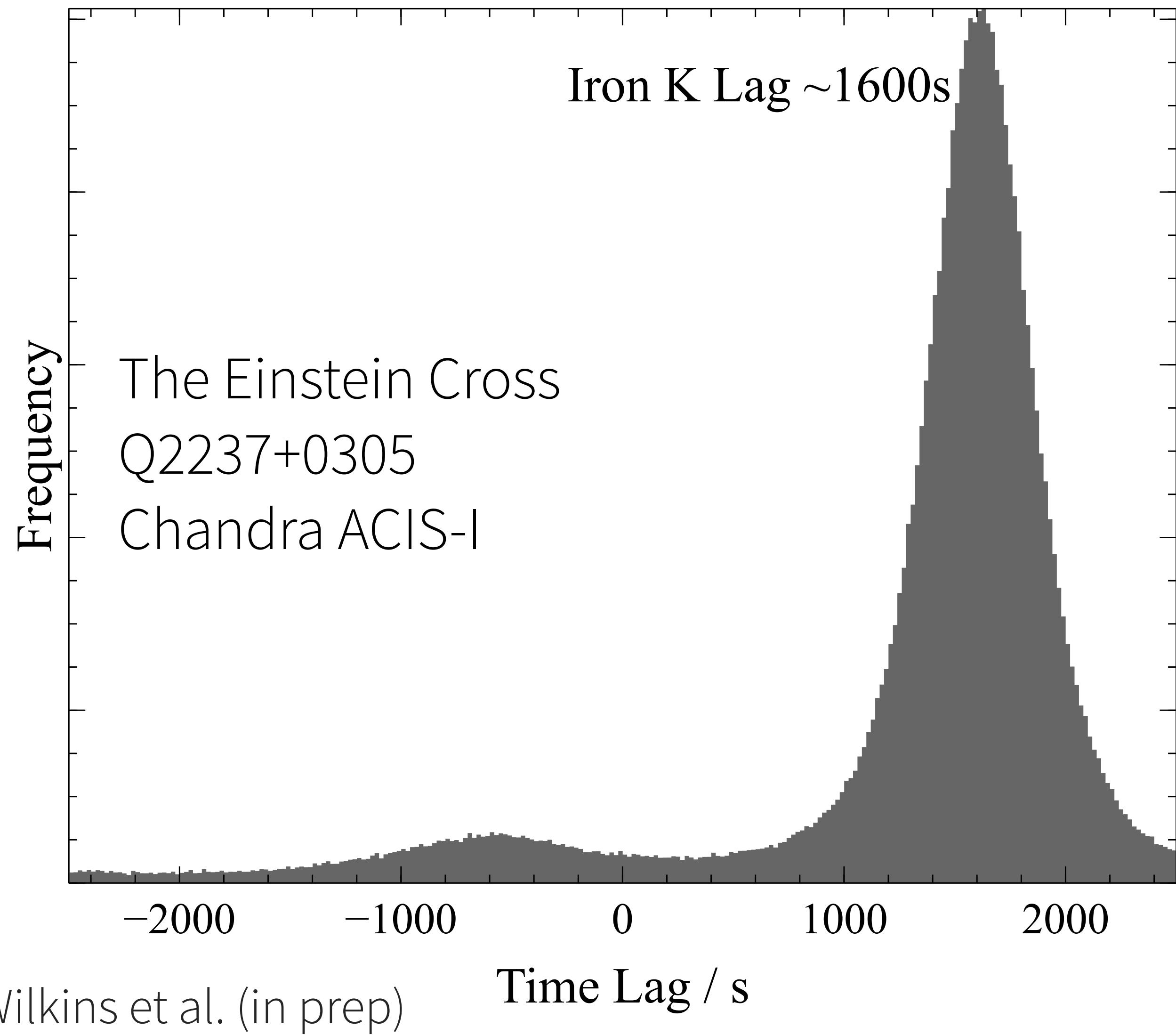
# Fitting the lag between two energy bands (Zoghbi et al. 2013)

- Stacked data vector of the two time series,  $x$  and  $y$
- Covariance matrix encodes autocorrelation of each time series, and the cross-correlation
- Time lag a parameter of cross-correlation model
- Use MCMC to obtain  $P(\text{lag}|\text{data})$

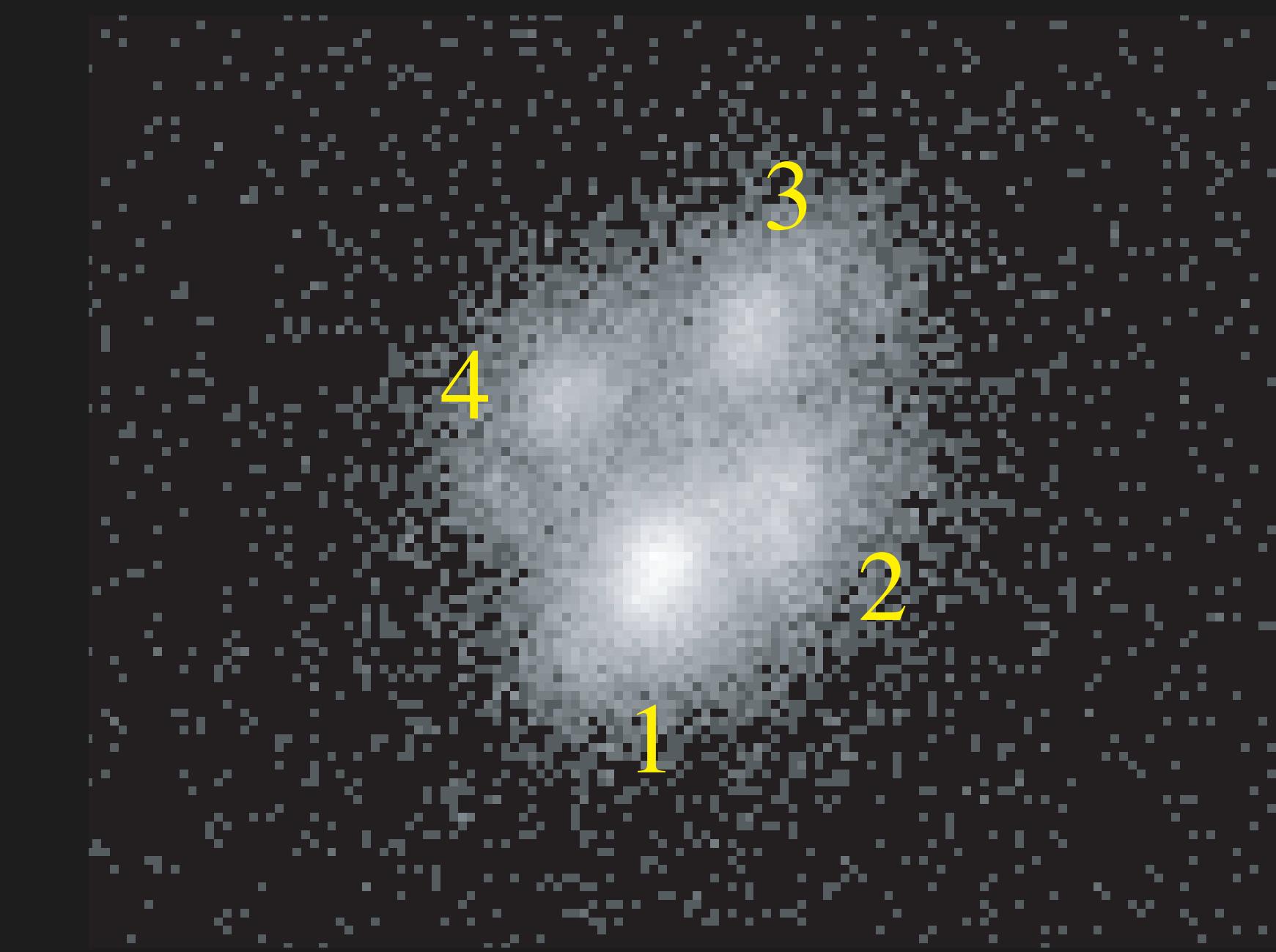
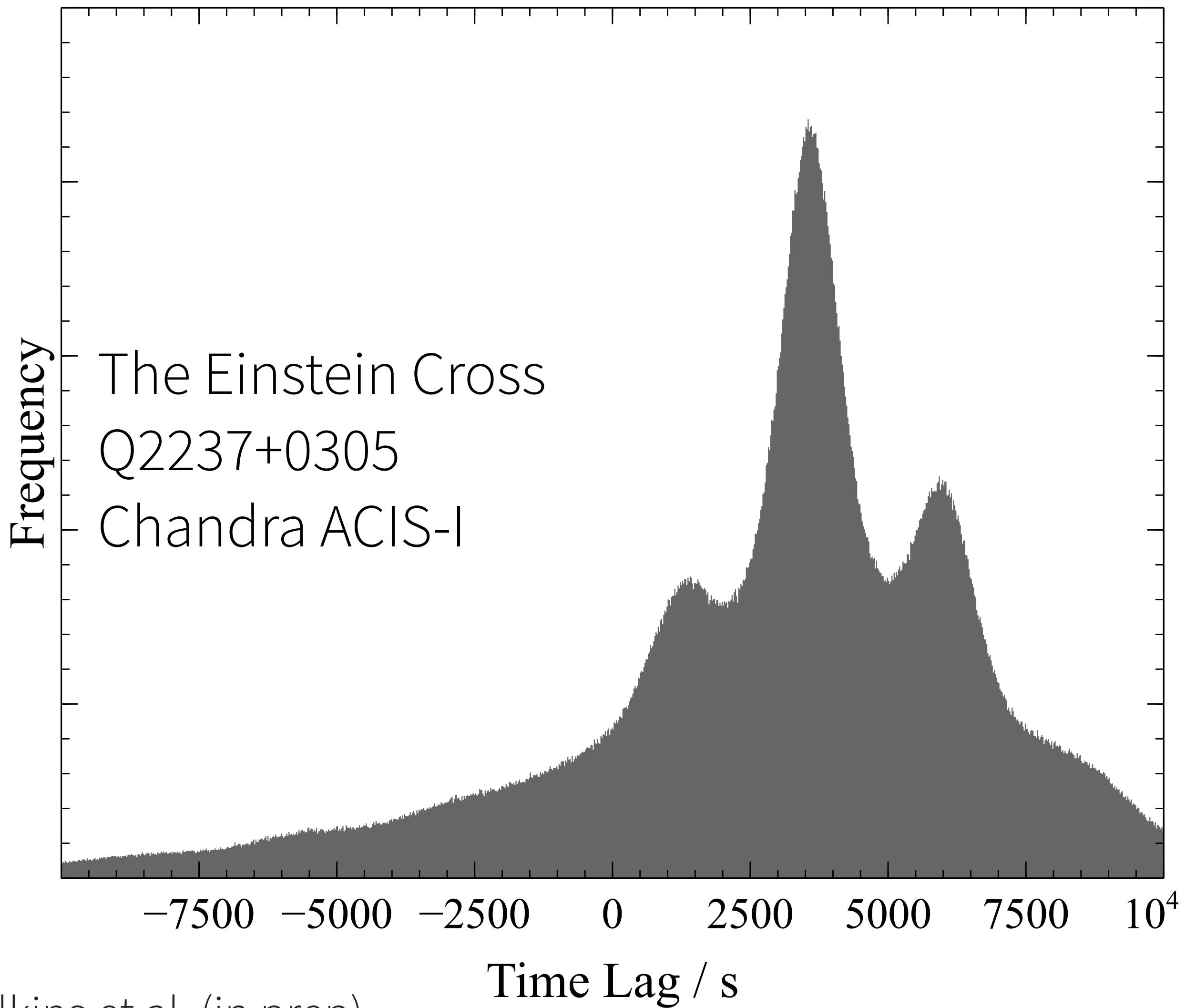
$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{pmatrix}$$

# Reverberation in the Einstein Cross



# Timing between images



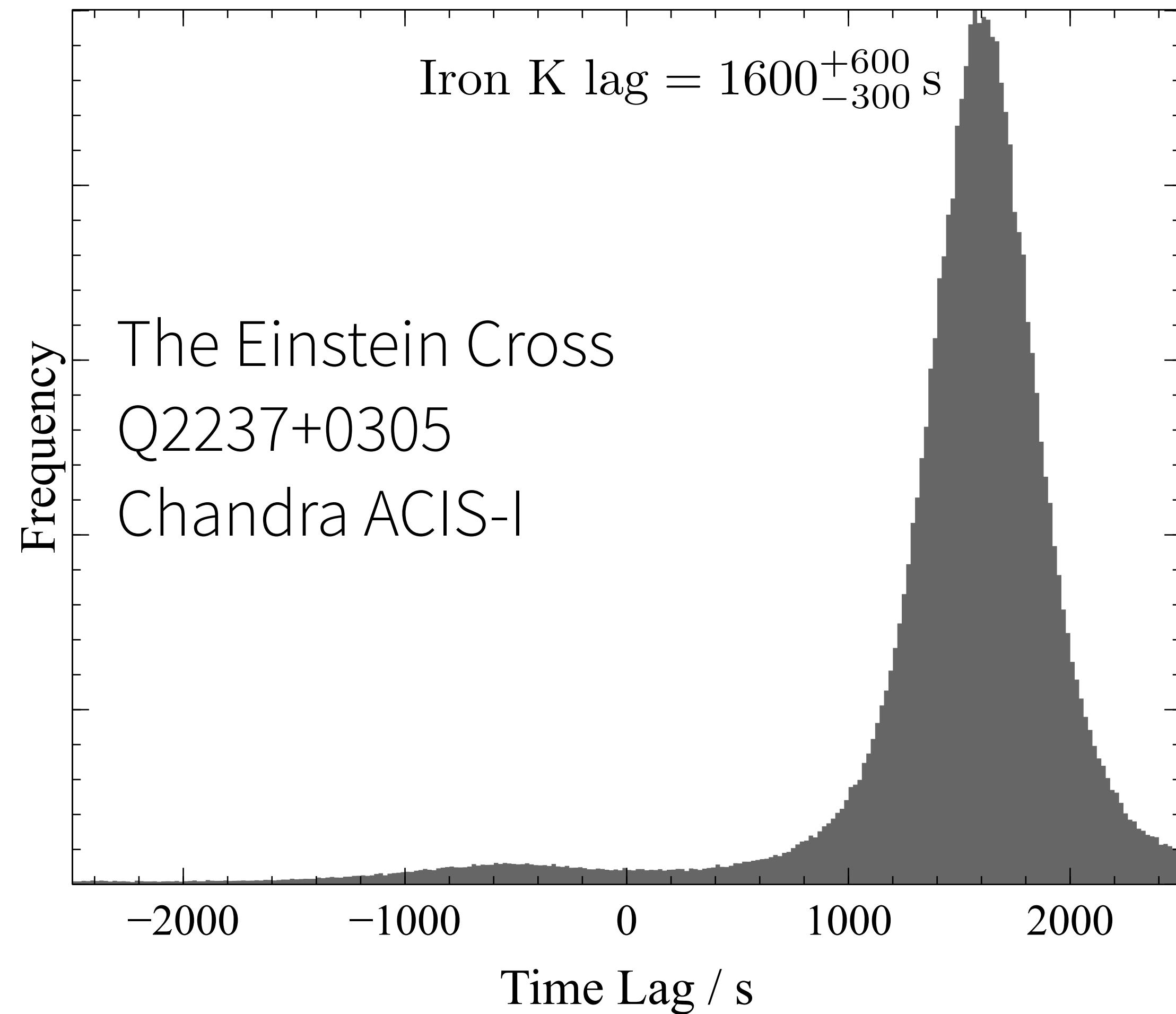
Fit time lag between continuum  
in image 3 and iron K in image 1

Need to account for lensing time  
delay between images

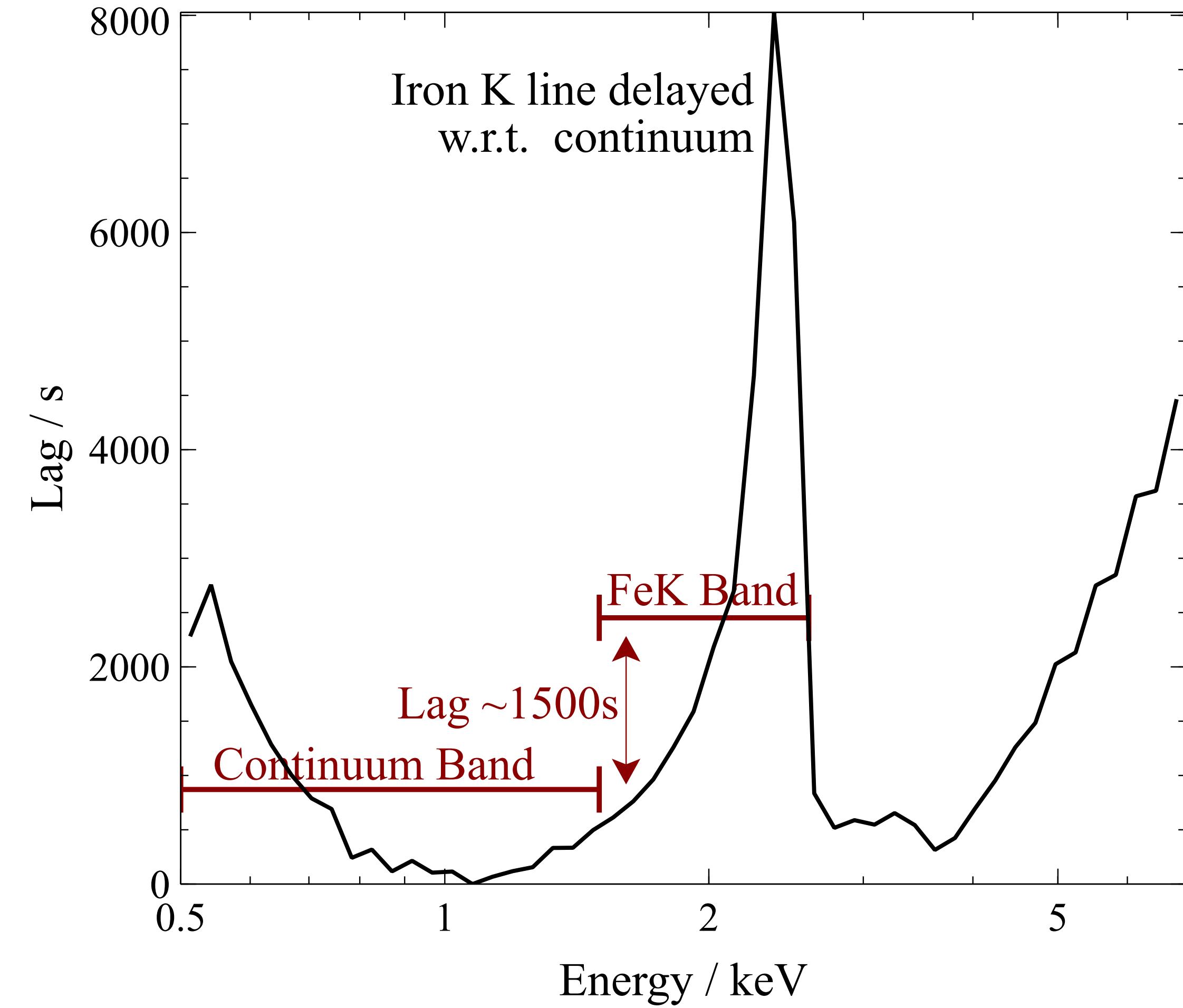
Simultaneously fit broadband lag  
between images 1 and 3

# Reverberation in the Einstein Cross

Simultaneous fit to image 1 and 1/3

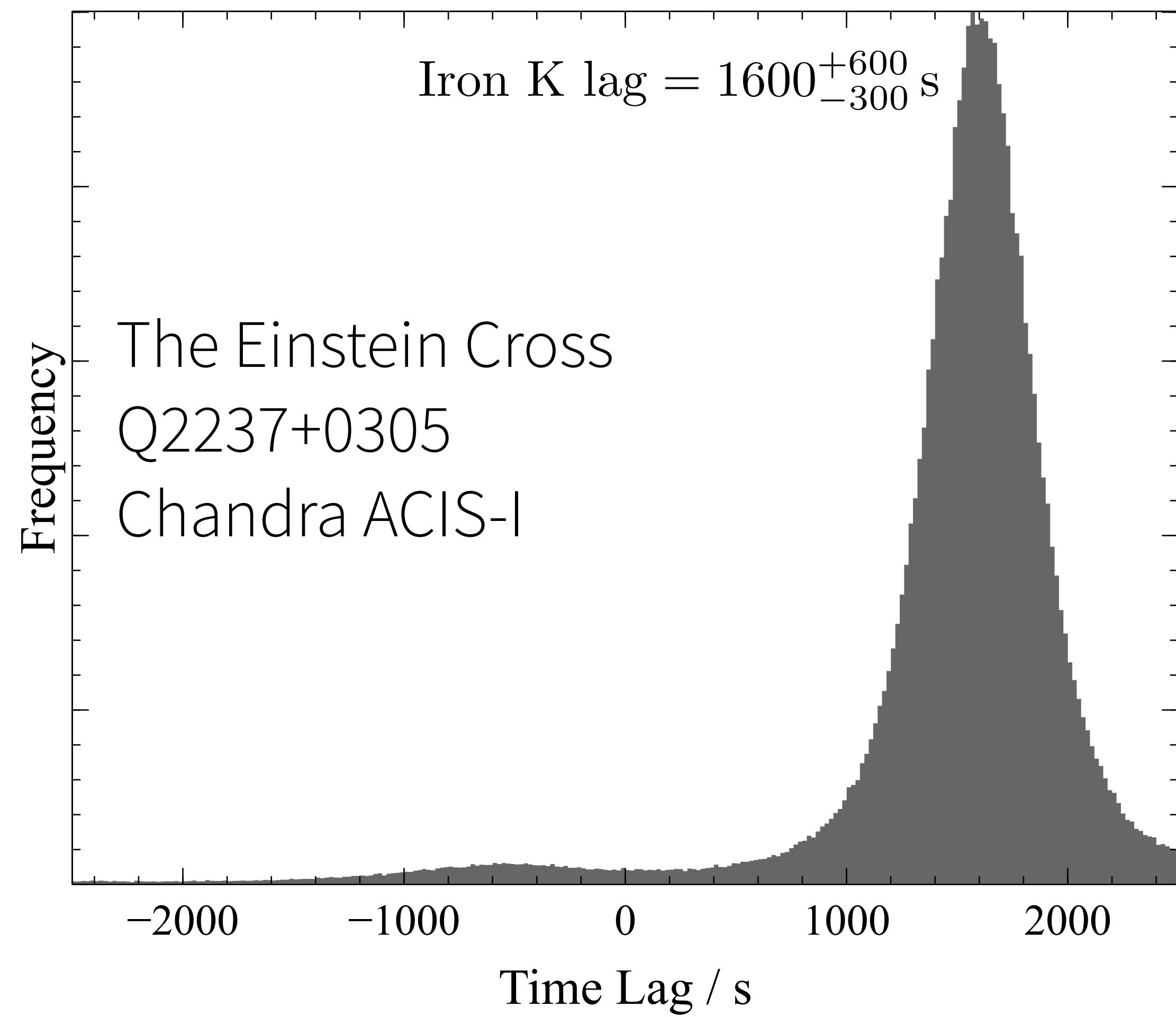


Prediction from spectral model

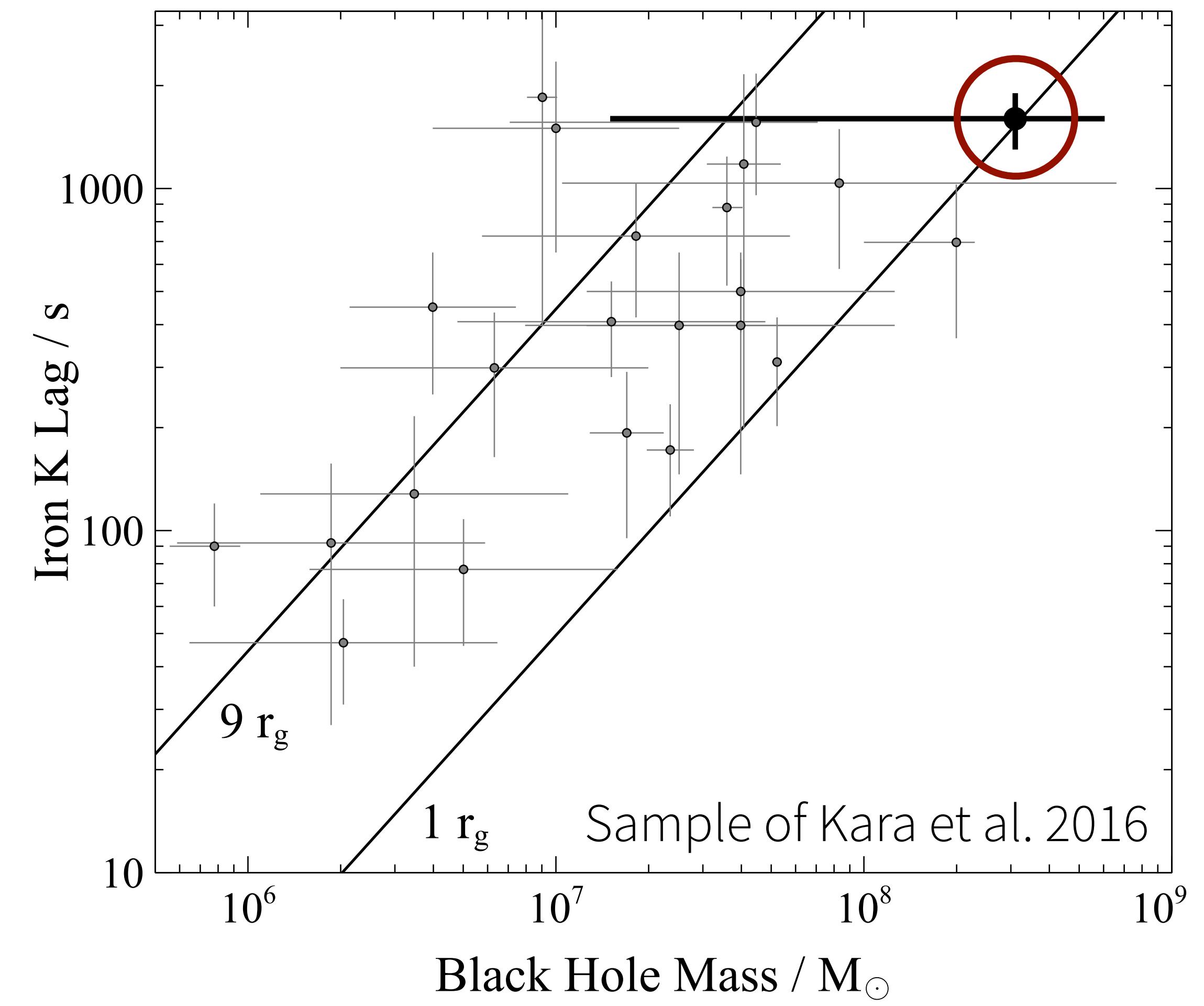


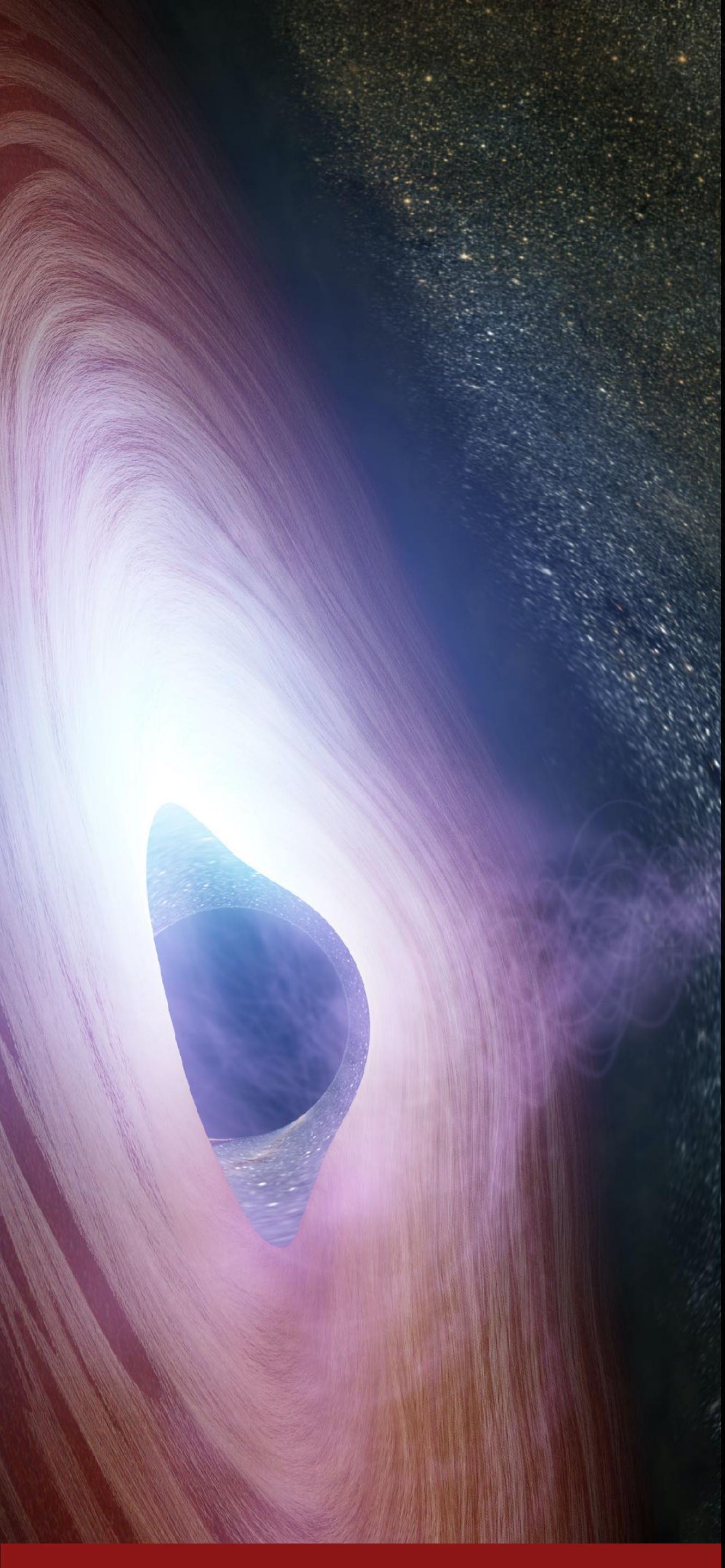
# Reverberation in the Einstein Cross

Simultaneous fit to image 1 and 1/3



Comparison to lags in Seyfert galaxies



- 
- Statistical frameworks based on Gaussian processes extend measurements of X-ray reverberation beyond Seyfert galaxies in the local Universe
  - Gaussian processes probe longer reverberation time scales around high mass black holes in radio galaxies
  - Obtain posterior distributions of time lags in low signal-to-noise observations of gravitationally lensed quasars
  - Leverage large collecting area, high spatial resolution X-ray missions (Lynx) to probe accretion physics onto growing black holes in high(er)-z quasars