Quasi-Oracle Estimation of Heterogeneous Treatment Effects

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Abstract
We propose R-learning for heterogeneous treatment effect estimation in observational studies. Assume we have a non-parametric treatment effect model:

$$ Y = f(X) + \tau(X)W + \epsilon, $$

where $$ f $$ is unknown. Recall from Robinson's Transformation:

$$ Y - m(X) = \tau(X) \cdot (W - e(X)) + \epsilon. $$

This suggests a natural oracle learner:

$$ \hat{\tau}(\cdot) = \arg\min_{\tau} \frac{1}{n} \sum_{i=1}^{n} (Y_i - m(X_i)) - (W_i - e(X_i)) \tau(X_i) \right)^2 + \lambda_n(\hat{\tau}(\cdot)). $$

Q: What about the plug-in version with $$ \hat{m}(\cdot) $$ ? Overfitting?

The Oracle Regret Bound (Mendelson and Neeman, 2010)

$$ \hat{\tau}(\cdot) = \arg\min_{\tilde{\tau}} \frac{1}{n} \sum_{i=1}^{n} (Y_i - m(X_i)) - (W_i - e(X_i)) \tilde{\tau}(X_i) \right)^2 + \lambda_n(\tilde{\tau}(\cdot)). $$

The Quasi-Oracle Regret Bound (in the case of regularized regression in RKHS)

The Oracle Regret Bound (Mendelson and Neeman, 2010)

$$ \hat{\tau}(\cdot) = \arg\min_{\tilde{\tau}} \frac{1}{n} \sum_{i=1}^{n} (Y_i - m(X_i)) - (W_i - e(X_i)) \tilde{\tau}(X_i) \right)^2 + \lambda_n(\tilde{\tau}(\cdot)). $$

The Quasi-Oracle Regret Bound

Theorem. (Nie and Wager, 2018) Suppose that

- Nuisance components $$ \hat{m}(\cdot) $$ and $$ \hat{e}(\cdot) $$ are $$ o(n^{-1/4}) $$-consistent
- The smoothness parameter is bounded by $$ 2\alpha \leq 1 $$

Then, the minimizer of the regularized plug-in loss satisfies the same regret bound.

Setup A

Difficult nuisance component $$ m^*(\cdot) $$ and $$ e^*(\cdot) $$, easy HTE function $$ \tau^* $$

$$ X_i \sim Unif(0,1), e^*(X_i) = \sin(\pi X_i), \tau^*(X_i) = 1/(1 + e^{x_i^2}). $$

Setup B

Randomized Trial

$$ X_i \sim N(0,1), e(X_i) = 0.5, \tau(X_i) = 1 $$

Setup C

Easy propensity score $$ e^*(\cdot) $$, difficult baseline $$ b^*(\cdot) $$

$$ X_i \sim N(0,1), e(X_i) = 1/(1 + e^{0.5X_i^2}), b^*(X_i) = 2 \log(1 + e^{0.5X_i^2}), \tau^*(X_i) = 1 $$

Setup D

Unrelated treatment and control arms

$$ X_i \sim N(0,1), e(X_i) = 0.5, \tau^*(X_i) = 1 $$

Simulation (with boosting)

$$ X_i \sim P(W_i | X_i \sim Bernoulli(0.5)), e(X_i) = 0.5 $$

Our proposal: R-learning

We assume a non-parametric treatment effect model:

$$ Y_i = f(X_i) + \tau(X_i)W_i + \epsilon_i, $$

where $$ f $$ is an unknown function. Recall from Robinson's Transformation:

$$ Y_i - m(\cdot) = \tau(X_i) \cdot (W_i - e(X_i)) + \epsilon_i. $$

This suggests a natural oracle learner:

$$ \hat{\tau}(\cdot) = \arg\min_{\tau} \frac{1}{n} \sum_{i=1}^{n} (Y_i - m(X_i)) - (W_i - e(X_i)) \tau(X_i) \right)^2 + \lambda_n(\tau(\cdot)). $$

Q: What about the plug-in version with $$ \hat{m}(\cdot) $$ ? Overfitting!

The R-learning framework:

1. Fit $$ \hat{m}(\cdot) $$ and $$ \hat{e}(\cdot) $$ via any black-box supervised learning for high predictive accuracy

2. Estimate treatment effects via a cross-fit estimator:

$$ \tilde{\tau}(\cdot) = \arg\min_{\tilde{\tau}} \frac{1}{n} \sum_{i=1}^{n} (Y_i - m^*(X_i)) - (W_i - e^*(X_i)) \tilde{\tau}(X_i) \right)^2 + \lambda_n(\tilde{\tau}(\cdot)). $$

The Quasi-Oracle Estimation

We propose R-learning for heterogeneous treatment effect estimation in observational studies.

Existing Literature

There are promising methods for this problem, based on BART (Chipman et al., 2010; Hill 2011); Boosting (Powell et al., 2017); Deep nets (Hartford et al., 2017; Shalit et al., 2017); Lasso (Jain and Rakhovski, 2013); Forests (Wager and Athey, 2018), Trees (Attey and Imbens, 2016; Su et al., 2009).

Q: Can we get something general that works with blackbox learners without per instance twiddling? YES!

Existing black-box model based approaches

- S-learning: $$ \mu(x,w):=E[Y_{1|w}|X=x,W=w] $$
- X-learning: $$ \nu(x,w):=E[Y_{0|w}|X=x,W=w] $$
- T-learning: $$ \nu(x,w):=E[Y_{0|w}|X=x,W=w] $$

Our inspiration: Robinson’s Transformation (1988)

Assume we have a partially linear treatment effect model,

$$ Y = f(X) + \tau W + \epsilon, $$

Rearrange, $$ Y - m(X) = \tau(W - e(X)) + \epsilon. $$

The induced estimator is OLS (Robinson, 1988).

$$ \hat{\tau}(\cdot) = \arg\min_{\tau} \frac{1}{n} \sum_{i=1}^{n} (Y_i - m(X_i)) - (W_i - e(X_i)) \tau(X_i) \right)^2 + \lambda_n(\tau(X_i)). $$

We extend this idea to non-parametric settings. * See also Chernozhukov et al. (2017), Zhao, Periafa, and Small (2017), Athey, Tibshirani, and Wager (2016)