Package ‘tsvd’

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Type Package

Title Thresholding-based SVD for multivariate reduced rank regression

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Author Xin Ma, Luo Xiao, and Wing H. Wong*

Maintainer Xin Ma <xm24@stanford.edu>

Description This package performs multivariate reduced rank regression with a sparse singular value decomposition on the coefficient matrix (T-SVD). Sparsity is achieved through iterative (hard) thresholding on the left and right singular vectors.

Depends MASS, glmnet

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Thresholding-based SVD for multivariate reduced rank regression.

The development of microarray and next-generation sequencing technologies has enabled rapid quantification of various genome-wide features (genomic sequences, gene expressions, noncoding RNA expressions, methylation etc.) in a population of samples. Large consortiums have compiled genetic and molecular profiling data in an enormous number of tumors samples.

There are some key characteristics in these regulatory relationships. First, multiple pathways are involved and can be viewed as independent programs of regulation. Second, combinatorial nature of the regulation in each program is likely to be sparse. Third, the number of potential predictors and responses often far exceed the sample size.

To address these challenges, we develop this computationally efficient multivariate response regression model T-SVD.

**Details**

This package performs multivariate reduced rank regression with a sparse SVD on the coefficient matrix. The model is

\[ Y = XC + E, \]

where \( C \) is an unknown coefficient matrix. Let \( UDV^T \) be the singular value decomposition (SVD) of \( C \). The method in this package intends to obtain sparse estimates of \( U \) and \( V \).

**Author(s)**

Xin Ma, Luo Xiao, Wing H. Wong*

**References**


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**MatrixMultiply**

**Matrices multiplication**

**Description**

This function gives the multiplication result of two matrices.

**Usage**

MatrixMultiply(A, B)

**Arguments**

A | i by j matrix
---|
B | j by k matrix
The multiplication result

Xin Ma, Luo Xiao, Wing H. Wong*


Y <- matrix(rnorm(3), 1, 3)
X <- matrix(rnorm(15), 3, 5)
product <- MatrixMultiply(Y, X)

select_rank(Y, X, control = list(sigma2 = NULL, constant = 4, abs.tol = 10^-7, rel.tol = 0.0001))

This function selects the rank of the coefficient matrix for multivariate response regression using the approach proposed by Bunea, She and Wegkamp (2011). Specifically, let $P = X(X^TX)^{-1}X^T$, where the generalized inverse is the Moore-Penrose pseudoinverse, and $\lambda_k(Y^TPY)$ be the $k$-th eigenvalue of $Y^TPY$. Then the selected rank is

$$K = \max\{k : \lambda_k(Y^TPY) > c \max(q, n) \sigma^2\},$$

where $c$ is the constant and $\sigma$ is the noise level of the data, estimated by $1.4826 \text{MAD}(\text{abs}(Y))$.

Y <- matrix(rnorm(3), 1, 3)
X <- matrix(rnorm(15), 3, 5)
product <- MatrixMultiply(Y, X)

select_rank(Y, X, control = list(sigma2 = NULL, constant = 4, abs.tol = 10^-7, rel.tol = 0.0001))

The selected rank

Xin Ma, Luo Xiao, Wing H. Wong*
**soda**  
*Sparse Orthogonal Decomposition Algorithm*

**Description**
This function is similar to the QR decomposition of a numeric matrix but provides potentially more sparse singular vectors if the input matrix is sparse. If the input has no sparsity, then one gets the same result as the QR decomposition.

**Usage**
soda(x)

**Arguments**
- **x**
  a numeric matrix whose left singular vectors are to be computed

**Value**
- **Q**
  left singular vectors with zero rows removed
- **R**
  R matrix as in the QR decomposition
- **index**
  indices of the rows of **Q**
- **d**
  singular values of *x*

**Author(s)**
Xin Ma, Luo Xiao, Wing H. Wong*

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**References**


**Examples**

```
n <- 100
p <- 200
q <- 100
rank <- 3
X <- matrix(rnorm(n*p),n,p)
U <- matrix(rnorm(p*rank),p,rank)
V <- matrix(rnorm(q*rank),q,rank)
D <- diag(rnorm(rank))
C <- U%*%D%*%t(V)
Y <- X%*%C + matrix(rnorm(n*q),n,q)
select_rank(Y,X)
```
References


See Also

qr

Examples

```r
## Example 1: no sparsity in the data
## same as the QR decomposition
x <- matrix(rnorm(200),40,5)
Q <- qr.Q(qr(x))
Q1 <- soda(x)$Q
f <- function(x,y){ min(sum((x-y)^2),sum((x+y)^2))}
diff <- 0
for(r in 1:5) diff <- diff + f(Q[,r],Q1[,r])
print(diff)

## Example 2: sparse data
## different from the QR decomposition
x <- matrix(rnorm(900),30,3)
x[1:10,2] <- 0
x[1:20,3] <- 0
Q <- qr.Q(qr(x))
Q1 <- soda(x)$Q
print(cbind(x,Q,Q1),digits=3)
```

Description

This function performs (hard) thresholding-based SVD for multivariate reduced rank regression proposed by Ma, Xiao and Wong (2014).

Usage

```r
tsvd(Y, X, rank, ini, BICtype = 2, thr = NULL,
    control = list(thr.lower = -1, thr.upper = -1, thr.by = 0.5))
```

Arguments

- `Y`: n by q response matrix
- `X`: n by p design matrix
- `rank`: rank of the coefficient matrix
- `ini`: initial estimate of the coefficient matrix
- `BICtype`: two BIC types
- `thr`: a pair of tuning parameters or a 2-column matrix of tuning parameters to search over
- `control`: a list of arguments for defining the sequence of tuning parameters; see details
Details

There are two types of BICs that can be used. BICtype 1:

$$\log(SSE) + \log(q \cdot n)/(q \cdot n)df,$$

where $df$ is the total number of non-zero entries in $U, V$ and $D$, subtracted by squared rank. BIC type 2:

$$\log(SSE) + \log(q \cdot n)/(q \cdot n)df_v + \log(r \cdot n)/(r \cdot n)df_u,$$

where $r$ is the rank, $df_v$ is the number of non-zero entries in $V$, subtracted by $r \cdot (r - 1)/2$, and $df_u$ is the number of non-zero entries in $U$, subtracted by $r \cdot (r - 1)/2$. The second BIC type performs better for variable selection on $U$.

Value

- est estimate of the coefficient matrix in “SparseSVD” structure
- C estimate of the coefficient matrix
- thr selected tuning parameters
- BICtype BICtype used
- BIC value of BIC

Author(s)

Xin Ma, Luo Xiao, Wing H. Wong

References


Examples

```r
n <- 100
p <- 150
q <- 150

## generate a sparse coefficient matrix
inorm <- function(a, l=1) (sum(abs(a)^2))^(1/2) #normaling function
U <- matrix(ncol= 3,nrow= p);
V <- matrix(ncol= 3,nrow= q);
V[,1] <- c(sample(c(-1,1),5,replace= TRUE),rep(0,20));
V[,2] <- c(rep(0,12),sample(c(-1,1),8,replace= TRUE),rep(0 ,5));
V[,3] <- c(rep(0,6),V[1:2,1],-V[3:4,1],sample(c(-1,1),2,replace= TRUE),
 -V[15:16,2],V[13:14,2],rep(0,9))
V[,1] <- V[,1]/inorm(V[,1],2);
V[,2] <- V[,2]/inorm(V[,2],2);
V[,3] <- V[,3]/inorm(V[,3],2);

U[,1] <- c(sample(c(1,-1),5,replace= TRUE)*runif(5,0.7,1),rep(0,20));
U[,2] <- c(rep(0,5),sample(c(1,-1),5,replace= TRUE)*runif(5,0.7,1),rep(0,15));
U[,3] <- c(rep(0,10),sample(c(1,-1),5,replace= TRUE)*runif(5,0.7,1),rep(0,10));
U[,1] <- U[,1]/inorm(U[,1],2);
U[,2] <- U[,2]/inorm(U[,2],2);
```
An initial estimator for multivariate reduced rank regression

This function provides an initial estimate for multivariate reduced rank regression.

**Usage**

```r
tsvd_ini(Y, X, rank, ini = NULL, method = 1)
```

**Arguments**

- `Y` n by q response matrix
- `X` n by p design matrix
- `rank` rank of the estimate
- `ini` an initial estimate of the coefficient matrix in "SparseSVD" structure
- `method` method = 1: rank-truncated ridge regression estimate; method = 2: assuming V is known from ini, estimate U by lasso (implemented by `glmnet`) for which the tuning parameter is selected by BIC.

**Value**

- `rank` rank
- `u` left singular vectors with zero rows removed
- `v` right singular vectors with zero rows removed
- `d` singular values
- `u.index` indices of rows of `u`
- `v.index` indices of rows of `v`

**Author(s)**

Xin Ma, Luo Xiao, Wing H. Wong*
References


Examples

n <- 100
p <- 150
q <- 150

## generate a sparse coefficient matrix
lnorm <- function(a,l=1) (sum(abs(a)^2))^(1/2) #normaling function
U <- matrix(ncol= 3,nrow= p);
V <- matrix(ncol= 3,nrow= q);
V[,1] <- c(sample(c(-1,1),5,replace= TRUE),rep(0,20));
V[,2] <- c(rep(0,12),sample(c(-1,1),8,replace= TRUE),rep(0,5));
V[,3] <- c(rep(0,6),V[1:2,1],-V[3:4,1],sample(c(-1,1),2,replace= TRUE),
        -V[15:16,2],V[13:14,2],rep(0,9))
V[,1] <- V[,1]/lnorm(V[,1],2);
V[,2] <- V[,2]/lnorm(V[,2],2);
V[,3] <- V[,3]/lnorm(V[,3],2);

U[,1] <- c(sample(c(1,-1),5,replace= TRUE)*runif(5,.7,1),rep(0,20));
U[,2] <- c(rep(0,5),sample(c(1,-1),5,replace= TRUE)*runif(5,.7,1),rep(0,15));
U[,3] <- c(rep(0,10),sample(c(1,-1),5,replace= TRUE)*runif(5,.7,1),rep(0,10));
U[,1] <- U[,1]/lnorm(U[,1],2);
U[,2] <- U[,2]/lnorm(U[,2],2);
U[,3] <- U[,3]/lnorm(U[,3],2);
D <- diag(c(5,1,5));

C <- U%*%D%*%t(V);
## generate data
X <- matrix(rnorm(n*p),n,p)
E <- matrix(rnorm(n*q),n,q)
sigma2 <- sum(diag(t(C)%*%C))/(n*q) ## so that the signal to ##noise ratio is 1
Y<-X%*%C+ sqrt(sigma2)*E

## obtain initial estimate
ini <- tsvd_ini(Y,X,rank=3)

---

**tsvd_wrapper**

Thresholding-based SVD for multivariate reduced rank regression

**Description**

This function is a wrapper for performing (hard) thresholding-based SVD for multivariate reduced rank regression proposed by Ma, Xiao and Wong (2014).
tsvd_wrapper

Usage

tsvd_wrapper(Y, X, rank=NULL, BICtype = 2, thr = NULL,
control = list(thr.lower = -10, thr.upper = -1, thr.by = 0.5),
rank.control = list(sigma2 = NULL, constant = 4,abs.tol = 10^(-7), rel.tol = 1e-04))

Arguments

Y  n by q response matrix
X  n by p design matrix
rank rank of the coefficient matrix; if NULL, estimated by select_rank.
BICtype two BIC types
thr a pair of tuning parameters or a 2-column matrix of tuning parameters to search over
control a list of arguments for defining the sequence of tuning parameters; see details
rank.control a list of arguments for rank selection; see select_rank

Details

The model is

\[ Y = XC + E, \]

where \( C \) is an unknown coefficient matrix. Let \( UDV^T \) be the singular value decomposition (SVD) of \( C \). The method in this package intends to obtain sparse estimates of \( U \) and \( V \).

If \( rank \) is NULL, an estimate is obtained from \( \text{select\_rank} \).

The wrapper has four steps: (1) obtain an initial estimate from rank-truncated ridge regression; (2) run \( \text{tsvd} \); (3) with \( V \) from step (2), obtain another estimate of \( U \) and \( D \) from lasso; (4) run \( \text{tsvd} \).

There are two types of BICs that can be used. BICtype 1:

\[
\log(\text{SSE}) + \log(q \ast n)/(q \ast n)df,
\]

where \( df \) is the total number of non-zero entries in \( U,V \) and \( D \), subtracted by squared rank. BIC type 2:

\[
\log(\text{SSE}) + \log(q \ast n)/(q \ast n)df_v + \log(r \ast n)/(r \ast n)df_u,
\]

where \( r \) is the rank, \( df_v \) is the number of non-zero entries in \( V \), subtracted by \( r \ast (r - 1)/2 \), and \( df_u \) is the number of non-zero entries in \( U \), subtracted by \( r \ast (r - 1)/2 \). The second BIC type performs better for variable selection on \( U \).

Value

est estimate of the coefficient matrix in “SparseSVD” structure
C estimate of the coefficient matrix
thr selected tuning parameters
BICtype BICtype used
BIC value of BIC

Author(s)

Xin Ma, Luo Xiao, Wing H. Wong*
References


Examples

```r
n <- 100
p <- 150
q <- 150

## generate a sparse coefficient matrix
lnorm <- function(a,l=1) (sum(abs(a)^2))^(1/2) #normaling function
U <- matrix(ncol= 3,nrow= p);
V <- matrix(ncol= 3,nrow= q);
V[,1] <- c(sample(c(-1,1),5,replace= TRUE),rep(0,20));
V[,2] <- c(rep(0,12),sample(c(-1,1),8,replace= TRUE),rep(0,5));
V[,3] <- c(rep(0,6),V[1:2,1],-V[3:4,1],sample(c(-1,1),2,replace= TRUE),
           -V[15:16,2],V[13:14,2],rep(0,9))
V[,1] <- V[,1]/lnorm(V[,1],2);
V[,2] <- V[,2]/lnorm(V[,2],2);
V[,3] <- V[,3]/lnorm(V[,3],2);

U[,1] <- c(sample(c(1,-1),5,replace= TRUE)*runif(5,0.7,1),rep(0,20));
U[,2] <- c(rep(0,5),sample(c(1,-1),5,replace= TRUE)*runif(5,0.7,1),rep(0,15));
U[,3] <- c(rep(0,10),sample(c(1,-1),5,replace= TRUE)*runif(5,0.7,1),rep(0,10));
U[,1] <- U[,1]/lnorm(U[,1],2);
U[,2] <- U[,2]/lnorm(U[,2],2);
U[,3] <- U[,3]/lnorm(U[,3],2);
D <- diag(c(20,10,5));

C <- UX*%*%t(V);

## generate data
X <- matrix(rnorm(n*p),n,p)
E <- matrix(rnorm(n*q),n,q)
sigma2 <- sum(diag(t(C)*%*%C))/(n*q) # so that the signal to noise ratio is 1
Y<-X*%*%C+ sqrt(sigma2)*E

## obtain estimate
ans <- tsvd_wrapper(Y,X,BICtype=2) #rank is estimated
print(sum((ans$C-C)^2))
```
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