Note: the problems “Lesson X, problem Y” are all from the textbook.

(1) (10 pt) Lesson 1, problem 1
(2) (10 pt) Lesson 1, problem 3
(3) (10 pt) Lesson 1, problem 5
(4) (20 pt) The Maxwell equations for electromagnetic fields are given by the following four equations:

\[
\begin{align*}
\nabla \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \quad \text{(Gauss's law)} \\
\nabla \cdot \vec{B} &= 0 \quad \text{(Gauss's law for magnetism)} \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law of induction)} \\
\nabla \times \vec{B} &= \mu_0 (\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) \quad \text{(Ampère's circuital law)}
\end{align*}
\]

where

- The electric field \( \vec{E} = (E_1(t, x, y, z), E_2(t, x, y, z), E_3(t, x, y, z)) \) is a vector that changes with time \( t \) and position \( (x, y, z) \)
- The magnetic field \( \vec{B} = (B_1(t, x, y, z), B_2(t, x, y, z), B_3(t, x, y, z)) \) is defined similarly
- The total electric charge density \( \rho = \rho(t, x, y, z) \) is a given function
- The total electric current density \( \vec{J} \) is a given vector
- \( \epsilon_0 \) and \( \mu_0 \) are universal constants
- \( \nabla \) is the gradient operator for 3D space; the \( \nabla \cdot \) symbol denotes the divergence operator, and the \( \nabla \times \) symbol denotes the curl operator.

We will only focus on the first two equations, which written out in coordinates are given by

\[
\begin{align*}
\partial_x E_1 + \partial_y E_2 + \partial_z E_3 &= \frac{\rho}{\epsilon_0} \quad \text{(Gauss's law)} \\
\partial_x B_1 + \partial_y B_2 + \partial_z B_3 &= 0 \quad \text{(Gauss's law for magnetism)}
\end{align*}
\]

Answer the following questions:

(a) Give the classification of the first equation in (2) based on chart 1.1 in the textbook. (Hint: even though we have more than one unknown functions, we can think of \( (E_1, E_2, E_3) \) as ONE unknown function \( \vec{E}(t, x, y, z) \).)

(b) Check if the following \( \vec{B} \) solves the second equation in (2):

\[
\vec{B} = (x^2 - 2xz + t, -2xy + t, z^2 + t^2).
\]

(5) (10 pt) Lesson 2, problem 1
(6) (10 pt) Lesson 2, problem 3
(7) (10 pt) Lesson 2, problem 4
(8) (10 pt) Lesson 3, problem 1
(9) (10 pt) Lesson 3, problem 2