(1) (10 pt) Lesson 17, problem 3
(2) (10 pt) Lesson 17, problem 4
(3) (10 pt) If both functions in the initial conditions \( f(x) \) and \( g(x) \) are odd functions,
i.e. \( f(-x) = -f(x) \), \( g(-x) = -g(x) \), show that the solution \( u(x, t) \) is also odd in \( x \), i.e. \( u(-x, t) = -u(x, t) \) for all \( t \).
(4) (20 pt) Consider the following initial value problem for the wave equation on \(-\infty < x < \infty, 0 < t < \infty:\)

\[
\begin{align*}
    u_{tt} &= c^2 u_{xx} \\
    u(0, x) &= f(x) \\
    u_t(0, x) &= g(x)
\end{align*}
\]

In class we solved this problem by using coordinate transformation and we got d’Alembert’s solution. In this problem, we will solve this problem using Fourier transform.

(a) Take Fourier transform in \( x \):

\[ U(\xi, t) = \mathcal{F}[u(x, t)] \]

Treat \( \xi \) as a parameter, transform the system into an ODE of \( U(t) \) with new initial conditions.

(b) Solve the second order ODE. Hint: your answer should be

\[ U(t, \xi) = \cos(ct|\xi|) F(\xi) + \frac{\sin(ct|\xi|)}{c|\xi|} G(\xi) \]

(c) Do inverse Fourier transform to get back \( u(x, t) \). Hint: you’ll need the inverse Fourier transform of

\[ W(t) = \cos(ct|\xi|), W'(t) = \frac{\sin(ct|\xi|)}{c|\xi|} \]

Use entries 8 and 10 in your inverse Fourier transform table.

(d) Does this solution agree with d’Alembert’s solution?

(5) (10 pt) Lesson 18, problem 1
(6) (20 pt) Lesson 18, problem 2. Note there is a typo in part (a), the correct version should be \( u(x, 0) = xe^{-x^2} \).
(7) (20 pt) Lesson 18, problem 3