Homework 6: due Wednesday, Feb. 20

• Section 7.7: 7.1, 7.8, 7.9, 7.14, 7.15

Additional problem:
(1) Let $F$ be a linear functional on a normed vector space $X$. Let $\text{Ker}(F)$ be the kernel of $F$. Show that $F$ is continuous if and only if $\text{Ker}(F)$ is closed.
(Hint: first show that if $F$ is unbounded, then there is a sequence $\{x_n\}_{n=1}^{\infty}$ such that $\|x_n\| \to 0$ as $n \to \infty$ and $F(x_n) = 1$. Then use linearity of $F$.)

(2) Let $f$ be a continuous linear functional on a subspace $Y$ of a Hilbert space $X$. Prove that $f$ has a unique norm-preserving extension to a continuous linear functional on $X$. 