A Dynamic Linear Programming Algorithm for Facilitated Charging and Discharging of Plug-In Electric Vehicles

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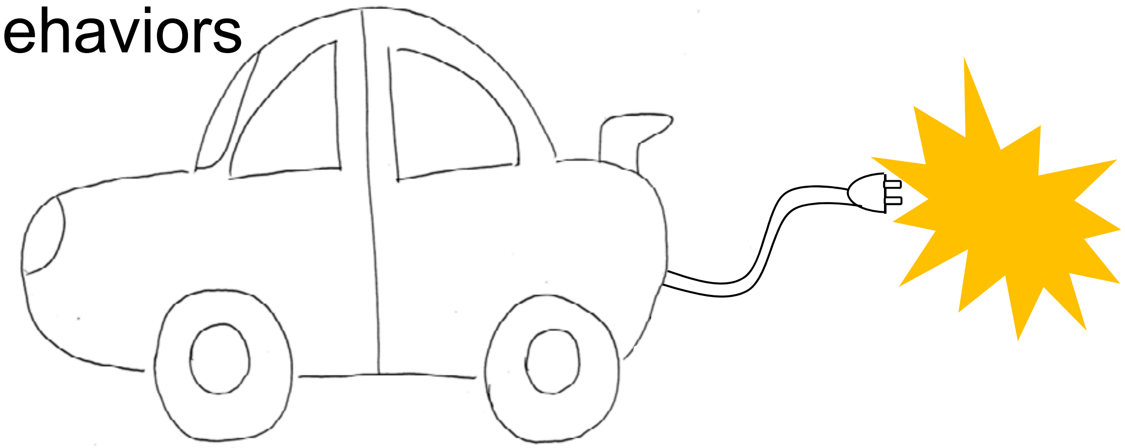
IBM, EPRI, and Stanford

Research supported by Precourt Energy Efficiency Center and Electric Power Research Institute

Outline

- Problem Motivation
- LP formulation and Shadow Prices
- Clustering Driving Behaviors
- Real Data
- Simulation Results
- Online Linear Programming Theory
- Future Work



Plug-In Electric Vehicle Network

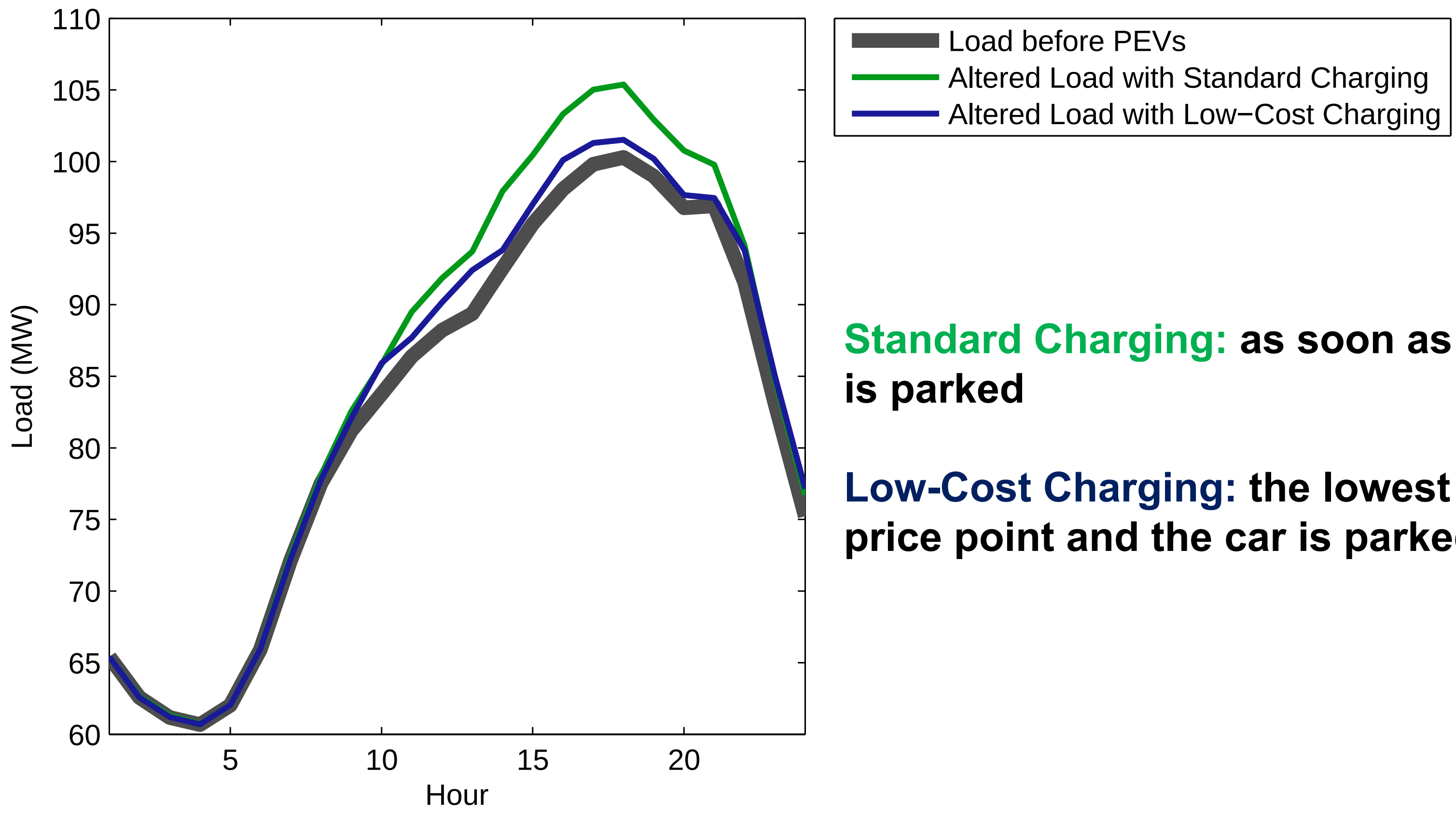
- Some estimates say there could be 100 million Plug-in Electric Vehicles (PEVs) on the road in the United States by 2030[1]
  - How will charging/discharging of PEVs add to the current load on the electricity grid?
  - Would smart management of these activities benefit both utility and consumer sides?

Motivated by these questions, we

- Construct a robust algorithm to dynamically assign low-cost, feasible, and satisfactory charging/discharging schedules for individual vehicles in a fleet
- Reduce the typical consumer cost of charging/discharging a PEV
- Lower the peak demand for electricity and benefit utility supplier to provide grid services

[1]EPRI PRISM Analysis, 2009

PEV Impact to the Electricity Grid



Standard Charging: as soon as the car is parked

Low-Cost Charging: the lowest PEG price point and the car is parked

A 30% penetration of Plug-in Electric Vehicles could impact the electricity grid.

Specific Problem Statement

The goal is to dynamically manage the charging/discharging of a fleet of PEVs so that:

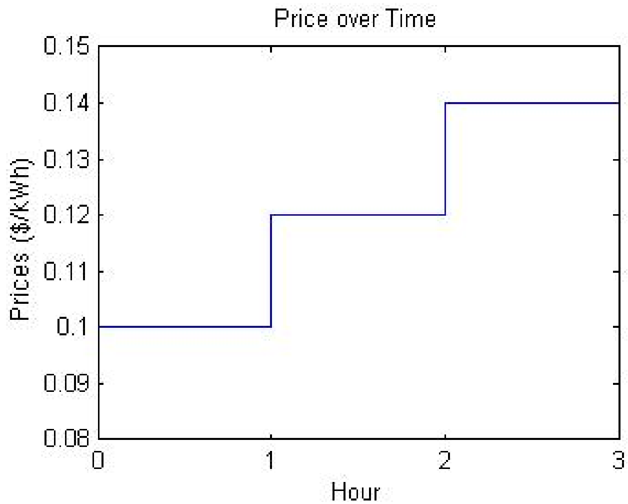
1. Every vehicle has enough energy in its battery to drive for a given period of time
2. The cost of charging is low
3. The peak electricity load does not increase and may even be reduced
4. The schedules are dynamic and robust to deal with uncertainty

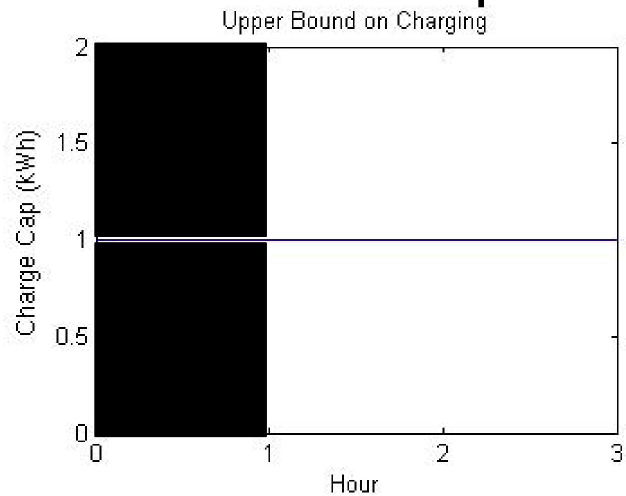
Using a linear program solution, one can make policy decisions about when to charge/discharge of every individual vehicle in a fleet based on:

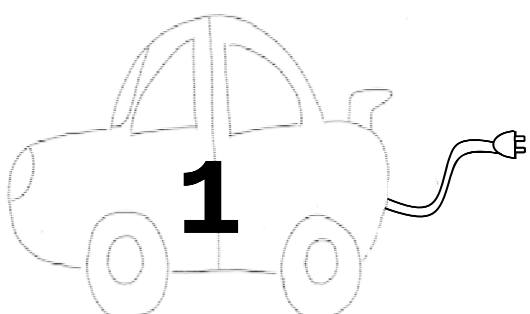
- Energy demand / time of each vehicle in a period
- Electricity load capacity and scheduling obligation
- Publicly available electricity and gasoline prices
- Individual vehicle characteristics / types

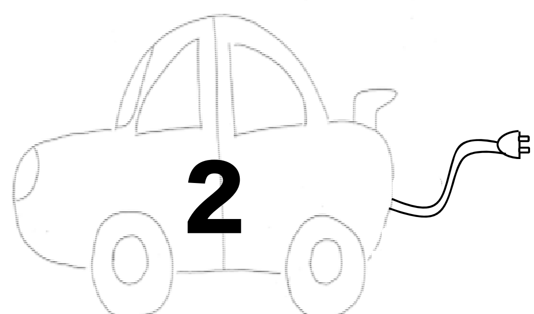
A Toy Example

There are two cars considered in a three hour period.









I need to charge1 kWh to drive in hour 3!

I need to charge 1 kWh to drive in hour 2!

LP Formulation Data

$\bar{p}$: electricity price vector at each hour

$\bar{p}^g$: gas price vector at each hour

$\bar{d}_{i,h}$: battery demand of car i at hour h

$\bar{L}_h$: basic electricity load

$s_{i,h}$: battery storage of car i at hour h

$s^g_{i,h}$: gasoline storage of car i at hour h

$c_{i,h}$: grid charge amount of car i at hour h

$f_{i,h}$: refuel amount of car i at hour h

$g_{i,h}$: gasoline charge amount of car i at hour h

c_{cap} : total grid charge cap

LP Formulation – Charging Only

If we had all the information ahead of time, we could solve a linear program to determine the optimal charging schedules:

$$\min \quad \sum_{i=1}^{k} (\bar{p}^T c_i + (\bar{p}^g)^T f_i) + \bar{\rho} c_{\text{cap}}$$

$$\text{s.t.} \quad s_{i,h} = s_{i,h-1} + \bar{c}_{eff} c_{i,h} + \bar{g}_{eff} g_{i,h} - \bar{d}_{i,h}, \forall i, h;$$

$$s^g_{i,h} = s^g_{i,h-1} + f_{i,h} - g_{i,h}, \qquad \forall i, h;$$

$$c_{i,h} = 0, f_{i,h} = 0, \text{or } g_{i,h} = 0, \qquad \forall \text{ some } i, h;$$

$$\bar{L}_h + \sum_{i=1}^{k} c_{i,h} - c_{\text{cap}} \le 0, \qquad \forall h;$$

lower and upper bound constraint

LP Formulation – Buy and Sell

Furthermore, if discharging is allowed, we could solve a linear program to determine the optimal charging schedules:

$$\min \quad \sum_{i=1}^{k} \left(\bar{p}^{T} (c_i - d_i) + (\bar{p}^{g})^{T} f_i \right) + \bar{\rho} c_{\text{cap}}$$

$$\text{s.t.} \quad s_{i,h} = s_{i,h-1} + \bar{c}_{eff} c_{i,h} - \bar{d}_{eff} d_{i,h} + \bar{g}_{eff} g_{i,h} - \bar{d}_{i,h}, \forall i, h;$$

$$s^{g}_{i,h} = s^{g}_{i,h-1} + f_{i,h} - g_{i,h}, \qquad \forall i, h;$$

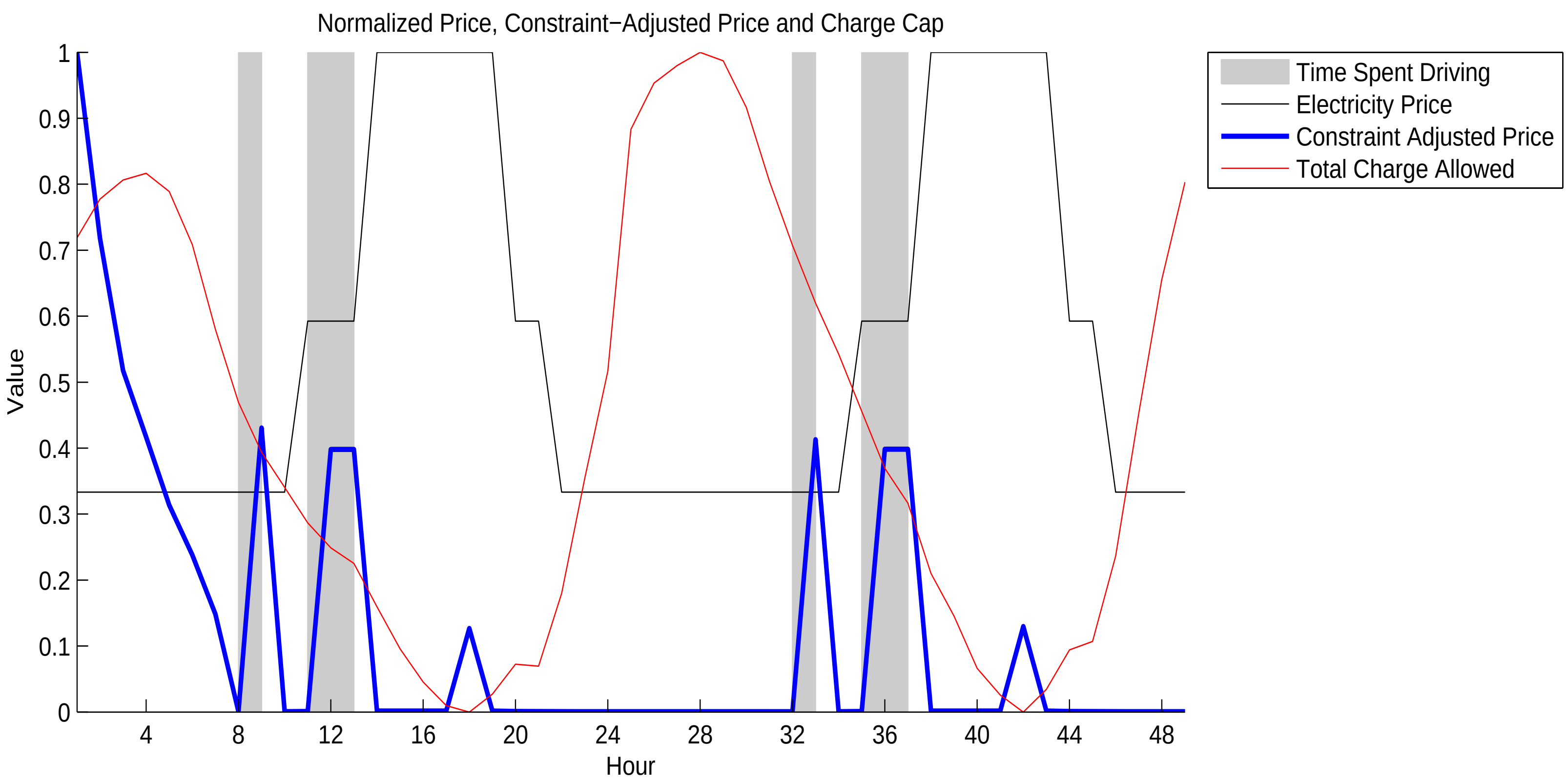
$$c_{i,h} = 0, f_{i,h} = 0, \text{or } g_{i,h} = 0, \qquad \forall \text{ some } \; i, h;$$

$$\bar{L}_h + \sum_{i=1}^{k} c_{i,h} - c_{\text{cap}} \leq 0, \qquad \forall h;$$

lower and upper bound constraint

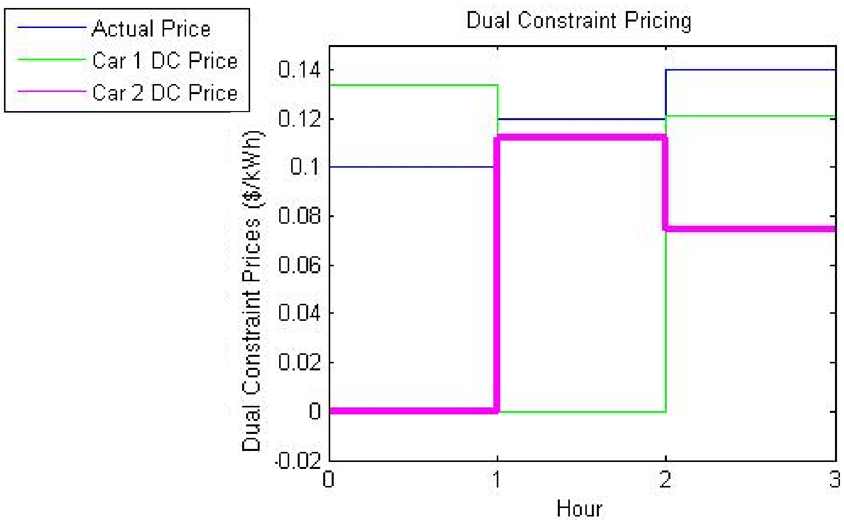
Shadow or Dual Pricing Mechanism

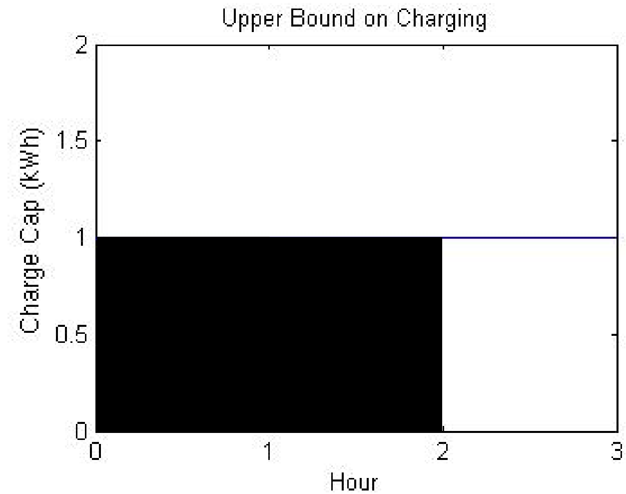
From the dual, one can construct a shadow price for each vehicle at each hour:

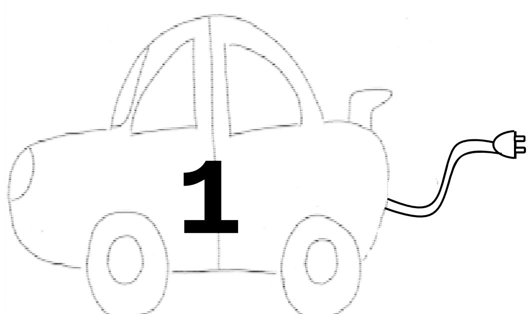


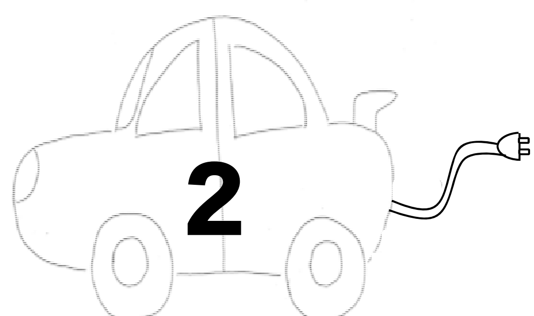
Shadow Price Two-Car Example

There are two cars considered in a three hour period.









I need to charge1 kWh to drive in hour 3! **I need to charge 1 kWh to drive in hour 2!**

Clustering

How could we assign a shadow price to each vehicle?

Clustering!

Say there are k types of driving patterns, and n_i drivers in each type, for $i = 1, \ldots, k$ Each type of driving pattern is classified as a *cluster*.

Then:

- Formulate a linear program, so that each cluster has its own dual constraint price.
- When a vehicle plugs in, determine its driving pattern and which cluster it belongs to
- Rank the dual constraint prices for this cluster
  - Charge the car when the dual constraint price is cheapest

Clustering Method

1. Format each transport load into the form:

$$[0, \; 0, \; 7, \; 10, \; 3, \; 0, \; \cdots, \; 0] \in \mathbb{R}^{24}$$

Not driving in these hours

Driving 7, 10 then 3 miles

Since we consider 24 hours

2. Normalized the vector

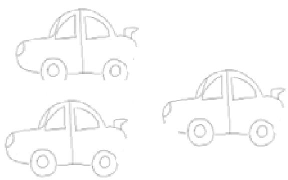
For example, the following two loads would have the same normalized differential:

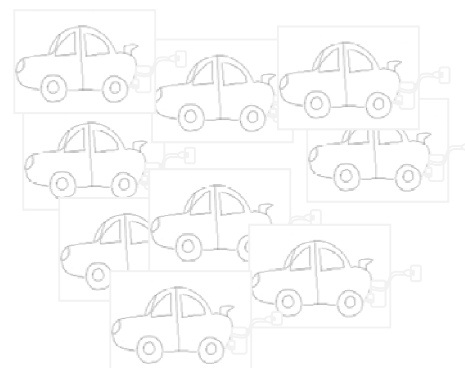
$$[0, \; 1, \; 1, \; 1, \; 0, \; 0, \; \cdots, \; 0]$$

$$[0, \; 10, \; 10, \; 10, \; 0, \; 0, \; \cdots, \; 0]$$

Clustering Method

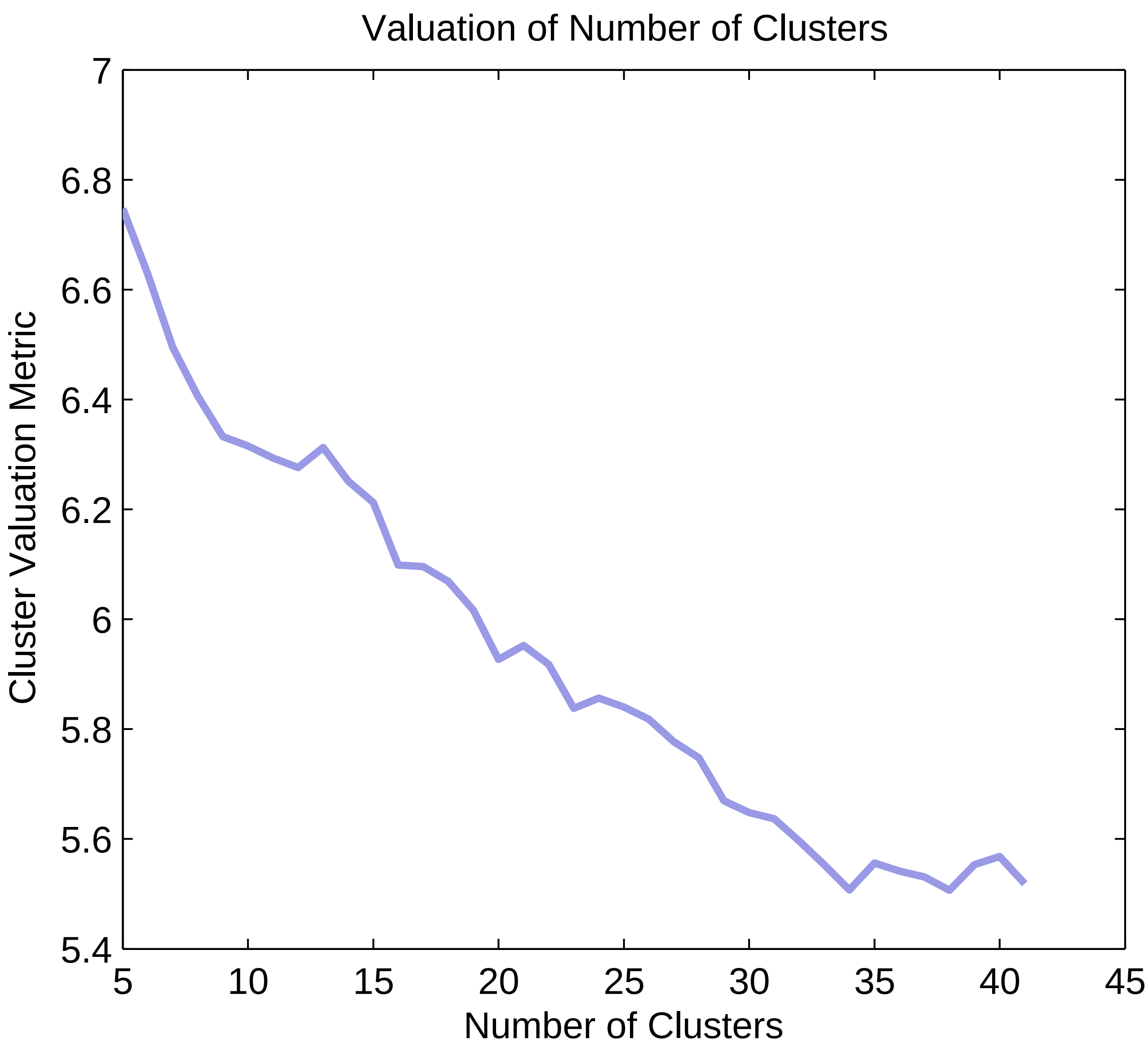
Basic Idea: Cars driving in the same hours, for the same *relative* amount should be in the same cluster





- Use the *k-means* clustering algorithm:
  - On the normalized load
  - With the Euclidean distance
- This algorithm attempts to put the vectors into clusters to minimize the average distance from the mean

Cluster Valuation



Test Based on Real Data

Vehicle Driving Behaviors

- Obtained from the 2009 National Household Transportation Survey (NHTS)
- Helpful discussions and filtered data from Morgan and Christine of EPRI
- The following results are based on data from urban California on a Monday

Electricity Load

- Obtained from CAISO OASIS (Open Access Same-Time Information System)
- Used the demand in the PG&E transmission access charge area for the week of August 22-28, 2011

Electricity and Gasoline Prices

- Electricity prices: PG&E baseline summer time of use rates
- Gasoline prices: mean gas price in the zip code 94305 on August 2011

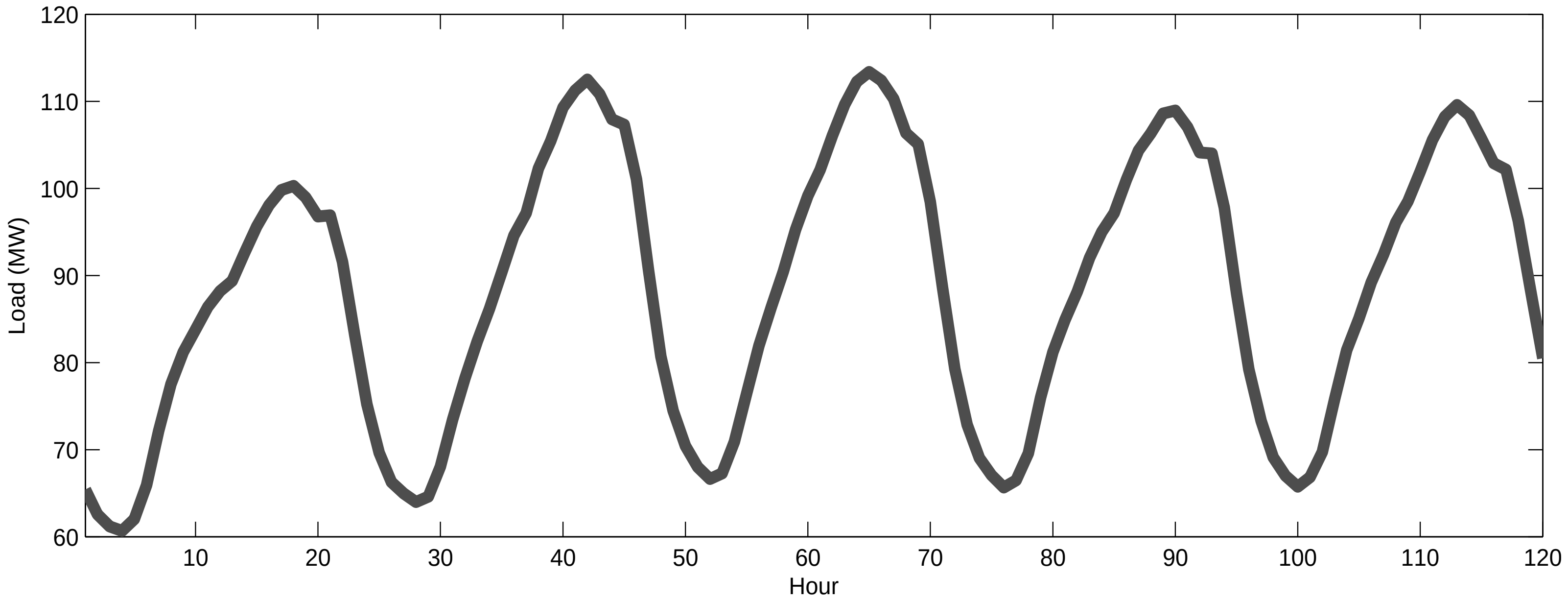
Data Characteristics

- **Vehicles**
  - Battery Electric Vehicle: 24 kWh battery (e.g., Nissan Leaf)
  - Hybrid Electric Vehicle: 16 kWh battery and 9.3 gallon fuel tank (e.g., Chevy Volt)

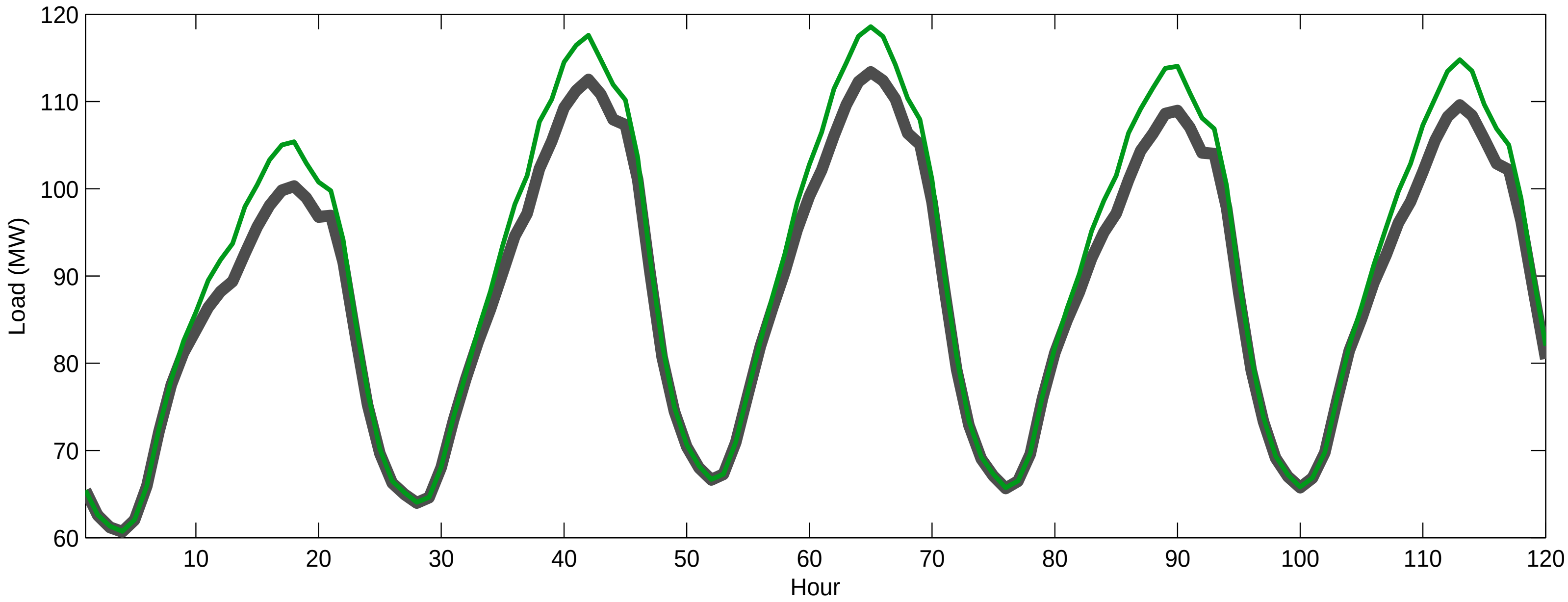
- Assign vehicles to drivers in our examples based on their daily driving patterns; each driver should be able to complete a day’s worth of driving with the assigned car
- Each car has the same:
  - Charge rate, 6.6 kW
  - Fueling rate,12*15 gallons (i.e., it takes 5 minutes to fuel 15 gallons)
  - Generation rate, 37 kW
- We simulate successive weekdays where drivers’ behavior is identical on each day.
- Assumption: each vehicle battery is full at the start of the first day, 30% penetration

Image Source: Cars.com

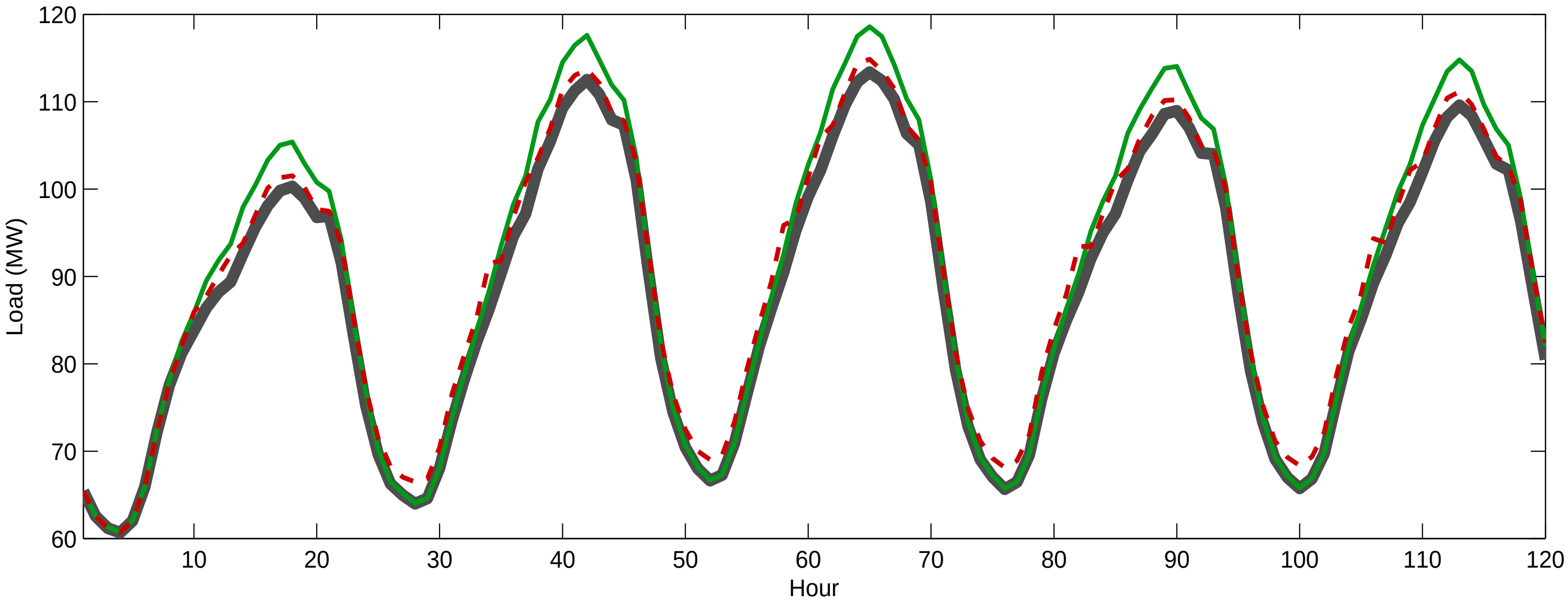
A Fleet of 10,000 Vehicles, 5 Days



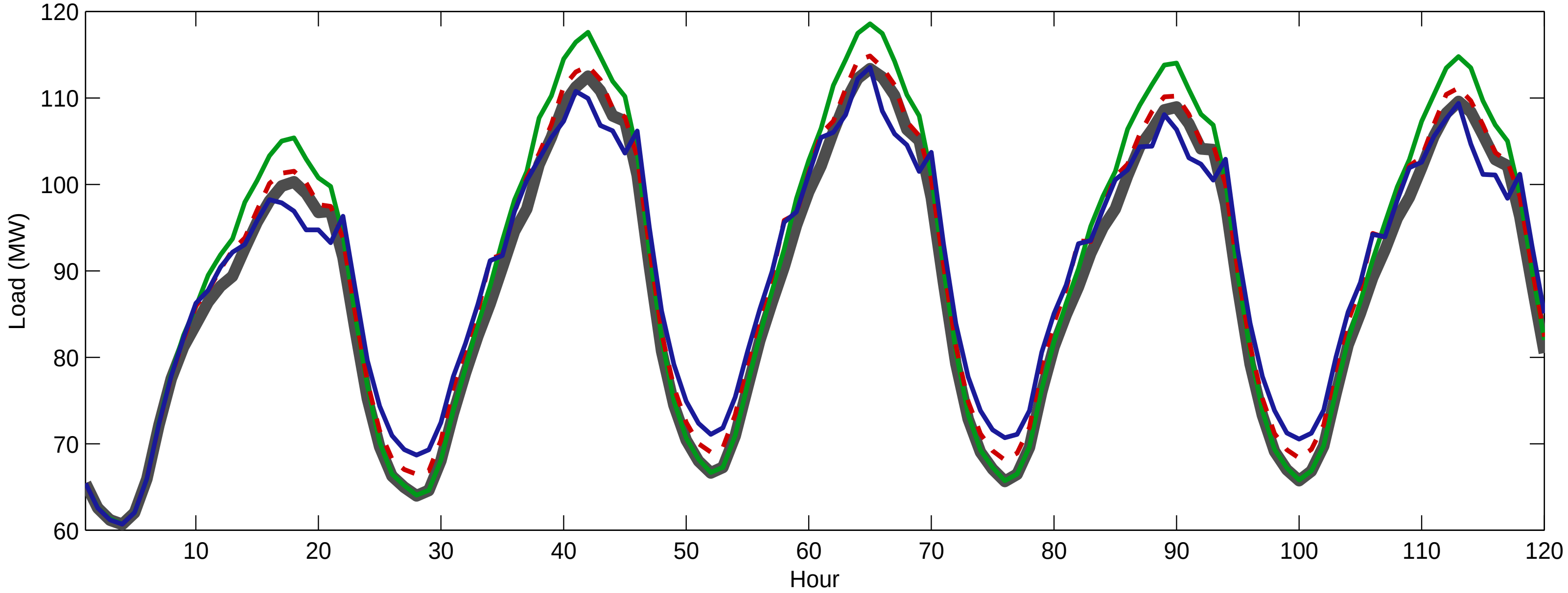
Standard Charging Policy



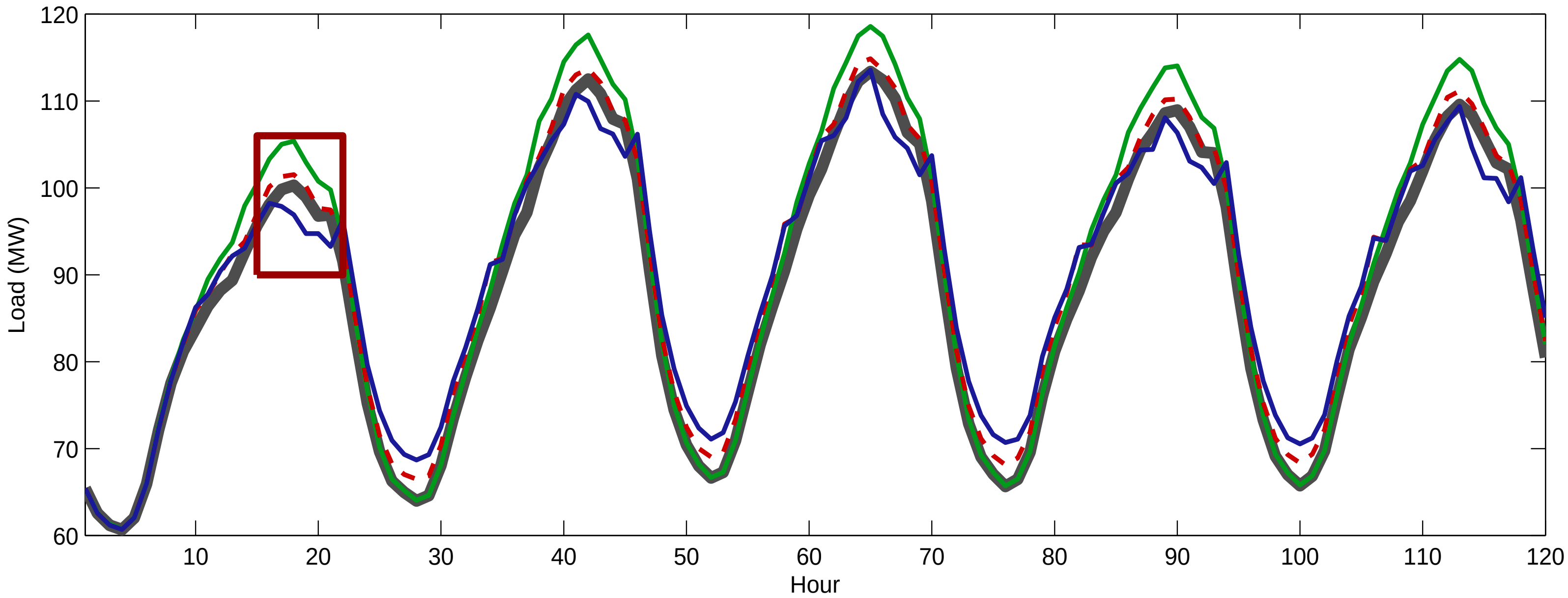
Lowest PGE Price Charging Policy



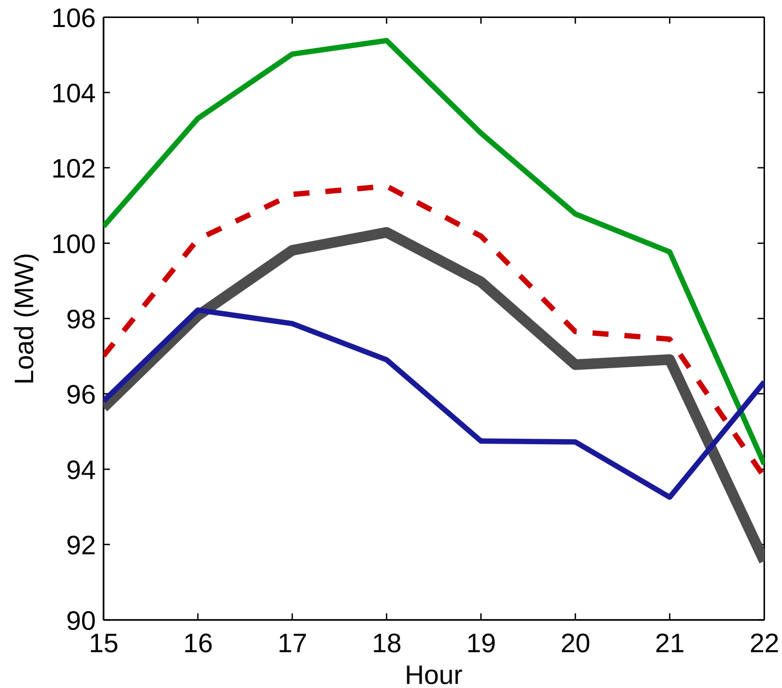
Lowest Shadow Price Charging Policy



Peak Reduction due to Discharging



Peak Reduction due to Discharging

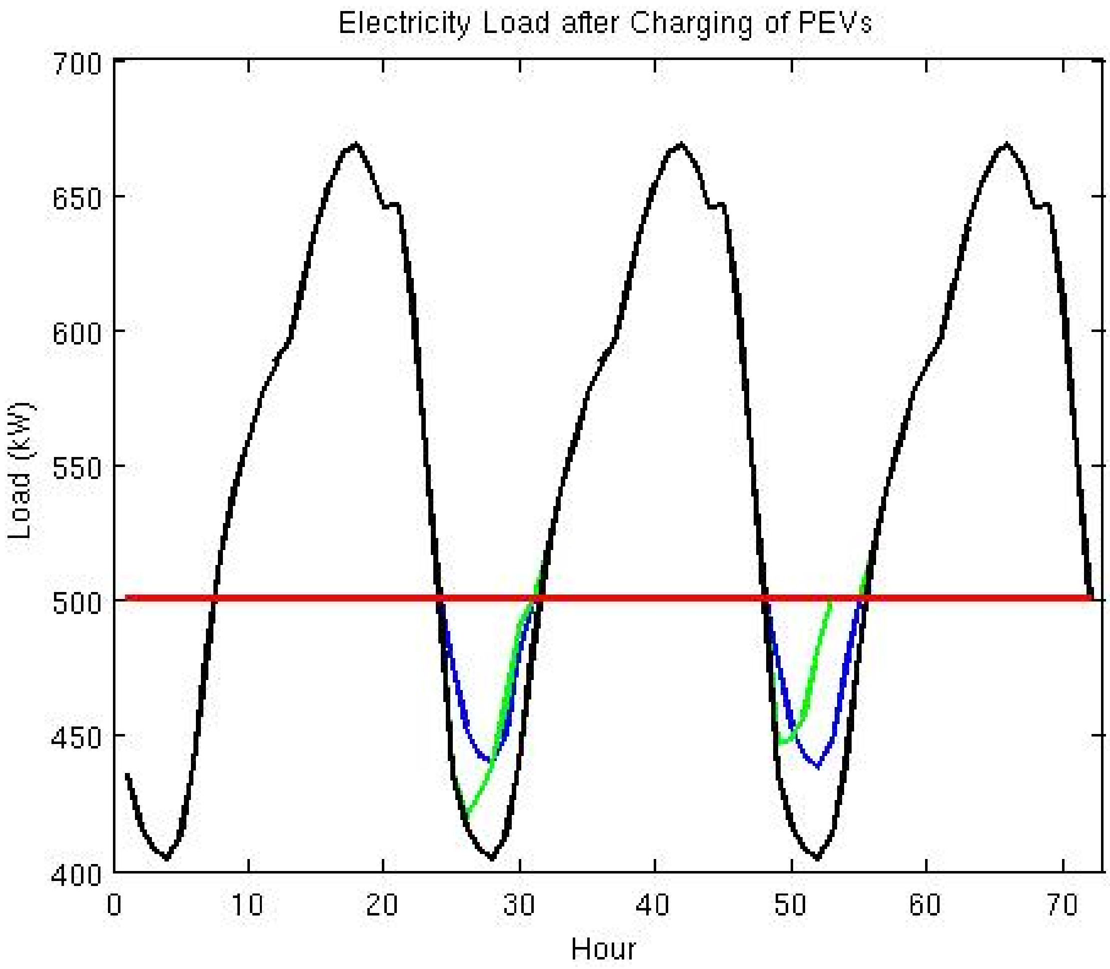


	Standard	Low PGE	Low Shadow
Total Fleet ($)	97,678	83,695	65,349
Mean Cost / Mile	0.068	0.044	0.0054
Increase in Peak	5.1%	1.4%	-0.25%

200 Simulated Fleets

	Standard	Low PGE	Low Shadow
Total Fleet ($)	91,987	76,653	61,580
Mean Cost / Mile	0.065	0.041	0.0067
Increase in Peak	4.7%	1.1%	-0.18%

Results – Charge Only



LP Formulation with Robustness

Furthermore, if discharging is allowed, we could solve a linear program to determine the optimal charging schedules:

Robust

$$\min \quad \sum_{i=1}^{k} \left(\bar{p}^T (c_i - d_i) + (\bar{p}^g)^T f_i \right) + \bar{\rho} c_{\text{cap}}$$

$$\text{s.t.} \quad s_{i,h} = s_{i,h-1} + \bar{c}_{eff} c_{i,h} - \bar{d}_{eff} d_{i,h} + \bar{g}_{eff} g_{i,h} - \alpha \bar{d}_{i,h}, \quad \forall i,h;$$

$$s^g_{i,h} = s^g_{i,h-1} + f_i - g_{i,h}, \qquad \forall i,h;$$

$$c_{i,h} = 0, f_{i,h} = 0, \text{ or } g_{i,h} = 0, \qquad \forall \text{ some } i,h;$$

$$\bar{L}_h + \sum_{i=1}^{k} c_{i,h} - \beta c_{\text{cap}} \le 0, \qquad \forall h;$$

lower and upper bound constraint s.

Online/Dynamic LP – Rolling Horizon

Weekly Rolling Horizon LP Shadow Price Mechanism

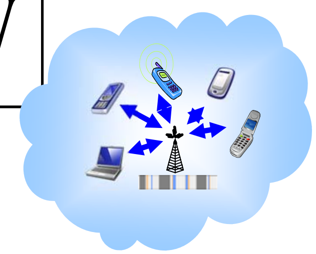
Essentially, use the data of the last period as samples to price resources and decide their allocation for the current period.

LP for Resource Allocation

$$\max \quad \sum_{k=1} \pi_k x_k$$

$$\text{s.t.} \quad \sum_{k} a_{ik} x_k \le b_i \qquad \forall\, i \in S$$

$$0 \le x_k \le 1 \qquad \forall\, k \in N$$



***b_i*: initial supply quantity of good/resource *i*;**

***a_{ik}*: demand rate of trader *k* on good *i* in the bundle**

***π_k*: bidding price per share from trade *k*;**

***x_k*: decision variable (0 or 1) of order fill for trader *k*.**

Online Linear Programming

- Traders come one by one sequentially, buy or sell, or combination, with a combinatorial order/bid **(** $\mathbf{a}_k, \pi_k$ **)**
- The seller/market-maker has to make an order-fill decision as soon as an order arrives
- The seller/market-maker faces a dilemma:
  - **To sell or not to sell?**
- Decision Policy or Mechanism Design

Regret for Online Linear Programming

$$\begin{aligned} \text{OPT}(A,\pi) = \max \quad & \sum_k \pi_k x_k \\ \text{s.t.} \quad & \sum_k a_{ik} x_k \le b_i \quad \forall\, i \in S \\ & 0 \le x_k \le 1 \quad \forall\, k \in N \end{aligned}$$

- We know the total number of customers, say *n;*
- Assume customers arrive in a random order.
- For a given mechanism/decision policy

$$Z(A,\pi) = E_\sigma[\ \sum_1^n \pi_k x_k\ ]$$

$$R(A,\pi) = 1 - \frac{Z(A,\pi)}{OPT(A,\pi)}$$

$$R = \sup_{(A,\pi)} R(A,\pi)$$

Regret Lower Bound for Online LP

Theorem: There is no decision policy/mechanism such that

$$R \leq O\left(\sqrt{\log(m)/B}\right), \quad B = \min_i b_i.$$

Corollary: If $B \leq \log(m)/\varepsilon^2$, then there is no decision policy/mechanism such that $R \leq O(\varepsilon)$.

Agrawal, Wang and Y, "A Dynamic Near-Optimal Algorithm for Online Linear Programming," 2010.

Previous Upper Bound Result

Theorem: There is an decision policy/mechanism such that

$$R \leq \mathrm{O}\left(\sqrt[3]{m\log(n)/B}\right), \quad B = \min_{\mathrm{i}} b_i.$$

Corollary: If $B > m\log(n)/\varepsilon^3$, then there is a decision policy/mechanism such that $R \leq \mathrm{O}(\varepsilon)$.

Kleinberg 2005 , Devanur and Hayes 2009, Feldman et al. 2010., etc.

New Regret Upper Bound for Online LP

Theorem: There is an decision policy/mechanism such that

$$R \leq \mathrm{O}\left(\sqrt{m\log(n)/B} \right), \quad B = \min_{\mathrm{i}} b_i .$$

Corollary: If $B > m\log(n)/\varepsilon^2$, then there is a decision policy/mechanism such that $R \leq \mathrm{O}(\varepsilon)$.

Agrawal, Wang and Y, "A Dynamic Near-Optimal Algorithm for Online Linear Programming," 2010.

Price Mechanism Again

$$
\begin{aligned}
\max \quad & \sum_k \pi_k x_k \\
\text{s.t.} \quad & \sum_k a_{ik} x_k \le b_i \ \forall\, i \in S \\
& 0 \le x_k \le 1 \quad \forall\, k \in N
\end{aligned}
$$

Given a price vector p for every resource, set $x_k=1$ if $\pi_k > a_k^T p$; otherwise $x_k=0$.

Shadow Prices of the Sample LP

Set the initial price very high, then solve the following sample LP and set price vector as the shadow prices.

$$\max \quad \sum_{k=1}^{t} \pi_k x_k$$

$$\text{s.t.} \quad \sum_{k=1}^{t} a_{ik} x_k \leq (1-h_t)\frac{t}{n} b_i \quad \forall\, i \in S$$

$$0 \leq x_k \leq 1 \qquad \forall\, k \in N$$

Shadow Prices Update

For a given ε, solve the sample LP at t=εn, 2εn, 4εn, …; and use the new shadow prices for the decision in the coming period.

$$\max \quad \sum_{k=1}^{t} \pi_k x_k$$

$$\text{s.t.} \quad \sum_{k=1}^{t} a_{ik} x_k \le (1-h_t)\frac{t}{n} b_i \quad \forall\, i \in S$$

$$0 \le x_k \le 1 \qquad \forall\, k \in N$$

Observations and Future Work

- A smart management of PEV can benefit both utility and consumer sides.
- Price mechanism could facilitate such a management
- Dual or shadow price of LP can serve the role of prices
- Repeated and online LP learning is better than one-time learning
- Close the gap on lower and upper bounds of the regret for online LP resource allocation