Hidden-City Ticketing: the Cause and Impact

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Abstract

Hidden-city ticket is an interesting airline ticket pricing phenomenon. It occurs when an itinerary connecting at an intermediate city is less expensive than a ticket from the origin to the intermediate city. In such a case, passengers traveling to the intermediate city have an incentive to pretend to be traveling to the final destination, deplane at the connection point and forgo the unused portion of the ticket. Hidden-city opportunities are not uncommon nowadays.

In this paper, we establish a mathematical model to shed some light on the cause of hidden-city opportunities and the impact of (the passengers') hidden-city ticketing practices on both the airlines' revenues and the consumer welfare. To perform our study, we adapt a flight network revenue management model. We illustrate that the hidden-city opportunity may arise when there is a large difference in the price elasticity of demand on related itineraries, therefore providing a plausible explanation for this phenomenon. To show the impact of the hidden-city ticketing practices on the airlines' revenues, we first argue that when the passengers take advantages of such opportunities whenever available, the airlines had better to react, and the optimal reaction will no longer contain any hidden-city opportunities. However, even if the airline optimally reacts, the revenue it can achieve still decreases from the optimal revenue it could have achieved when passengers do not practice hidden-city ticketing. Moreover, under our model, the revenue decrease could be as much as half of the optimal revenue when passengers do not use hidden-city tickets, but it cannot be more if the airline's network has a hub-and-spoke structure. From the passenger's perspective, we show that when they practice hidden-city ticketing, the prices of certain itineraries would rise and somewhat surprisingly, their overall consumer surplus may fall. Therefore, our results indicate that the practices of hidden-city ticketing by the passengers not only hurt the airlines, but could also hurt their own welfare.

1 Introduction

Since the deregulation of the airline industry in the 1970s, airlines have employed more and more sophisticated pricing strategies to strive for more revenue from passengers. Many interesting pricing phenomena are frequently observed. Among those is the *hidden-city opportunity* in which the price quoted by an airline for an itinerary from city A to city B is more expensive than the price quoted by the same airline for an itinerary from city A to city C with a connection at city B. When such a situation happens, passengers from A to B will have an incentive to pretend to be traveling from A to C, deplane at city B and forgo the unused portion of the ticket. In this case, we call city B the "hidden-city". And the practice of using such ticketing strategies by the passengers is called "hidden-city ticketing". An example of a hidden-city opportunity is

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illustrated as follows[1]:

A one-way flight from San Francisco (SFO) to Minneapolis-St Paul (MSP) on Feb. 4th, 2013 is quoted as \$715.90 *by Delta Air Lines. However, if one searches for a one-way flight from SFO to Milwaukee (MKE) on the same day, Delta Air Lines will provide a connecting flight with a connection at MSP (using the same flight let as the \$715.90 one) but only ask for \$275.80 (we call MSP the hidden-city in this case).*

The above example may not make sense to many passengers. How can it be that the entire flight is less expensive than a shorter segment of it? However, this pricing phenomenon prevails in today's airline pricing, especially in the modern hub-and-spoke airline networks[2]. According to GAO (2001), about 17% of the markets among major U.S. airlines exhibit hidden-city opportunities (which is defined in GAO (2001) as 100 dollars' difference in high fare classes and 50 dollars' difference in low fare classes). For the itineraries between major cities, hidden-city opportunities exist in nearly all of them (see Airfareiq.com (2009)). In the mean time of saving passenger's money, the practice of using hidden-city tickets hurts the airlines' revenues (in the example above, the airline expects to earn \$715.90 for a passenger traveling from SFO to MSP, but if the passenger uses the hidden-city ticket, the airline can only collect \$275.80). Therefore, most airlines explicitly prohibit the practice of hidden-city ticketing in the terms and conditions of a ticket. However, few passengers are adversely sanctioned for such practices because it is difficult and costly for airlines to pursue individual passengers. Moreover, such practices may not be sufficiently widespread yet to justify expensive controls.

Although it is debatable whether the hidden-city ticketing practice is legal or ethical, there are several questions that are worth studying. First, why such seemingly unreasonable pricing policies are established by the airline in the first place. Or in other words, how can it be that a bundle of goods (the whole itinerary) is sold at a cheaper price than a component of it (a single leg). Second, even though the number of passengers who practice hidden-city ticketing in the past may be small, the developments of several new travel search tools are making it easier for the passengers to find such opportunities (e.g., there are websites now that are specifically designed to achieve this task, such as airfareiq.com and flyshortcut.com). Therefore, it is important for the airlines to understand the potential implications on their revenues if hidden-city ticketing is practiced by more passengers, and how they should react. Finally, even for the passengers, though it appears that practicing hidden-city ticketing is a smart trick to save their money occasionally, it is important to understand the potential (long term) effects of such practices on their welfare, if the airlines more actively react to such practices by changing their pricing strategies. Understanding these issues is fundamental for the decision makers involved, such as the law makers, the airline executives as well as the passengers.

For these purposes, we establish a mathematical model in this paper to shed some light on this problem. In our study, we first consider a base model, in which the airline does not anticipate the passengers to practice hidden-city ticketing. The base model is adapted from the classical network revenue management model, in which the airline decides the optimal routes and prices to offer at each time during the selling period. We show that the hidden-city opportunity could exist in the optimal solution to this model when there is a significant difference in the price elasticity of demand on related itineraries. In practice, these differences in the price elasticity of demand can occur due to factors such as different competition intensities on different itineraries, the seasonality of demand and the very nature of the destinations (e.g., business or vacation destinations). Therefore, our study provides a theoretical explanation for this seemingly puzzling phenomenon.

[1] Prices are found on Delta Air Lines website (`www.delta.com`) on Feb. 3rd, 2013.

[2] A hub-and-spoke network consists hub cities and spoke cities. Itineraries between spoke cities are connected at a certain hub. We will formally define this term in Section 4.

After understanding the cause of the hidden-city opportunity, we study the impact of the hidden-city ticketing practice to the airlines' revenues if passengers practice it whenever possible. First we show that if the airlines do not react to such practices, their revenues could be seriously affected. Then we establish a modified network revenue management model for the airlines in which the hidden-city ticketing practices are taken into account explicitly. We prove that using the modified model, the optimal pricing strategies of the airlines will no longer contain any hidden-city opportunities. However, even with the optimal reaction, the airlines still cannot fully mitigate the negative revenue effect of such ticketing practices. We show that the airlines will always lose revenue if passengers practice hidden-city ticketing. Furthermore, for the popular hub-and-spoke network, we show that the revenue loss could be as much as half of the original optimal revenue (when the passengers do not practice hidden-city ticketing) but no more. As far as we know, our result is the first one to quantitatively study the revenue impact to the airlines of the hidden-city ticketing practice, and it indicates that such practices could be quite detrimental to the airlines' revenues.

On the other hand, although the airlines have a strong incentive to prohibit hidden-city ticketing, it appears that such practices can save passenger's money. However, we argue that, the practice of hidden-city ticketing may also *hurt* the passengers through the externalities that the behavior causes. We show that when the passengers choose to practice hidden-city ticketing and the airlines optimally react to it, the fares to the spoke cities will increase under certain conditions. And under mild conditions, we show that the total consumer surplus of the passengers will decrease (compared to if the passengers do not practice hidden-city ticketing). Moreover, a game between the airline and the passengers is studied, and we show that if they interact repeatedly, it might be optimal for the passengers to avoid using hidden-city tickets even it could save their money in the current period. Therefore, our results convey that even from the passenger's perspective, practicing hidden-city ticketing may not be a good strategy in a long term. We believe that this somewhat unintuitive message is of value to the air travel community.

The remainder of the paper is organized as follows. In the rest of this section, we review the literature that is related to our study. In Section 2, we build our base airline pricing model for our study. We show in Section 3 that hidden-city opportunity could arise in the optimal solution to the base model. In Section 4, we study the optimal responses of the airlines when the passengers practice hidden-city ticketing and the impact of such practices on the airlines' revenues. In Section 5, we study the changes of prices and consumer surplus when the hidden-city ticketing is practice by the passengers and build a game to analyze the equilibrium strategies of both the airlines and the passengers. In Section 6, we illustrate our results by using some numerical experiments. And we conclude our paper in Section 7.

1.1 Literature Review

Since the deregulation of the airline industry in the 1970s, there has been extensive empirical research on the airline industry. Much of these research focus on the change of airline networks, its impact on the competitions of airlines and the resulting airfares. We refer the readers to Borenstein and Rose (2007) and Bamberger and Carlton (2003) for comprehensive reviews of the related literature. In the seminal work by Borenstein (1989), the authors verify the importance of route and airport dominance in determining the degree of market power exercised by an airline, and consequently the prices offered. One of the major findings in his work is that in a hub-and-spoke flight network, the fares to or from the hubs for an airline are significantly higher due to its competitive advantages at the hub. Similar findings can also be found in Borenstein (1991), Evans and Kessides (1993), Brueckner and Spiller (1994) and GAO (2001). This finding is related to the hidden-city opportunities since most such opportunities are found on the O-D pairs to a hub. In addition, the phenomenon is found in many studies that for certain markets, the round-trip fares are cheaper than the one-way fares (see Evans and Kessides (1993), GAO (2001)). This phenomenon can be viewed as a special case of the hidden-city opportunity with the destination of the hidden-city ticket being its origin. In this work, we complement the empirical studies by providing a plausible theoretical argument for the phenomenon observed in practice, and specifically focus on the hidden-city opportunities. In addition to explain the empirical observations, we take one step further to shed some light on the interactions between the consumer choice behaviors and the airline pricing strategy. This step, which is not taken in the prior studies, provides new understandings and insights for the airline pricing problems as well as consumer behaviors, which is important as consumers become more and more sophisticated when equipped with modern technologies.

From an economics point of view, hidden-city ticket is one example of nonlinear pricing, see Wilson (1997). It is well understood in the economics literature that the difference of price elasticity plays a vital role in the nonlinearity of prices (we refer the readers to the book by Frank (2008) for a comprehensive review of the role of price elasticities in pricing). However, there are several features of the hidden-city ticket phenomenon that distinguish itself such that an independent study of it is worthwhile. First, in a typical nonlinear pricing example, the marginal price of an additional item (in the plane ticket context, the additional item is a flight leg) is lower than the price when it is sold separately. However, it is rare that the marginal price is negative, as in the hidden-city ticket case. In particular, due to the potential large difference in the price elasticity of demand in a hidden-city ticket case, the marginal price of an additional flight leg could be fairly large (in the example at the beginning of this section, it could be as large as several hundred dollars or more than 60% of the fare). The large negative marginal prices are very counterintuitive and may cause confusion among the consumers, thus require special illustrations. More importantly, the special characteristics of the airline industry make it possible for the airlines to *fence* customers from purchasing hidden-city tickets (e.g., by checking passenger's bags, rerouting passengers under certain conditions or forfeiting the frequent flyers miles), however, the current existing fence is controversial and sometimes cannot be fully enforced (GAO (2001)). This leaves the important question that how valuable it is to build and enforce the fence in the first place. Answering this question is quite instrumental for the decision makers involved and is one of the goals of our study.

In the revenue management literature, there has been extensive studies on the flight revenue management problems in the past several decades, see Talluri and van Ryzin (2004b), Bitran and Caldentey (2003), Phillips (2005) and references therein for comprehensive reviews on this subject. The earliest papers in this literature consider leg-based controls, e.g., the classical Littlewood model by Littlewood (1972) and the EMSR-a and EMSR-b models by Belobaba (1987b,a, 1989). Since the fares in the leg-based models are additive, using such models will not lead to hidden-city opportunities. However, the leg-based models are not sophisticated enough to obtain good revenue in a modern airlines network. Simulations have shown that notable revenue benefits can be obtained by using network methods over leg-based methods (see e.g., Williamson (1992)). And industrial studies have shown that almost all airlines nowadays use some kind of network revenue management controls, see Chapter 8.7 in Phillips (2005). Among the studies of network revenue management, Gallego and van Ryzin (1997) lay a foundation by introducing a dynamic program with remaining time and capacity as the state variables. This idea, taken by Adelman (2007), Cooper (2002), Talluri and van Ryzin (1998), Talluri and van Ryzin (1999) and others, is also adopted in our paper. However, unlike most prior studies which focus on how to obtain solutions from this model, we use this model as a tool to explore a new problem - to study the phenomenon of hidden-city ticketing. Thus our focus is quite different from most prior study in this literature.

In the traditional revenue management models, it is assumed that the demand over different origin-destination pairs (O-D pairs) are independent. However, in practice, passengers choose from all itinerary/price combinations available to them according to certain choice behaviors, and the demand functions across different itineraries might be correlated. To capture this choice behavior, Talluri and van Ryzin (2004a) propose a choice-based revenue management model in which a "product set" is offered at each time period. Several properties of optimal solutions are examined in their work. Later, Liu and van Ryzin (2008) extend the choice-based model to network case and propose a decomposition method to solve it. In some sense, our work can be viewed as a special case of the model in Liu and van Ryzin (2008), in which the "product set" contains a certain connecting flight for each O-D pair at a certain price, and in addition to the traditional customer choice behavior, we add "hidden-city" ticket as one choice of the passengers. Although sharing the same framework, this distinguishing consideration makes our analysis more focused and many interesting results are obtained.

Another line of literature which is related to our paper is in the study of bundle pricing. We refer the readers to Venkatesh and Mahajan (2009) for a recent literature review. In the bundle pricing literature, the main focus is on which form of bundle the seller should sell and how to price those bundles. Especially, the valuations of consumers, the complementarity or substitutability of goods, the nature of competition, and other marketing considerations are usually considered for obtaining bundling strategies in the literature. In contrast, in our paper, we study the flight network pricing problem in which the bundle components are constrained to be the connecting flights for each O-D pair and the demand function for each O-D pair is exogenously given. Also, in the bundle pricing literature, it is frequently so that the price of a bundle is cheaper than the sum of prices of its component, however, to the best of our knowledge, none of them discusses the case in which the price of a bundle is cheaper than *one* of its component. We note that given the development of internet technology which enables more personalized pricing strategies, our work may be of interest to the bundle pricing literature as well.

2 Network Revenue Management Model

In this section, we build a base model to study the airline pricing problem. The model is adopted and modified from the network revenue management model introduced in Gallego and van Ryzin (1997). In the base model, the airlines do not anticipate the passengers to practice hidden-city ticketing. The model will be extended to incorporate the practice of hidden-city ticketing in later sections.

We consider an airline network consisting of N cities and define $\mathcal{E}$ to be all the flight legs this airline serves and $\mathcal{O}$ to be all the O-D pairs served using $\mathcal{E}$. The airline sells tickets for all the O-D pairs it serves and the goal is to maximize its revenue over the entire flight network.

Assume there is a fixed capacity on each flight leg, which we use a vector $\boldsymbol{C}$ to denote. In this paper, we assume that $\boldsymbol{C}$ already takes into account the overbooking pad and no further overbooking is allowed. We consider a discrete time model with T periods indexed backwards. That is, we start from time T and all flights depart at time 0. At each time t, the airline has to provide a list of itineraries for each O-D pair, each associated with a price. In this paper, we make the following assumption:

Assumption 1. *We only allow the airlines to provide itineraries with at most one stop.*

The main reason for making this assumption is for the ease of analysis. In practice, an itinerary with two or more stops are rarely attractive for the passengers. Moreover, most major airlines in the U.S. nowadays have the capacity of serving most pairs of its destinations via a one-stop (or direct) flight. Therefore, this assumption is reasonable in practice.

Next we define how passengers choose among the options provided by the airline. We make the following assumption in our paper:

Assumption 2. *For a passenger of the O-D pair $i \to j$, he is only interested in the cheapest itinerary among all itineraries provided by the airline that can send him from i to j.*

Assumption 2 is not generally true in practice. For example, a direct flight is obviously more attractive than a one-stop flight and many passengers would be willing to pay extra to get on a direct flight. However, there are several reasons that we believe it is a reasonable assumption. First, in reality, the airline's network is usually fixed, and whether a direct flight or a connecting flight can be provided for a certain O-D pair usually has been determined by the network structure. For example, in the hub-and-spoke network adopted by many airlines nowadays, two spoke cities can only be served by a connecting flight (though there might be several connection cities to choose from), while an itinerary involving at least one hub is usually served by a direct flight. And among all the one-stop flights, as long as the connection time and location are reasonable, it is not unrealistic to assume that the passengers will choose the cheaper one. Our model could be extended to allow more sophisticated customer choice models[3], however, we restrain from explicitly doing so as we want to keep our models simple to focus on our main objective, that is, to unveil the causes of the hidden-city ticket phenomenon and the potential impact of hidden-city ticketing practices to the airlines as well as the passengers.

Under Assumption 1 and 2, the airline's decisions in each time period t for each O-D pair $i \to j$ are effectively:

1. Which route to provide, i.e., which connection node (or a direct flight) to provide,
2. What price to offer for that itinerary.

In this section, we consider the case in which the passenger's purchase choice only depends on the price of the O-D pair he travels, and is independent of the prices of all other O-D pairs. In other words, the passenger does not explore hidden-city opportunities even if they are present. In this case, when p_{ij}^t is offered for the O-D pair $i \to j$ at time period t, the probability that some passenger will purchase this itinerary during this time period can be captured by a function $\lambda_{ij}^t(p_{ij}^t)$. Here we assume that $\lambda_{ij}^t(\cdot)$ has already taken into account the arrival probabilities of the passengers as well as the options offered by other airlines. In addition, we assume that $\lambda_{ij}^t(\cdot)$ is known in advance and it is small enough so that at each period, the probability that there are more than one purchases can be ignored. Given the historical data and future forecasts an airline typically has, this assumption may not be too unrealistic and is adopted in many prior studies, e.g., Gallego and van Ryzin (1997).

Now we build a dynamic programming formulation to capture the decision problem faced by the airline. We use $V^t(\boldsymbol{x})$ to denote the optimal expected revenue when there are t periods left and the remaining capacity (seats on each flight leg) is $\boldsymbol{x}$. At each period t and facing a remaining capacity $\boldsymbol{x}$, the decision of the airline is a set of connection cities and prices $(k_{ij}^t, p_{ij}^t)_{(i,j)\in\mathcal{O}}$ which means that the airline will serve the O-D pair (i,j) through a connection at k_{ij}^t, and asking for a price p_{ij}^t. Specifically, we use $k_{ij}^t = 0$ to denote a direct flight. A dynamic programming model can be formulated as follows:

$$\begin{aligned} V^t(\boldsymbol{x}) &= \max_{k_{ij}^t, p_{ij}^t, \forall (i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij}^t)\left(p_{ij}^t + V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t})\right) + \left(1 - \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij}^t)\right) V^{t-1}(\boldsymbol{x}) \right\} \\ &= V^{t-1}(\boldsymbol{x}) + \max_{k_{ij}^t, p_{ij}^t, \forall (i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij}^t)\left(p_{ij}^t + V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t}) - V^{t-1}(\boldsymbol{x})\right) \right\} \end{aligned} \quad (1)$$

with boundary conditions

$$V^0(\boldsymbol{x}) = 0 \quad \forall \boldsymbol{x}, \qquad V^t(0) = 0 \quad \forall t,$$

and

$$V^t(\boldsymbol{x}) = -\infty \quad \text{if } \boldsymbol{x} \text{ contains negative entry.}$$

[3]For example, if we want to achieve a choice model for an O-D pair such that two itineraries are chosen with probability p and q, it can be done by alternating the offer between these two itineraries in different time periods.

In (1), we use A_{ij}^k to denote a vector that has 1 at the ikth and kjth entries (recall that $\boldsymbol{x}$ is the remaining capacity for each flight leg, therefore A_{ij}^k is a capacity consumption vector in which each entry corresponds to one flight leg). When $k = 0$, A_{ij}^k denotes the vector that has 1 only at the ijth entry. One can view the dynamic programming problem (1) as one special case of the dynamic program proposed in Gallego and van Ryzin (1997)[4]. Next we discuss some basic properties of the dynamic program (1). Note that in (1), the optimization over k_{ij}^t and p_{ij}^t are separated. Therefore, given the values of $V^{t-1}(\cdot)$, for each $(i,j) \in \mathcal{O}$, the optimal k_{ij}^t can be solved as:

$$\hat{k}_{ij}^t = \arg\max_{k_{ij}^t} V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t}). \tag{2}$$

And the the optimal p_{ij}^t is the one that maximizes

$$\lambda_{ij}^t(p_{ij}^t)\left(p_{ij}^t + V^{t-1}(\boldsymbol{x} - A_{ij}^{\hat{k}_{ij}^t}) - V^{t-1}(\boldsymbol{x})\right). \tag{3}$$

However, even with such structures, solving the dynamic program (1) exactly is computationally intractable, due to the curse of dimensionality. Several heuristics have been proposed in the literature to approximately solve such problems. We refer the readers to those literature (for example, Adelman (2007), Zhang and Adelman (2009)) for viable approaches to solve this DP problem approximately. In the following, instead of studying how to solve this dynamic programming problem, we focus on how the hidden-city opportunities may arise in the optimal solution of this model, and what would be the effect if passengers take advantages of them. We make the following assumptions on the demand functions in our following discussions.

Assumption 3. *For any $(i,j) \in \mathcal{O}$ and any t,*

1. *$\lambda_{ij}^t(p)$ is continuously differentiable and non-increasing in p.*
2. *For any $c > 0$, $\lambda_{ij}^t(p)(p-c)$ is quasiconcave in p and there exists a unique maximizer for $\lambda_{ij}^t(p)(p-c)$ on (c,∞).*

Assumption 3 is quite mild. The first condition is a basic regularity condition. The second condition guarantees that there is always a unique optimal price for each O-D pair. One can easily verify that some most commonly-used demand models such as the linear demand model or the multinomial logit choice model all satisfy Assumption 3.

3 Cause of the Hidden-City Opportunities

In this section, we study the cause of the hidden-city opportunities using the dynamic programming model we introduced in Section 2. We start with a simple example to illustrate the intuition.

Example 1. Consider a two-city network as shown in Figure 1. The airline serves two O-D pairs on this network, $A \to B$ and $A \to C$ (assume there is no demand on other O-D pairs). There are one period remaining and one seat left on each of the leg $A \to B$ and $B \to C$. The demand functions for the two O-D pairs are[5]:

$$\lambda_{AB}(p) = \max\{2-p, 0\},\ \lambda_{AC}(p) = \max\{1-p, 0\}.$$

[4]More precisely, our DP model can be viewed as a discretized version of that in Gallego and van Ryzin (1997). And the purchase probability for an itinerary I for the O-D pair $i \to j$ equals to $\lambda_{ij}^t(p_{ij}^t)$ if I is the cheapest itinerary among all the itineraries offered for this route. Otherwise, the purchase probability is 0.

[5]There could be a constant factor for the demand function. We omit it in the example to make the notation simpler.

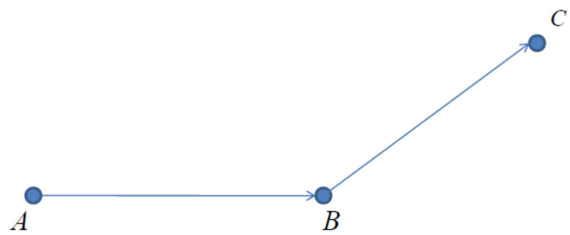

Figure 1: Illustration of Example 1.

Now we can compute the optimal prices for the two O-D pairs. In this case, since there is only one period left, the airline will choose prices to maximize the expected revenue on both O-D pairs. Thus the optimal prices are:

$$\hat{p}_{AB} = \arg\max_{p}\{\lambda_{AB}(p)p\} = 1, \quad \hat{p}_{AC} = \arg\max_{p}\{\lambda_{AC}(p)p\} = 0.5,$$

with an expected revenue of 1.25. One can see that a hidden-city opportunity arises in this situation (the price of the flight to the intermediate city B is more expensive than a flight to C with connection at B). To argue that this is indeed necessary for achieving the maximal revenue, one can compute the optimal revenue without hidden-city opportunity (i.e., with the constraint $p_{AB} \leq p_{AC}$) and find that the optimal prices are $p_{AB} = p_{AC} = 0.75$ with an expected revenue of 1.125. As one can see, in this case, the reason behind this phenomenon is because of the difference in the demand function. Namely, the demand for the O-D pair $A \to B$ is higher than that of $A \to C$ for any given price. And therefore, the optimal price tends to be higher despite that $A \to B$ consumes less inventory. $\square$

In the following, we capture the idea in Example 1 in our DP model and extend the discussion to multi-period and general network problems. As we have discussed in Section 2, at time period t and with remaining capacity $\boldsymbol{x}$, the optimal decision entails the airline to first solve the optimal connection point for each O-D pair $i \to j$:

$$\hat{k}^t_{ij} = \arg\max_{k_{ij}} V^{t-1}(\boldsymbol{x} - A^{k_{ij}}_{ij}).$$

In the following, we define $c^t_{ij} = V^{t-1}(\boldsymbol{x}) - V^{t-1}(\boldsymbol{x} - A^{\hat{k}^t_{ij}}_{ij})$. Intuitively, c^t_{ij} can be viewed as the opportunity cost of using the capacity $A^{\hat{k}^t_{ij}}_{ij}$ when there are t periods left and $\boldsymbol{x}$ capacity remaining. Then as in (3), the optimal price for the O-D pair $i \to j$ can be determined by finding:

$$\hat{p}^t_{ij} = \arg\max_{p} \lambda^t_{ij}(p)(p - c^t_{ij}). \tag{4}$$

In the remainder of this section, we omit the superscript t for the simplicity of notation. By the optimality condition of (4), at optimal $\hat{p}_{ij}$, we have:

$$1 - \frac{c_{ij}}{\hat{p}_{ij}} = -\frac{\lambda_{ij}(\hat{p}_{ij})}{\hat{p}_{ij}\lambda'_{ij}(\hat{p}_{ij})} = -(E_{ij}(\hat{p}_{ij}))^{-1},$$

where $E_{ij}(p) = \frac{p\lambda'_{ij}(p)}{\lambda_{ij}(p)}$ is the price elasticity of demand at price p for the O-D pair $i \to j$. Suppose the hidden-city opportunity exists on the route $i \to j \to k$, i.e., for the O-D pair $i \to j$, the airline offers an itinerary with price p_{ij} (either direct or 1-stop), and for the O-D pair $i \to k$, the airline offers a connecting flight with a connection at j, however, with price $p_{ik} < p_{ij}$. By the above argument, p_{ij} and p_{ik} must satisfy:

$$1 - \frac{c_{ij}}{p_{ij}} = -(E_{ij}(p_{ij}))^{-1} \quad \text{and} \quad 1 - \frac{c_{ik}}{p_{ik}} = -(E_{ik}(p_{ik}))^{-1}. \tag{5}$$

However, since the airline chooses $\hat{k}_{ij}^t$ over a direct flight to offer for the O-D pair $i \to j$, we must have

$$V^{t-1}(\boldsymbol{x} - A_{ij}^{\hat{k}_{ij}^t}) \geq V^{t-1}(\boldsymbol{x} - A_{ij}^0).$$

Moreover, because of the monotonicity of $V^t(\boldsymbol{x})$ in $\boldsymbol{x}$ (which is easy to see), we have $V^{t-1}(\boldsymbol{x} - A_{ij}^0) \geq V^{t-1}(\boldsymbol{x} - A_{ik}^j)$ and thus $c_{ij} \leq c_{ik}$. Combined with that $p_{ij} > p_{ik}$ and (5), we must have $E_{ij}(p_{ij}) > E_{ik}(p_{ik})$. Since the price elasticity of demand is always negative, it is equivalent as having $|E_{ij}(p_{ij})| < |E_{ik}(p_{ik})|$. In the following, when we say "larger price elasticity of demand", we mean larger in absolute value. We have the following proposition on the conditions when the hidden-city ticket may exist.

Proposition 1. *If j is a hidden-city such that a direct flight from i to j is more expensive than a connecting flight from i to another city k with connection at j, then the price elasticity of demand of the O-D pair $i \to k$ is greater than that of the O-D pair $i \to j$ at optimal prices.*

The result states that the difference in price elasticity of demand is a necessary condition for the hidden-city opportunity to exist. However, it is not a sufficient condition. In order for a hidden-city opportunity to exist, the difference in the price elasticity must be large enough to compensate the extra (opportunity) cost of using an additional leg.

Now we link the result of Proposition 1 to what we observe in practice. In practice, there are many situations when there is a large difference in the price elasticity of demand between two O-D pairs. For example, one destination may attract mostly business travelers while the other destination attracts mainly leisure travelers. As shown in Pindyck and Rubinfeld (2001), there is a large difference between the price elasticity of demand of business travelers ($-0.9 \sim -0.3$) and that of leisure travelers (about -1.5). In order to capture this difference, the airlines sometimes have to offer fares that exhibit hidden-city opportunities. Another common reason for such a difference in price elasticity is due to different competition level in different routes. As classical economics theory explains, higher competition typically results in a higher level of price elasticity of demand, and vice versa. In many examples where the hidden-city opportunity occurs, the hidden-city is the airline's hub city (e.g., in the example in Section 1, Minneapolis/St Paul (MSP) is one of the current hub of Delta Air Lines). Since airlines usually enjoy dominant market positions for the itineraries to/from their hubs, they usually face less competition for those routes. In contrast, for the O-D pairs that connect two spoke cities of an airline, there are usually many competitors who offer similar capacities for those routes (through different connection cities) and therefore the competition level is higher. Other factors such as seasonality sometimes also contribute or exacerbate such phenomena.

From the passenger's perspective, the reason that most of them find such phenomena unreasonable or even think it is a mistake of the airline pricing system is the (wrong) impression that airplane seats are sold in a cost-based approach, rather than a demand driven approach. In addition to that, other industries rarely have such phenomena since it would be much easier for the consumers to exploit the price difference[6]. Although simple, we hope that the analysis in this section help people understand this somewhat puzzling phenomenon.

[6] Although it is less common, the hidden-city phenomena do exist in other industries occasionally. For example, when we searched for a period of hotel stays starting from Feb 27th 2013 (the search was done on Feb 10th 2013), for the same Hotel (Hilton St. Louis Airport), we find that for an exactly same room with all conditions equal, a four days stay (costs \$611 including tax) is more expensive than a five days stay (costs \$500 including tax).

4 Revenue Effects when Passengers Practice Hidden-City Ticketing

In this section, we assume that all the passengers will practice hidden-city ticketing whenever such an opportunity is available and we study the revenue implications for the airlines in such cases. Indeed, in practice, only a small portion of the passengers would take advantage of such opportunities, however, with the growth of the internet, it becomes easier and easier for the passengers to find such opportunities (e.g., there are websites now that are specifically designed to achieve this task, such as airfareiq.com and flyshortcut.com) and therefore more and more passengers have the opportunities to practice hidden-city ticketing. Moreover, the airlines sometimes place fences nowadays to prevent passengers from practicing such ticketing (such us checking the baggages, rerouting passengers or revoking frequent flyer miles), however, such fences are often controversial and hard to enforce. Therefore, it is important for the airline to understand the full potential effect of such practices on its revenue, which will help them to make better decisions.

In this section, we build on the base model that is introduced in Section 2. In addition, to capture the practice of hidden-city ticketing practice, we assume that when a passenger purchases a flight ticket for the O-D pair $i \to j$, he will search for all the O-D pairs $i \to l$ with $k_{il} = j$. Then he compares the cheapest price to the price offered to the O-D pair $i \to j$. If the former one is cheaper, the passenger will purchase a hidden-city ticket (without letting the airline know his true intention). In the following, we study the revenue implications to the airlines in this case. We first show that if the airline does not react to such behaviors and keeps its original pricing policy, its revenue could be seriously affected. Then we develop a model for the airlines to react to hidden-city ticketing practices and study how the revenue would be affected by hidden-city ticketing if the airline can anticipate it and respond accordingly. We start with the following example to show that if the airline does not react properly, its revenue could be seriously affected:

Example 2. (If the airline does not react) We consider the same network as shown in Figure 1. The only two O-D pairs the airline serves are A to B and A to C with connection at B. We assume there are 1 period left before departure and 1 seat left on both $A \to B$ and $B \to C$ legs. In the following, we first consider a series of demand functions that are piecewise constant. However, we can use continuous functions to approximate them with arbitrary degree of accuracy. Therefore the result of this example also applies when restricted to continuous demand functions.

Let the demand functions be:

$$\lambda_{AC}(p) = \begin{cases} \beta, & \text{if } p \le 100 \\ 0, & \text{if } p > 100 \end{cases} \quad \text{and} \quad \lambda_{AB}(p) = \begin{cases} 10/\gamma, & \text{if } p \le \gamma \\ 0, & \text{if } p > \gamma, \end{cases}$$

where $\beta < 0.1$, $\gamma > 100$ are constants chosen beforehand. It is easy to see that the optimal prices without taking into account of hidden-city ticketing will be to offer 100 for the O-D pair $A \to C$ and γ for the O-D pair $A \to B$ and the optimal expected revenue is $r_1 = 10 + 100\beta$. However, if hidden-city tickets are used by the passengers, then the airline's revenue will be $r_2 = \frac{1000}{\gamma} + 100\beta$. By choosing γ sufficiently large and β sufficiently small, the ratio between r_1 and r_2 can be made arbitrarily large. Therefore, if the airline does not change its price in response to the hidden-city ticketing practice, the revenue can be reduced by a factor that is arbitrarily large. □

The above example indicates that the airline may suffer serious loss if they do not react to hidden-city ticketing practices. Next we study how the airline should react to this practice and what would be the revenue effects in that case. In the following, we modify the model introduced in Section 2 to take into account the hidden-city ticketing practices.

We use the same notations as in (1) and use $\bar{V}$ to denote the value function when passengers practice hidden-city ticketing. In this case, the airline's optimization problem when facing remaining capacity $\boldsymbol{x}$ at time t becomes:

$$\begin{aligned}\bar{V}^t(\boldsymbol{x}) &= \max_{k_{ij}^t, p_{ij}^t, \forall (i,j)\in\mathcal{O}} \left\{ \sum_{(i.j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij}^t)\left(\bar{p}_{ij}^t + \bar{V}^{t-1}(\boldsymbol{x} - \bar{A}_{ij}^t)\right) + \left(1 - \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij}^t)\right)\bar{V}^{t-1}(\boldsymbol{x})\right\} \\ &= \bar{V}^{t-1}(\boldsymbol{x}) + \max_{p_{ij}^t, k_{ij}^t, \forall(i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij}^t)\left(\bar{p}_{ij}^t + \bar{V}^{t-1}(\boldsymbol{x} - \bar{A}_{ij}^t) - \bar{V}^{t-1}(\boldsymbol{x})\right)\right\}, \qquad (6)\end{aligned}$$

where

$$\bar{p}_{ij}^t = \min\{p_{ij}^t, \min_{l:k_{il}^t=j} p_{il}^t\}, \qquad \bar{k}_{ij}^t = \begin{cases} \arg\min_{l:k_{il}^t=j} p_{il}^t, & \text{if } \min_{l:k_{il}^t=j} p_{il}^t < p_{ij}^t \\ 0, & \text{if } \min_{l:k_{il}^t=j} p_{il}^t \geq p_{ij}^t \end{cases}$$

and

$$\bar{A}_{ij}^t = \begin{cases} A_{ij}^{k_{ij}^t}, & \text{if } \bar{k}_{ij}^t = 0 \\ A_{i\bar{k}_{ij}^t}^j, & \text{if } \bar{k}_{ij}^t \neq 0 \end{cases}.$$

The boundary conditions are the same as before. Here $\bar{A}_{ij}^t$ is the actual capacity consumption when a unit of demand for the O-D pair $i \to j$ is satisfied and $\bar{p}_{ij}^t$ is the actual revenue collected from that O-D pair. If $\bar{k}_{ij}^t$ is non-zero, then a hidden-city opportunity exists and the passenger would buy a ticket from i to $\bar{k}_{ij}^t$ and get off when connecting at j; if $\bar{k}_{ij}^t$ is zero, then no hidden-city opportunity exists in this O-D pair. Note that in (6), the optimization problem is no longer separable for each O-D pair because the price in one O-D pair may affect the price/demand for other O-D pairs. Therefore, the properties (2) and (3) will no longer hold in this case. Next we show an important property of the solution to the above dynamic program.

Theorem 1. *There exists a solution to (6) that does not contain any hidden-city opportunity, i.e., when all the passengers take advantage of hidden-city opportunities, the best response of the airline will eliminate all such opportunities.*

Proof. We prove by contradiction. Assume the optimal pricing policy for (6) still contains hidden-city opportunities. Then there exists city i and j (hidden-city) such that

$$p_{ij}^t > \min_{l:k_{il}^t=j} p_{il}^t.$$

Next we discuss four cases and show that we can always modify the current pricing policy such that the revenue earned is at least as large as before however the hidden-city opportunity is eliminated. In the proof, we will use the following claim multiple times, which is intuitive and thus its proof is omitted.

Claim 1. *For any t, any vector $\boldsymbol{x} \geq \boldsymbol{y}$ (we define all vector inequality as componentwise), we have $\bar{V}^t(\boldsymbol{x}) \geq \bar{V}^t(\boldsymbol{y})$.*

1. $k_{ij}^t = 0$. In this case, we just lower the price p_{ij}^t to $\bar{p}_{ij}^t$. Then for the right hand side of (6), the only thing that is changed is $\bar{A}_{ij}^t$ and it is decreased. Therefore, by Claim 1, the expected revenue is at least as high as before.
2. $k_{ij}^t \neq 0$ and k_{ij}^t is not a hidden-city in the solution to (6). In this case, instead of offering the route $i \to k_{ij}^t \to j$ for the O-D pair $i \to j$, we offer a direct flight from i to j with price $\tilde{p}_{ij}^t = \min_{l:k_{il}^t=j} p_{il}^t$. Comparing this new pricing policy to the original one, the only term in

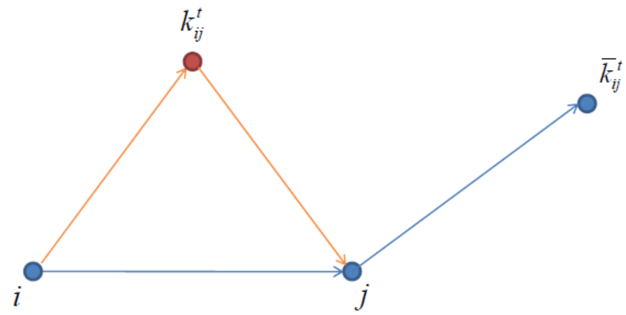

Figure 2: An illustration of the proof of Theorem 1.

(6) that is changed is for the O-D pair i to j. However, the value $\bar{V}^{t-1}(\boldsymbol{x} - \bar{A}_{ij}^t) - \bar{V}^{t-1}(\boldsymbol{x})$ in the new pricing policy is no less than the old one since the new policy only consumes one unit of leg $i \to j$ while the old one consumes one unit of leg $i \to j$ and $j \to \bar{k}_{ij}^t$. Meanwhile, the effective price for the O-D pair $i \to j$ remains the same. Therefore, the revenue achieved by the new pricing policy is at least as high as the old one.

3. k_{ij}^t is also a hidden-city in the solution to (6) but the cheapest route for traveling from i to k_{ij}^t is not $i \to k_{ij}^t \to j$ (or precisely, $\bar{k}_{ik_{ij}^t} \neq j$). Then again, instead of offering the route $i \to k_{ij}^t \to j$, we offer a direct flight $i \to j$ with price $\tilde{p}_{ij}^t = \min_{l:k_{il}^t = j} p_{il}^t$. Then by the same argument as in the previous case, this modification achieves at least the same revenue.

4. k_{ij}^t is also a hidden-city and the route $i \to k_{ij}^t \to j$ is the cheapest route to travel from i to k_{ij}^t (or more precisely, $p_{ij}^t = \min\{p_{ik_{ij}^t}, \min_{l:k_{il}^t = k_{ij}^t} p_{il}^t\}$). We draw this case in Figure 2. In this case, we offer a direct flight for O-D pair $i \to j$ with price $\tilde{p}_{ij}^t = \min_{l:k_{il}^t = j} p_{il}^t$ and a direct flight for O-D pair $i \to k_{ij}^t$ with price $\tilde{p}_{ik_{ij}^t} = p_{ij}^t$. Now we consider the terms in (6):

   - For the O-D pair $i \to j$, the effective price is the same as the original pricing policy, but the consumption of the capacity is lower (changed from $i \to j \to \bar{k}_{ij}^t$ to $i \to j$), thus according to Claim 1 the revenue achieved is at least as high as before.
   - For the O-D pair $i \to k_{ij}^t$, the effective price is the same as the original policy, but the consumption of the capacity is lower (changed from $i \to k_{ij}^t \to j$ to $i \to k_{ij}^t$), thus the revenue achieved is at least as high as before.
   - For all other O-D pairs, the prices and capacity consumptions are the same.

To summarize, if there is a hidden-city opportunity in the network, there is always a modification to the prices such that the revenue achieved is at least as high as before and that the hidden-city opportunity is eliminated. Note that the above modification can always be terminated in a finite number of steps, since at each step, the modification only modifies some prices in the network to a lower price that also exists in the network. And when this modification terminates, there must be no hidden-city opportunities in the network. Thus the theorem is proved. □.

Theorem 1 is intuitive, since if the passengers will exploit the fare differences in the hidden-city routes anyway, intuitively the airline should not offer the difference in the first place. Yet Theorem 1 is an important result for our further study. A direct consequence of Theorem 1 is a simplification of the dynamic program in (6). According to Theorem 1, the dynamic program

(6) can be equivalently written as:

$$\bar{V}^t(\boldsymbol{x}) = \max_{p_{ij}^t, k_{ij}^t, \forall (i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij}^t)\left(p_{ij}^t + \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t})\right) + \left(1 - \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij}^t)\right)\bar{V}^{t-1}(\boldsymbol{x}) \right\}$$
$$\text{s.t.} \qquad p_{ij}^t \le p_{il}^t, \quad \forall\, k_{il}^t = j, \quad \forall\, i,j. \tag{7}$$

Comparing to the dynamic program in (1), the only difference in (7) is that it has some additional constraints. The following theorem follows quickly from this observation.

Theorem 2. *Let $V^t(\boldsymbol{x})$ and $\bar{V}^t(\boldsymbol{x})$ be as defined in (1) and (6). Then $V^t(\boldsymbol{x}) \ge \bar{V}^t(\boldsymbol{x})$ for all t and $\boldsymbol{x}$.*

Proof. We prove by induction on t. The statement is trivial for $t = 0$. Assume it is true for $s = t-1$ for all $\boldsymbol{x}$, then for time t, as argued above, $V^t(\boldsymbol{x})$ and $\bar{V}^t(\boldsymbol{x})$ can be computed by (1) and (7).

Now we compare (1) and (7). By induction assumption, the objective value of (7) (the part inside the optimization) is less than that of (1). And also (7) has more constraints. Thus the theorem holds. □

Theorem 2 is also quite intuitive. It states that when the passengers practice hidden-city ticketing, the airline will always suffer losses, even if it chooses the optimal reaction. Another way to look at it is that when hidden-city ticketing are practiced by passengers, the airline loses one way to differentiate passengers in pricing, and thus loses revenue. This result could also explain why in practice airline always wants to forbid passengers from using hidden-city tickets. Now knowing that the airlines' revenues always decrease due to the hidden-city ticketing practices, the next immediate question is how significant the decrease could be. In Example 2, we have shown that if the airline does not react to the practice, the decrease could be up to the original revenue. However, in the following, we show that the airline can at least recover some of its revenue if it optimally reacts to the hidden-city ticket practice.

We first define some terms which will be useful in our later discussions.

Definition 1. *(Pricing Network): At any time t, we define the current pricing network PN to be the set of (p_{ij}^t, k_{ij}^t), for all $(i,j) \in \mathcal{O}$.*

Definition 2. *(Hidden-City Pair): For any pricing network PN, we call an ordered pair of cities $(i,j) \in \mathcal{O}$ to be a hidden-city pair if there exists another city l such that $k_{il}^t = j$ and $p_{ij}^t > p_{il}^t$.*

Definition 3. *(Hidden-City Branch): We define a hidden-city branch in a pricing network PN to be a hidden-city pair (i,j) along with the set of all cities l_s, $s = 1,...,n$ such that $k_{il_s}^t = j$ and $p_{ij}^t > p_{il_s}^t$. We denote such a hidden-city branch by $((i,j),\{l_1,..l_n\})$. And we call all l_s the final destinations of the hidden-city pair (i,j).*

Now we make the following assumption of our pricing network.

Assumption 4. *(Destination of Hidden-City Pair is not a Hub): The final destinations of hidden-city pairs are not used as connections at any time. To be more precise, assume the pricing network PN is the pricing network solved from (1) for some t and $\boldsymbol{x}$. If l_s is a final destination of a hidden-city pair of PN, then l_s is not used as a connection point in PN.*

The assumption that the destinations of hidden-city pairs are not connection points seems quite strong, however it is quite reasonable in practice. In practice, the hidden-city is usually a hub and the destinations of hidden-city pairs are usually non-hub cities. Therefore, the destinations of hidden-city pairs will usually not be connection points since in such network, nearly all connections take place in the hub (in fact, there may not be outbound flights from the destinations of hidden-city pairs other than towards hub cities). Now we make the discussion formal below:

Definition 4. *(Hub-and-Spoke Network) We call a flight network a hub-and-spoke network if the cities in this network can be divided into two categories: "hubs" and "spokes". The flights between spokes and hubs (also between two hubs) are direct and the flights between spoke cities are connected at one of the hubs.*

As several studies suggest (see e.g. GAO (2001, 2006)), most major airlines in the U.S. (as well as many smaller airlines) adopt a hub-and-spoke network strategy nowadays, because of its efficiency and capability to serve more O-D pairs. We have:

Proposition 2. *Any hub-and-spoke network satisfies Assumption 4.*

Therefore, Assumption 4 is not very restrictive in practice. With this assumption, we prove the following theorem showing that when the airlines optimally react to the hidden-city ticketing practice, the revenue loss can not exceed half of the revenue it can earn when the passengers do not practice hidden-city ticketing.

Theorem 3. *Let Assumption 3 and 4 hold and $V^t(\boldsymbol{x})$ and $\bar{V}^t(\boldsymbol{x})$ be as defined in (1) and (7). Then $\bar{V}^t(\boldsymbol{x}) \geq \frac{1}{2}V^t(\boldsymbol{x})$, for all t and $\boldsymbol{x}$.*

Proof. We prove by induction on t. The theorem is trivial for $t = 0$. Now we assume that the theorem holds for $t-1$ and will show that $\bar{V}^t(\boldsymbol{x}) \geq \frac{1}{2}V^t(\boldsymbol{x})$ for all $\boldsymbol{x}$.

To this end, we will construct a feasible solution $(\bar{p}_{ij}, \bar{k}_{ij})_{(i,j)\in\mathcal{O}}$ to (7) (i.e., a pricing network at time t). We will show that the constructed solution combined with the induction assumption can infer that $\bar{V}^t(\boldsymbol{x}) \geq \frac{1}{2}V^t(\boldsymbol{x})$ for all $\boldsymbol{x}$.

To construct $(\bar{p}_{ij}, \bar{k}_{ij})$, we start with the optimal solution to (1) when there are t time periods left and $\boldsymbol{x}$ capacity remaining. We denote it by $(p_{ij}, k_{ij})_{(i,j)\in\mathcal{O}}$. In our construction, we maintain all the connection points, i.e., we let $\bar{k}_{ij} = k_{ij}$ for all $(i,j) \in \mathcal{O}$. Then we make some adjustments to the prices p_{ij} such that the constraints in (7) are satisfied (therefore $(\bar{p}_{ij}, \bar{k}_{ij})$ will be a feasible solution to (7)). We do this by redefining prices in each hidden-city branch in the pricing network (p_{ij}, k_{ij}). For those O-D pairs that are not in any of the hidden-city branches, we keep the original prices p_{ij}. By Assumption 4, no two hidden-city branches contain the same O-D pair. This allows us to modify prices separately in different hidden-city branches. In the following, we focus on a single hidden-city branch $((i,j), \{l_1, l_2, ...l_n\})$ in the pricing network (p_{ij}, k_{ij}). By the definition of hidden-city branch, we know that $k_{il_s} = j$ and $p_{ij} > p_{il_s}$ for all $s = 1, ..., n$. Without loss of generality, we assume that $p_{ij} > p_{il_1} \geq ... \geq p_{il_n}$. We try to achieve the following goals when we redefine the prices $\bar{\boldsymbol{p}} = \{\bar{p}_{ij}\}_{(i,j)\in\mathcal{O}}$ (we also write $\boldsymbol{p} = \{p_{ij}\}_{(i,j)\in\mathcal{O}}$):

1. $\bar{p}_{il_s} \geq \bar{p}_{ij}$, i.e., the prices are feasible for (7);
2. The expected revenue earned in this branch using $\bar{\boldsymbol{p}}$ during period t is at least half as much as that earned by using $\boldsymbol{p}$, i.e.,

$$\lambda_{ij}^t(\bar{p}_{ij})\bar{p}_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(\bar{p}_{il_s})\bar{p}_{il_s} \geq \frac{1}{2}\left(\lambda_{ij}^t(p_{ij})p_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})p_{il_s}\right); \quad (8)$$

3. The expected opportunity costs due to the capacity consumption (related to the legs of this hidden-city branch) during period t when $\bar{\boldsymbol{p}}$ is used is no more than that when $\boldsymbol{p}$ is used, i.e.,

$$\begin{aligned} &\lambda_{ij}^t(\bar{p}_{ij})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})\right) + \textstyle\sum_{s=1}^{n} \lambda_{il_s}^t(\bar{p}_{il_s})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})\right) \\ \leq\; &\lambda_{ij}^t(p_{ij})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})\right) + \textstyle\sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})\right) \quad (9) \end{aligned}$$

In Appendix A, we show that we can always find $\bar{\boldsymbol{p}}$ such that the three goals above are satisfied for each hidden-city branch. With such $\bar{\boldsymbol{p}}$, we can conclude that:

$$\begin{aligned}
\bar{V}^t(\boldsymbol{x}) &\geq \bar{V}^{t-1}(\boldsymbol{x}) + \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij}) \left(\bar{p}_{ij} + \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{\bar{k}_{ij}}) - \bar{V}^{t-1}(\boldsymbol{x})\right) \\
&\geq \bar{V}^{t-1}(\boldsymbol{x}) + \frac{1}{2}\sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})p_{ij} + \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij}) \left(\bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}}) - \bar{V}^{t-1}(\boldsymbol{x})\right) \\
&= \frac{1}{2}\sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})p_{ij} + \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})\bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}}) + \left(1 - \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})\right)\bar{V}^{t-1}(\boldsymbol{x}) \\
&\geq \frac{1}{2}\left(\sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})p_{ij} + \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}}) + \left(1 - \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})\right)V^{t-1}(\boldsymbol{x})\right) \\
&= \frac{1}{2}\left(V^{t-1}(\boldsymbol{x}) + \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(p_{ij})\left(p_{ij} + V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}}) - V^{t-1}(\boldsymbol{x})\right)\right) = \frac{1}{2}V^t(\boldsymbol{x}),
\end{aligned}$$

where the first inequality is because that $\bar{\boldsymbol{p}}$ is a feasible solution to (7), the second inequality is due to the properties (8) and (9) of $\bar{\boldsymbol{p}}$, and the last inequality is because of the induction assumption and the assumption that λ's are sufficiently small. Therefore, the theorem is proved. □

Theorem 3 shows that by optimally reacting to hidden-city practice, the revenue of the airline could be reduced by at most a half (under Assumption 3 and 4). The following example shows that this bound is tight, that is, there exists such cases in which when the passengers practice hidden-city ticketing, the revenue will decrease by half even if the airline optimally responds to it.

Example 3. (The revenue can be reduced by half) We consider the same example as in Example 2 with $\beta = 0.1$ (it is easy to check that this example satisfies Assumption 4). The optimal pricing without hidden-city ticketing will be $p_{AC} = 100$ and $p_{AB} = \gamma$ with an optimal expected revenue of 20. However, if hidden-city tickets are used by the passengers, the optimal pricing strategy will be to price 100 for both O-D pairs and the expected revenue is $10 + 1000/\gamma$. By choosing γ sufficiently large, the ratio can be made arbitrarily close to 1/2. □

5 Changes in Prices and Consumer Surplus when Hidden-City Ticketing is Practiced and the Game between Passengers and Airlines

In Section 4, we studied the effect of hidden-city ticketing practices on the airlines' revenues. We showed that the airlines should react to such practices by taking them into account when making price decisions, however, even with optimal reactions, the airlines still suffer revenue losses. Now there is one important question remaining. That is, given that the airline would anticipate such practices and change its prices accordingly, whether the passengers still have the incentive to practice the hidden-city ticketing? In this section, we analyze the price changes when the hidden-city tickets are practiced (and anticipated by the airlines), as well as the changes of the consumer surplus. We also form a game to analyze the equilibrium behaviors of the passengers and the airlines. We show that even from the passenger's perspective, practicing hidden-city ticketing may not be a preferred strategy in the long term.

5.1 The changes of prices

In this section, we compare the optimal prices of the airlines between two cases: 1) the passengers do not practice hidden-city ticketing and 2) the passengers practice hidden-city ticketing and the airlines can anticipate it and optimally react to it. We first prove a theorem for a special case when the passengers practice hidden-city only in one period (the first period) and then discuss the general scenarios.

We consider the dynamic programming model we set up in (1). Let $(p_{ij}^t, k_{ij}^t)_{(i.j)\in\mathcal{O}}$ be the optimal solutions to (1) when there are t periods left and $\boldsymbol{x}$ capacity remaining (remember that these are the optimal decisions when the passengers do not use hidden-city tickets). Now we assume that the passengers would take advantage of any hidden-city opportunities available in this period (period t only). To respond, the airline's optimal decisions $(\tilde{p}_{ij}^t, \tilde{k}_{ij}^t)_{(i,j)\in\mathcal{O}}$ in this period are:

$$\begin{aligned}
&\arg\max_{p_{ij},k_{ij},\forall(i,j)\in\mathcal{O}} \left\{ \sum_{(i.j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij})\left(\bar{p}_{ij} + V^{t-1}(\boldsymbol{x} - \bar{A}_{ij})\right) + \left(1 - \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij})\right) V^{t-1}(\boldsymbol{x}) \right\} \\
= &\arg\max_{p_{ij},k_{ij},\forall(i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \lambda_{ij}^t(\bar{p}_{ij})\left(\bar{p}_{ij} + V^{t-1}(\boldsymbol{x} - \bar{A}_{ij}) - V^{t-1}(\boldsymbol{x})\right) \right\},
\end{aligned}$$

where

$$\bar{p}_{ij} = \min\{p_{ij}, \min_{l:k_{il}=j} p_{il}\}, \qquad \bar{k}_{ij} = \begin{cases} \mathrm{argmin}_{l:k_{il}=j} p_{il}, & \text{if } \min_{l:k_{il}=j} p_{il} < p_{ij} \\ 0, & \text{if } \min_{l:k_{il}=j} p_{il} \geq p_{ij} \end{cases}$$

and

$$\bar{A}_{ij} = \begin{cases} A_{ij}^{k_{ij}}, & \text{if } \bar{k}_{ij} = 0 \\ A_{i\bar{k}_{ij}}^{j}, & \text{if } \bar{k}_{ij} \neq 0 \end{cases}.$$

We have the following theorem regarding the changes of prices.

Theorem 4. *Let Assumptions 3 and 4 hold. Consider any hidden-city pair (i, j) in the pricing network $(p_{ij}^t, k_{ij}^t)_{(i,j)\in\mathcal{O}}$, and any destination l of this hidden-city pair. Then we have 1) $\tilde{p}_{il}^t \geq p_{il}^t$, 2) $\tilde{p}_{ij}^t \leq p_{ij}^t$.*

The proof of Theorem 4 is relegated to Appendix B. Theorem 4 states that the fares between the hidden-city pair will decrease and the fares from the origin to the destinations of the hidden-city pair will increase if hidden-city tickets are used by the passengers. The latter part of this finding, although only under a one-period model, is consistent with the discussions in the literature. For example, in GAO (2001), the authors conclude "... if legislation required airlines to permit hidden-city ticketing, airfares in certain markets (i.e., for travel between certain spoke communities connecting over a hub) could increase immediately... ". A similar conclusion is also hinted in a letter from American Airlines to its passengers (see American Airlines (2013)).

It may be tempting to extend Theorem 4 to the full multi-period model, i.e., comparing the optimal prices in (1) to that in (6). Remember that we have proved in Theorem 1 that the optimal prices to (6) should have eliminated all hidden-city opportunities. And intuitively, to eliminate such opportunities, prices to the hidden-city destination should increase and the prices between the hidden-city pair should decrease. Unfortunately, as we show in an example in Appendix C, this is not necessarily true. However, the example we find is not well-behaved since it requires drastically different demand functions for the same O-D pair in different periods. Exploring to what extent Theorem 4 holds is one interesting direction of future studies.

5.2 Consumer Surplus

Now we consider how the consumer surplus (the surplus of the passengers) would change if they practice hidden-city ticketing and the airlines react to it.

In this section, we confine our study to one period and a single hidden-city branch. This is partly because it is extremely difficult to study the consumer surplus in a multi-period model, since it involves a recursive computation of the surplus using the optimal solutions of the dynamic program (1) to each state, which is not possible to compute. More importantly, as will be shown soon, a one period model is already versatile enough to capture the main insights of this problem and yield nontrivial results. Also since we are interested in how hidden-city ticketing practices affect the consumer surplus, it suffices to focus on those routes whose prices contain hidden-city opportunities. Therefore it suffices for us to consider each hidden-city branch as defined in Section 4.

In the following, we consider a hidden-city branch $((i,j),\{l_1,...,l_n\})$ (under the optimal price solved from (1)) and assume there is sufficient capacity on all the legs on this branch[7]. We are interested in the total consumer surplus obtained on this hidden-city branch. We denote the demand function of the O-D pair $i \to j$ by $\lambda_0(\cdot)$, and the demand functions of the O-D pair $i \to l_i$ by $\lambda_i(\cdot)$. Define $p_i = \arg\max_p\{p\lambda_i(p)\}$, $i = 1,...,n$. Since there is only one period left, it is easy to see that p_i's are the optimal prices for each O-D pair when passengers do not practice hidden-city ticketing. And by the definition of hidden-city branch, we have $p_0 > p_i$, for $i = 1,...,n$.

Now we consider the optimal prices when the passengers practice hidden-city ticketing and the airlines optimally react to such practices. By Theorem 1 and its ensuing discussions, the optimal prices $\{\tilde{p}_i\}_{i=0}^n$ should be the optimal solution to the following optimization problem:

$$\begin{aligned} \max_{\tilde{p}_0,\ldots,\tilde{p}_n} \quad & \textstyle\sum_{i=0}^n \tilde{p}_i\lambda_i(\tilde{p}_i) \\ \text{s.t.} \quad & \tilde{p}_0 \le \tilde{p}_i, \qquad \forall i = 1,...,n. \end{aligned}$$

Now we define the consumer surplus on each O-D pair. We follow the standard economics literature (see Frank (2008)). For any good, given the demand function $\lambda(p)$ and the selling price p^*, the consumer surplus of this good is:

$$CS = \int_{p^*}^{\infty} \lambda(p)dp.$$

By this definition, our remaining task is to compare $CS_1 = \sum_{i=0}^n \int_{p_i}^{\infty} \lambda_i(p)dp$ (the consumer surplus when passengers do not practice hidden-city ticketing) and $CS_2 = \sum_{i=0}^n \int_{\tilde{p}_i}^{\infty} \lambda_i(p)dp$ (the consumer surplus when passengers practice hidden-city ticketing and the airlines optimally react to it). In fact, this is a special case of studying the effect of monopoly third-degree price discrimination on consumer surplus. To see this, one can view the passengers to different destinations as different segments of consumers (in the one period model, the fact that the two segments of consumers use different amount of capacity is not important, since the inventory cost is sunk and we have assumed that there is always enough capacity). In particular, the comparison between CS_1 and CS_2 is equivalent to study the effect of third-degree price discrimination on consumer surplus when $n = 1$ (only one hidden-city destination). This is because that when $n = 1$, it is easy to see that $\tilde{p}_0 = \tilde{p}_1$, therefore CS_2 is the consumer surplus without third-degree price discrimination (and CS_1 is the consumer surplus with third-degree price discrimination). Recently, this problem is studied by Cowan (2012). We here include the results for the completeness. We first define some notations.

[7] This is without loss of generality since one can scale the demand rate to make it small enough and thus satisfy our model assumption in Section 2 (the probability of having a purchase is small).

In the following, we define $p(\lambda)$ to be the inverse demand function (here we omit subscripts for the ease of notation) and $E(\lambda) = \frac{p}{p'(\lambda)\lambda}$ to be the price elasticity of demand when the demand is λ. Define

$$P(c) = \arg\max_p (p-c)\lambda(p)$$

to be the optimal price when a cost c is embedded in the objective function. And we call $P'(c)$ the cost pass-through coefficient at c. As shown in Bulow and Pfleiderer (1983), $P'(c) = \frac{1}{2-\sigma(\lambda^*)}$ where $\sigma(\lambda) = -\frac{\lambda p''(\lambda)}{p'(\lambda)}$ and λ^* is the optimal sales quantity at $P(c)$. We also define $\bar{p}$ to be the non-discriminatory price, that is, $\bar{p}$ maximizes $p(\lambda_0(p) + \lambda_1(p))$. Then the following theorem compares the consumer surplus CS_1 and CS_2:

Theorem 5. *Assume that* $-E(\lambda) \geq \frac{P(c)P''(c)}{[P'(c)]^2}$ *for all* c. *Then*

1. $CS_1 \geq CS_2$ *if the ratio of the cost pass-through at* 0 *to the absolute value of price elasticity of demand at the non-discriminatory sales quantity, is at least as high on the O-D pair* $i \to l_1$ *as on the O-D pair* $i \to j$, *i.e.,*

$$P_1'(0)/|E_1(\lambda_1(\bar{p}))| \geq P_0'(0)/|E_0(\lambda_0(\bar{p}))|;$$

2. $CS_1 \leq CS_2$ *if*

$$P_0(0)P_0'(0)\frac{\lambda_0'(p_0)}{\lambda_0'(\bar{p})} \geq P_1(0)P_1'(0)\frac{\lambda_1'(p_1)}{\lambda_1'(\bar{p})}.$$

The proof of the theorem is referred to Proposition 1 in Cowan (2012). Two corollaries follow immediate from Theorem 5.

Corollary 1. *If the demand functions on both O-D pairs are linear, i.e.,* $\lambda_i(p) = a_i - b_i p$, *for some* a_i, b_i, *then* $CS_1 \leq CS_2$.

Proof of Corollary 1. According to Bulow and Pfleiderer (1983), the cost-pass through is 0.5 for linear demand functions. Therefore, $P_0'(0) = P_1'(0) = 0.5$ and also the assumption in Theorem 5 holds ($P''(c) = 0$). Also for linear demand functions, $\frac{\lambda_0'(p_0)}{\lambda_0'(\bar{p})} = \frac{\lambda_1'(p_1)}{\lambda_1'(\bar{p})} = 1$. Moreover, by definition, we have $P_0(0) = p_0 \geq p_1 = P_1(0)$. Therefore, according to case 2 of Theorem 5, $CS_1 \leq CS_2$. □

Corollary 2. *If the demand functions on both O-D pairs are logit demand functions, i.e.,* $\lambda_i(p) = \frac{b_i}{1+e^{(p_i - a_i)}}$ *for some* a_i, b_i, *and moreover,* $a_i < 2$, *then* $CS_1 \geq CS_2$.

Proof of Corollary 2. For a logit demand function, the optimal price can be solved by the identity $e^{(p_i - a_i)}(p_i - 1) = 1$. When $a_i < 2$, the optimal p_i must be greater than a_i. And as shown in Cowan (2012), the assumption in Theorem 5 holds in this case. To show that the condition in case 1 in Theorem 5 holds, we have for logit demand functions, the cost pass through is $1 - \lambda(P(0))$ and the price elasticity of demand is $-P(0)(1 - \lambda(P(0))$. Therefore, we have

$$P_1'(0)/|E_1(\lambda_1(\bar{p}))| = 1/p_1 \geq 1/p_0 = P_0'(0)/|E_0(\lambda_0(\bar{p}))|$$

and thus $CS_1 \geq CS_2$ by Theorem 5. □

Two examples corresponding to the cases in Corollary 1 and 2 are shown in Appendix D. The examples verify the relationship between the consumer surplus as indicated in Theorem 5. And since Theorem 5 concerns a special case of the general problem, the result that the consumer surplus could be either lower or higher when the passengers practice hidden-city ticketing also holds in general.

The results in Theorem 5 are important for understanding the consequences of the hidden-city ticketing practice. It provides conditions under which the consumer surplus when hidden-city ticketing is practiced by the passengers is greater or less than that when hidden-city ticketing is not practiced by the passengers. In particular, it shows that under some conditions, the consumer surplus when the passengers practice hidden-city ticketing is lower. This is quite unintuitive, since typically the consumer surplus is higher if they are endowed with more freedom in purchase choices (in this case, more freedom means that they could purchase a hidden-city ticket and throw away the latter half of it). However, this theorem shows that if the passengers do that, the externality the practices exert (through the reactions of the airlines) in fact hurts the passengers themselves, given certain forms of demand functions. And such demand functions include very commonly-used demand functions such as logit demand functions. Therefore, even from the passengers perspective, it could also be harmful to practice hidden-city ticketing, as their ticketing strategy. We will formalize this point in the next section.

5.3 Game between passengers and airlines and the potential impacts of hidden-city ticketing practice

In this section, we further extend the analysis earlier and build a game between the passengers and the airlines to study the equilibrium strategy for each player. We also comment on the potential caveat when using our analysis.

The game we build is a straightforward one. There are two players in the game, the airline and the passengers. Here we assume that the passengers is the whole group of travelers, instead of each individual ones. For the passengers, their intended O-D pairs are fixed, the only decision (strategy) is whether to practice hidden-city ticketing when given such opportunities (P) or not (N). And the strategy for the airlines is simply its pricing decisions. For simplicity, we only allow airlines to choose between 1) not incorporating hidden-city practice in their pricing model (i.e., to solve (1) which is denoted by strategy N) and 2) incorporating hidden-city practice into their pricing model (i.e., to solve (7), which is denoted by strategy I). The payoff matrix of this game is shown in Table 1:

		Passengers	
		N	P
Airlines	N	(R_1, CS_1)	(R', CS')
	I	(R_2, CS_2)	(R_2, CS_2)

Table 1: Payoff matrix of the game between the airline and the passengers

In Table 1, the payoffs are airlines' revenues and consumer surplus. For airlines' revenues, $R_1 = V^t(\boldsymbol{x})$ and $R_2 = \bar{V}^t(\boldsymbol{x})$ are the optimal values of (1) and (7) respectively, when there are t periods and $\boldsymbol{x}$ capacity left. R' is the expected revenue when the airline uses the solution to (1) in the dynamic program (7). We have proved in Section 4 that $R' \leq R_2 \leq R_1$. For the consumers, CS_1 and CS_2 are defined in Section 5.2, and CS' equals to CS_1 plus the money saved by using hidden-city tickets. It can be seen that no matter which strategy the airline takes, the passengers will always prefer to practice hidden-city ticketing. Therefore, practicing hidden-city ticketing is a dominating strategy for the passengers. And given that strategy, the airline's best response is to react, that is to use the prices solved from (7). Therefore, (I, P) is the unique equilibrium in this game with payoff (R_2, CS_2). In the following, we discuss the equilibrium in two cases.

The first case is when $CS_1 \geq CS_2$. In this case, although the unique equilibrium outcome is (I, P), the equilibrium payoff is not Pareto optimal. Both the airline and the passengers would prefer the outcome (N, N) whose payoff is (R_1, CS_1). In fact, one may identify that this is exactly the same situation as the classic prisoner's dilemma in game theory. If the game is played only once (or a finite number of times), the players may not cooperate, and the equilibrium would be (R_2, CS_2). However, if the game is played repeatedly, then a cooperative solution may arise (see Fudenberg and Tirole (1991) for the related discussions) in which (N, N) is played. This can be viewed as a practical explanation to this problem, since in reality, the interactions between passengers and airlines are usually in a long term and thus passengers may have the incentive to *abide the rules* of the airlines in order to obtain long term benefits from the low fares. And by such cooperations, both players can achieve long term benefits.

The second case is when $CS_1 \leq CS_2$. In this case, the equilibrium outcome (I, P) is also Pareto optimal. Therefore, it appears that the passengers will always prefer to practice hidden-city ticketing and the airlines will eventually change its price to respond such practice. However, this analysis is only confined to the case in which the airline's only strategy is the fares. In reality, there are other decisions that the airlines can make, e.g., which route to offer. If the decrement of revenue (from R_1 to R_2) is large enough such that offering a certain route is no longer profitable, the airlines may eventually suspend or reduce the capacity of that service. If that happens, the airline may raise the fares due to the reduction of capacity. And eventually, this might also hurt the passengers as air travels become less convenient and more expensive.

Lastly, we want to point out some caveats of the above analysis. The above analysis is based on the assumption that the passengers act as a single group. That is, either all the passengers practice hidden-city ticketing or none of them do. And we use the total consumer surplus as the payoff for the passengers. In reality, each individual passenger has his own surplus function, and may take different actions based on it. For example, if a passenger frequently travels on a mix of hub-to-hub and spoke-to-spoke routes, he might be less willing to practice hidden-city tickets as the optimal reaction of the airlines (increasing the fare to the spoke cities) will also negatively impact his welfare. However, if a passenger only travels between two hubs, then he might not worry as much. Also, in practice, only a handful passengers may be aware of the hidden-city opportunities and are able to practice it and the above analysis has to be used with caution in that case. We provide some basic analysis for this case (when only a portion of passengers use hidden-city tickets) in Appendix E, and leave the rest for future studies.

6 Numerical Results

In this section, we provide some numerical test results. Our numerical experiments are based on some small test problems, therefore by no means would emulate the complexity of the problems faced in practice. Yet they are useful in illustrating our results in the earlier sections and providing intuitions for more comprehensive empirical studies in the future.

In our numerical tests, we consider a small hub-and-spoke network with four cities and three flight legs as shown in Figure 3. We assume there are demands between A to B, A to C and A to D. And flights from A to C and D are connected at B. We assume the capacity on the flight legs $A \to B$, $B \to C$, $B \to D$ are 45, 15 and 10 respectively and there are totally 200 time periods before departure.

In our numerical study, we assume that in each period, there is a probability η that a passenger intended for each O-D pair arrives. We vary η = 0.1, 0.2, 0.3 and 0.4 to represent different level of the load factor[8]. And given a passenger arrives, we use a logit function to characterize the purchase probability. Combined them together, the demand functions are defined as follows:

[8] In our model, we don't have a straightforward definition of the load factor as in some of the prior work, since at each time period, there is also a choice function, which typically is much less than 1. Therefore, in this section, we simply use the load factor to refer to the relative intensity of the passenger arrivals.

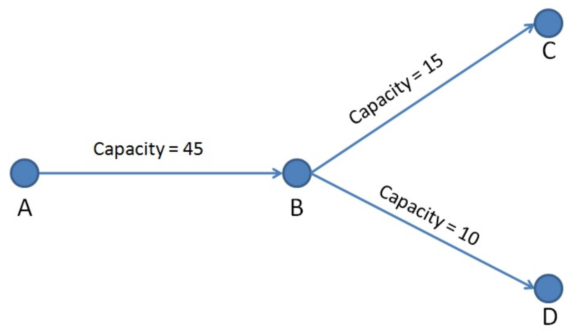

Figure 3: A Small Flight Network.

$$\lambda_{AB}^{t}(p_{AB}) = \frac{\eta \exp(-\beta_1 p_{AB})}{\alpha_1 + \exp(-\beta_1 p_{AB})}$$

$$\lambda_{AC}^{t}(p_{AC}) = \frac{\eta \exp(-\beta_2 p_{AC})}{\alpha_2 + \exp(-\beta_2 p_{AC})}$$

$$\lambda_{AD}^{t}(p_{AD}) = \frac{\eta (\exp -\beta_3 p_{AD})}{\alpha_3 + \exp(-\beta_3 p_{AD})}.$$

In our experiments, we fix $\alpha_2 = 1$, $\alpha_3 = 1.5$, $\beta_1 = 0.01$, $\beta_2 = 0.01$ and $\beta_3 = 0.008$. Then we choose different values of α_1 to see the effect of different market environments on the hidden-city opportunities on this network. Note that a smaller α_1 means that the passenger has a higher chance to choose this flight for a given price, or it could be interpreted as the market competition on the O-D pair $A \rightarrow B$ is relatively low (or the price elasticity of demand is low). The opposite is true when α_1 is large. In our experiments, we choose α_1 to be 0.05, 0.1, 0.25, 0.5 and 1. All the computation results are shown in Table 2.

In Table 2, V^* is the optimal revenue of the airline when hidden-city ticketing is not practiced by the passengers, i.e., the optimal value of the dynamic program (1). $\tilde{V}$ is the revenue obtained by using the policy solved by (1), however, all the passengers take advantage of hidden-city opportunities whenever possible. $\bar{V}$ is the revenue obtained by the optimal response of the airlines when hidden-city ticketing is practiced by the passengers, i.e., the optimal value of the dynamic program (6). In the next six rows, p^*'s show the optimal prices for the three O-D pairs at the first time period ($t = 200$) when hidden-city tickets are not used; and $\bar{p}$'s show the corresponding optimal prices when passengers practice hidden-city ticketing. The letter "H" in the parenthesis identifies hidden-city opportunities in the price network. And since the size of this problem is relatively small, the numbers in Table 2 are all solved from the exact dynamic program, not from an approximation.

In Table 2, we can see that when α_1 is small, there could be hidden-city opportunities existing in the network. As discussed earlier, a smaller α_1 means a lower price elasticity of demand on the O-D pair $A \rightarrow B$, comparing to that of $A \rightarrow C$ and $A \rightarrow D$. Therefore, this observation is in line with our discussions of the cause of hidden-city opportunities in Section 3. Also, for each case where hidden-city opportunities exist, the airline's revenue could be hurt quite badly if it does not make price adjustments ($\tilde{V}$). In our tests, the revenue could be reduced by as much as

$\eta = 0.1$ $\alpha_1 =$	0.05	0.1	0.25	0.5	1
V^*	4162.0	3366.7	2488.6	1978.9	1609.9
$\tilde{V}$	3227.6(−22.45%)	2938.1(−12.73%)	2401.4(−3.50%)	1967.8(−0.56%)	1609.9(−0.00%)
$\bar{V}$	3924.8(−5.70%)	3251.0(−3.16%)	2462.6(−1.08%)	1974.8(−0.21%)	1609.9(−0.00%)
p^*_{AB}	256.0	216.3	172.0	148.3	128.0
p^*_{AC}	128.1(H)	128.1(H)	128.1(H)	128.1(H)	128.1
p^*_{AD}	150.7(H)	150.7(H)	150.7(H)	150.7	150.7
$\bar{p}_{AB}$	230.2	192.0	157.9	140.4	128.0
$\bar{p}_{AC}$	230.2	192.0	157.9	140.4	128.1
$\bar{p}_{AD}$	230.2	192.0	157.9	150.7	150.7

$\eta = 0.2$ $\alpha_1 =$	0.05	0.1	0.25	0.5	1
V^*	8258.5	6695.8	4950.8	3932.3	3194.2
$\tilde{V}$	6889.0(−16.58%)	5997.1(−10.43%)	4799.3(−3.26%)	3915.3(−0.43%)	3194.2(−0.00%)
$\bar{V}$	7839.4(−5.07%)	6489.3(−3.08%)	4903.5(−0.96%)	3925.5(−0.17%)	3194.2(−0.00%)
p^*_{AB}	261.2	219.2	172.5	148.0	128.0
p^*_{AC}	140.7(H)	135.6(H)	131.9(H)	131.6(H)	131.6
p^*_{AD}	173.3(H)	168.1(H)	165.5(H)	165.3	165.3
$\bar{p}_{AB}$	232.0	194.5	159.3	140.6	128.0
$\bar{p}_{AC}$	232.0	194.5	159.3	140.6	131.6
$\bar{p}_{AD}$	232.0	194.5	165.5	165.3	165.3

$\eta = 0.3$ $\alpha_1 =$	0.05	0.1	0.25	0.5	1
V^*	11478.8	9488.8	7188.4	5750.0	4655.0
$\tilde{V}$	10677.4(−6.98%)	9033.5(−4.80%)	7073.9(−1.59%)	5741.7(−0.14%)	4655.0(−0.00%)
$\bar{V}$	11223.3(−2.22%)	9340.6(−1.56%)	7147.5(−0.59%)	5746.7(−0.06%)	4655.0(−0.00%)
p^*_{AB}	296.4	248.3	187.2	151.8	128.2
p^*_{AC}	206.8(H)	182.3(H)	157.3(H)	148.8(H)	146.8
p^*_{AD}	238.5(H)	219.0(H)	201.2	195.9	194.5
$\bar{p}_{AB}$	271.6	227.2	176.5	149.7	128.2
$\bar{p}_{AC}$	271.6	227.2	176.5	149.7	146.8
$\bar{p}_{AD}$	271.6	227.2	201.2	195.9	194.5

$\eta = 0.4$ $\alpha_1 =$	0.05	0.1	0.25	0.5	1
V^*	13615.4	11428.2	8903.4	7279.5	5933.9
$\tilde{V}$	13148.9(−3.43%)	11148.7(−2.45%)	8828.9(−0.84%)	7279.5(−0.00%)	5933.9(−0.00%)
$\bar{V}$	13496.9(−0.87%)	11348.9(−0.69%)	8876.7(−0.30%)	7279.5(−0.00%)	5933.9(−0.00%)
p^*_{AB}	332.3	281.2	217.0	169.4	133.2
p^*_{AC}	266.4(H)	231.8(H)	193.2(H)	175.5	169.9
p^*_{AD}	296.1(H)	267.0(H)	240.7	229.6	223.9
$\bar{p}_{AB}$	316.3	267.3	208.2	169.4	133.2
$\bar{p}_{AC}$	316.3	267.3	208.2	175.5	169.9
$\bar{p}_{AD}$	316.3	267.3	240.7	229.6	223.9

Table 2: Revenues and optimal prices with/without hidden-city ticketing

22.4%. When the airlines choose to react to the hidden-city ticketing practices ($\bar{V}$), in average, more than 70% of the loss can be recovered. Yet, for all cases, $\bar{V}$ is less than V^*, meaning that the airlines would still suffer losses even with the optimal reaction. However, as α_1 increases, the difference of the price elasticity of demand between the O-D pair $A \to B$ and the other two becomes smaller, and the hidden-city opportunities gradually diminish. Meanwhile, the revenue losses are also much smaller in those cases. When α_1 achieves a certain level, the hidden-city opportunities will partially or completely disappear. These observations are all in accordance with our discussions in Section 3 and 4.

Now we study the changes of optimal prices when hidden-city ticketing are practiced by the passengers and the airlines optimally react to it. We see that in all cases, hidden-city opportunities will no longer exist in the airline's optimal reaction. Moreover, one can see that in the optimal reaction, the prices to the spoke cities rise ($A \to C$ and $A \to D$) while the prices to the connection city ($A \to B$) fall. This result is consistent with the claim in Theorem 4.

Lastly, we study the effect of the load factor on the hidden-city phenomenons. We see that when the load factor is higher (larger η), the hidden-city opportunity is less obvious and the revenue losses are smaller. This can be partially explained as follows. When the load factor is high, the demand is ample, therefore, the airline does not need to offer very low prices to the spoke city in the early period, which would otherwise cause large differences in the price. Also, since the overall prices will be higher when the load factor is higher, the magnitude of the relative revenue loss would be smaller. Overall, we find that higher load factor helps mitigate the effect of hidden-city ticketing.

7 Conclusion

In this paper, we build, for the first time, a mathematical model to study the hidden-city ticketing phenomenon in airline ticket pricing. We consider a network revenue management model and show that the hidden-city opportunities could arise when there is a large difference in the price elasticity of demand in different O-D pairs. We show that if passengers take advantage of the hidden-city opportunities, the airlines had better to react in price, otherwise their revenue could be seriously affected. However, even with the optimal reaction, the airline's revenue will still be reduced, and we show that in our model, the reduction could be as much as half of the original optimal revenue but not more. We also study the changes of prices and consumer surplus when the passengers practice hidden-city ticketing and the airlines anticipate such practices and react optimally. Our results indicate that even from the passengers perspective, it might not be of their best interests to practice hidden-city ticketing. We believe that these results could provide important information for the decision makers involved such as the airlines, passengers and the law makers.

There are many possible directions for future research. First, in reality, not all of the passengers are aware of hidden-city opportunities and not all of them are able to use them. Therefore, a thorough study of how the fraction of passengers who practice hidden-city ticketing affects the current result would be useful. Secondly, in practice, the network is not necessarily a perfect hub-and-spoke network, and passengers occasionally have to take flights with more than two connections. It would be interesting to see to what extent our model still works in that case. Thirdly, it is really of great interest to see that in a network of real scale, what would be the actual impact of hidden-city ticketing on the airlines' revenues, as well as the consumer surplus. Currently, the related literature is quite limited, both empirical ones and theoretical ones. And we hope that our paper could provide a starting point for further studies on this problem.

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A Proof of Theorem 3

We show that for any hidden-city branch $((i,j),\{l_1,...,l_n\})$, and any $\boldsymbol{p}$, there always exists a modification $\bar{\boldsymbol{p}}$ on this hidden city branch such that

1. $\bar{p}_{il_s} \geq \bar{p}_{ij}$, for all s, i.e., $\bar{\boldsymbol{p}}$ is feasible for (7);
2. The expected revenue earned in this branch using $\bar{\boldsymbol{p}}$ during period t is at least half as much as that earned by using $\boldsymbol{p}$, i.e.,

$$\lambda_{ij}^t(\bar{p}_{ij})\bar{p}_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(\bar{p}_{il_s})\bar{p}_{il_s} \geq \frac{1}{2}\left(\lambda_{ij}^t(p_{ij})p_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})p_{il_s}\right) \tag{10}$$

3. The expected opportunity costs due to the capacity consumption (related to the legs of this hidden-city branch) during period t when $\bar{\boldsymbol{p}}$ is used is no more than that when $\boldsymbol{p}$ is used, i.e.,

$$\begin{aligned} & \lambda_{ij}^t(\bar{p}_{ij})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})\right) + \textstyle\sum_{s=1}^n \lambda_{il_s}^t(\bar{p}_{il_s})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})\right) \\ \leq\ & \lambda_{ij}^t(p_{ij})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})\right) + \textstyle\sum_{s=1}^n \lambda_{il_s}^t(p_{il_s})\left(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})\right) \end{aligned} \tag{11}$$

In the following, we consider two cases:

- Case 1

$$\lambda_{ij}^t(p_{ij})p_{ij} \geq \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})p_{il_s}.$$

This is the case when the revenue generated from the O-D pair (i,j) dominates the revenue generated from all other O-D pairs in this hidden-city branch. In this case, we define $\bar{p}_{il_s} = \bar{p}_{ij} = p_{ij}$ for all $s = 1,...,n$. By definition, $\bar{\boldsymbol{p}}$ is feasible for (7) in this hidden-city branch. Furthermore, we have

$$\lambda_{ij}^t(\bar{p}_{ij})\bar{p}_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(\bar{p}_{il_s})\bar{p}_{il_s} \geq \lambda_{ij}^t(p_{ij})p_{ij} \geq \frac{1}{2}\left(\lambda_{ij}^t(p_{ij})p_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})p_{il_s}\right),$$

i.e., (10) holds. Furthermore, (11) must also hold since $\bar{p}_{ij} = p_{ij}$, $\bar{p}_{il_s} \geq p_{il_s}$ and that $\lambda_{il_s}^t(\cdot)$'s are decreasing functions. Therefore, $\bar{\boldsymbol{p}}$ satisfies the above three properties in this hidden-city branch.

- Case 2

$$\lambda_{ij}^t(p_{ij})p_{ij} < \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})p_{il_s}. \tag{12}$$

In this case, we choose price p that satisfies the following equation:

$$\lambda_{ij}^t(p) + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s} \vee p) = \lambda_{ij}^t(p_{ij}) + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s}), \tag{13}$$

where $a \vee b = \max\{a, b\}$. Intuitively, we choose a price p such that the capacity consumption on leg $i \to j$ is the same as that when no hidden-city tickets are used. Then we define $\bar{p}_{ij} = p$ and $\bar{p}_{il_s} = \max\{p, p_{il_s}\}$.

First we claim that such p always exists. This follows from the Rolle's Mean Value Theorem. Note that the left hand side of (13) is smaller than the right hand side when $p = p_{ij}$, and it is larger when $p = p_{il_n}$. Also by Assumption 3, the left hand side of (13) is a continuous function of p. Therefore there must exist p such that (13) holds.

Now we study the expected revenue and the opportunity costs due to capacity consumption when $\bar{\boldsymbol{p}}$ is used. Define $p_{il_0} = p_{ij}$. Let $0 \le k \le n-1$ be the index such that $p_{il_k} > p \ge p_{il_{k+1}}$. Note that by the way we choose p, such a k must exist. For the expected revenue, we have:

$$\begin{aligned}
& \lambda_{ij}^t(\bar{p}_{ij})\bar{p}_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(\bar{p}_{il_s})\bar{p}_{il_s} \\
= \;& \lambda_{ij}^t(p)p + \sum_{s=1}^{k} \lambda_{il_s}^t(p_{il_s})p_{il_s} + \sum_{s=k+1}^{n} \lambda_{il_s}^t(p)p \\
= \;& (\lambda_{ij}^t(p) + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s} \vee p)) \cdot p + \sum_{s=1}^{k} \lambda_{il_s}^t(p_{il_s})(p_{il_s} - p) \\
= \;& (\lambda_{ij}^t(p_{ij}^t) + \sum_{s=1}^{n} \lambda_{il_S}^t(p_{il_s})) \cdot p + \sum_{s=1}^{k} \lambda_{il_s}^t(p_{il_s})(p_{il_s} - p) \\
\ge \;& \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})(p_{il_s} \vee p) \\
\ge \;& \frac{1}{2}\left(\lambda_{ij}^t(p_{ij})p_{ij} + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})p_{il_s}\right).
\end{aligned}$$

where the third equality is because of (13) and the last inequality is because of the case assumption (12). Therefore, (10) holds. For the expected opportunity costs due to the capacity consumptions (related to the legs in this hidden-city branch), we have

$$\begin{aligned}
& \lambda_{ij}^t(\bar{p}_{ij})(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})) + \sum_{s=1}^{n} \lambda_{il_s}^t(\bar{p}_{il_s})(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})) \\
= \;& \lambda_{ij}^t(p)(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})) + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s} \vee p)(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})) \\
= \;& \lambda_{ij}^t(p_{ij})(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})) + \sum_{s=1}^{n} \lambda_{il_s}^t(p_{il_s})(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})) \\
& + (\lambda_{ij}^t(p) - \lambda_{ij}^t(p_{ij}))(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})) \\
& + \sum_{s=1}^{n}(\lambda_{il_s}^t(p_{il_s} \vee p) - \lambda_{il_s}^t(p_{il_s}))(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j})).
\end{aligned}$$

However, we have

$$\begin{aligned}
&(\lambda_{ij}^t(p) - \lambda_{ij}^t(p_{ij}))(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})) \\
= &\sum_{s=1}^{n}(\lambda_{il_s}^t(p_{il_s}) - \lambda_{il_s}^t(p_{il_s} \vee p))(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}})) \\
\leq &\sum_{s=1}^{n}(\lambda_{il_s}^t(p_{il_s}) - \lambda_{il_s}^t(p_{il_s} \vee p))(\bar{V}^{t-1}(\boldsymbol{x}) - \bar{V}^{t-1}(\boldsymbol{x} - A_{il_s}^{j}))
\end{aligned}$$

where the first equality is because of (13). The second inequality is because the choice of the connection k_{ij} and the monotonicity of the value function in the remaining capacity. Therefore, (11) must hold for $\bar{\boldsymbol{p}}$.

Thus we have shown that for any hidden-city branch and any $\boldsymbol{p}$, we can find $\bar{\boldsymbol{p}}$ that satisfies the three properties. □

B Proof of Theorem 4

Before we prove Theorem 4, we introduce the following technical lemma.

Lemma 1. *Let Assumption 3 holds. Then for any $c_2 \geq c_1$, $p_1 = arg\max_p \lambda(p)(p - c_1)$ and $p_2 = arg\max_p \lambda(p)(p - c_2)$, we have $p_2 \geq p_1$.*

Proof. By the optimality condition, we have $\lambda'(p_1)(p_1 - c_1) + \lambda(p_1) = 0$. Therefore, since $\lambda'(p) \leq 0$, we have $\lambda'(p_1)(p_1 - c_2) + \lambda(p_1) \geq 0$. Then by Assumption 3 that $\lambda(p)(p - c_2)$ is quasiconcave and there exists a unique optimal solution, we must have $p_2 \geq p_1$. □

Proof of Theorem 4. We first prove that $\tilde{p}_{il}^t \geq p_{il}^t$. We prove by contradiction. Assume $\tilde{p}_{il}^t < p_{il}^t$. We will show contradictions in two cases.
Case 1: $\tilde{k}_{il}^t$ still equals to j. In this case, we show that by raising $\tilde{p}_{il}^t$ to p_{il}^t while maintaining all other prices, the revenue will increase. First, we argue that doing this will not create any new hidden-city opportunity for the pricing network $(\tilde{p}, \tilde{k})$. This is true because by Theorem 1, the pricing network $(\tilde{p}, \tilde{k})$ does not contain any hidden-city opportunity. And by Assumption 4, l can not be used as a connection point. Therefore, increase the fare of $i \to l$ will not create any hidden-city opportunity in this network. Then we claim that the revenue would increase. This is because $p_{il}^t = arg\max_p \lambda_{il}^t(p)(p + V^{t-1}(\boldsymbol{x} - A_{il}^j) - V^{t-1}(\boldsymbol{x}))$. Therefore, choosing p_{il}^t will increase the revenue over $\tilde{p}_{il}^t$.
Case 2: $j' = \tilde{k}_{il}^t$ no longer equals to j. In this case, we claim that $\tilde{p}_{il}^t$ must be no less than p_{il}^t. By definition, we have

$$p_{il}^t = arg\max_p \lambda_{il}^t(p)\left(p + V^{t-1}(\boldsymbol{x} - A_{il}^j) - V^{t-1}(\boldsymbol{x})\right).$$

Define

$$\bar{p}_{il}^t = arg\max_p \lambda_{il}^t(p)\left(p + V^{t-1}(\boldsymbol{x} - A_{il}^{j'}) - V^{t-1}(\boldsymbol{x})\right). \tag{14}$$

Since $k_{il}^t = j$, we must have $V^{t-1}(\boldsymbol{x} - A_{il}^j) \geq V^{t-1}(\boldsymbol{x} - A_{il}^{j'})$. Therefore, by Lemma 1, we have $\bar{p}_{il}^t \geq p_{il}^t$. Now we claim that $\tilde{p}_{il}^t \geq \bar{p}_{il}^t$. If not, we increase $\tilde{p}_{il}^t$ to $\bar{p}_{il}^t$. First, by Assumption 4, this will not create any new hidden-city opportunity for the pricing network $(\tilde{p}, \tilde{k})$ since l is not used as any connection point. And also, since $\bar{p}_{il}^t$ is optimal to (14), the revenue is increased. Therefore, we reach a contradiction in this case too and we have proved $\tilde{p}_{il}^t \geq p_{il}^t$.

Next we consider the second part. Suppose $\tilde{p}_{ij}^t > p_{ij}^t$. Then we modify the price network by lowering the price $\tilde{p}_{ij}^t$. First we claim that this will not create any new hidden-city opportunities because as stated in Assumption 4, j cannot be the destination of any hidden-city pair. Also k_{ij}^t must remain the same, because $V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t})$ is the maximum one among all possible connections from i to j. Using the fact that p_{ij}^t is the optimal solution to

$$\lambda_{ij}^t(p_{ij}^t)\left(p_{ij}^t + V^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t}) - V^{t-1}(\boldsymbol{x})\right),$$

we know that the revenue would increase if we lower $\tilde{p}_{ij}^t$ in this case. □

C A counterexample of Theorem 4 for multi-period model

Example 4. (Theorem 4 may not hold for the multi-period model) Consider the same network as shown in Figure 1. Assume there are 2 periods left and there is one seat on both flight leg $A \to B$ and $B \to C$. There are demands only for the itinerary $A \to B$ and $A \to C$.

Now we consider the following setup: in period $t = 1$ (the last time period), $\lambda_{AB}^1(p) = 0.2\max\{1 - p/200, 0\}$ and $\lambda_{AC}^1(p) = 0.2\max\{1 - p/100, 0\}$. In this case, when hidden-city tickets are not used, we have the optimal prices are $p_{AB}^1 = 100$ and $p_{AC}^1 = 50$, and the value function $V^1((1,1)) = 0.2 \cdot 0.5 \cdot 100 + 0.2 \cdot 0.5 \cdot 50 = 15$. In the case when hidden-city opportunity is taken by passengers (solve (7)), the optimal prices $\bar{p}_{AB}^1$ and $\bar{p}_{AC}^1$ can be computed by solving:

$$\begin{array}{cc} \max_{\bar{p}_{AB}^1, \bar{p}_{AC}^1} & \bar{p}_{AB}^1 \lambda_{AB}^1(\bar{p}_{AB}^1) + \bar{p}_{AC}^1 \lambda^1(\bar{p}_{AC}^1) \\ \text{s.t} & \bar{p}_{AB}^1 \leq \bar{p}_{AC}^1. \end{array}$$

The optimal solutions are $\bar{p}_{AB}^1 = \bar{p}_{AC}^1 = 200/3$ and the optimal revenue is $\bar{V}^1((1,1)) = 40/3$. Note that these prices are consistent with the results in Theorem 4 since it is the last period. Now we consider the following setup for period $t = 2$ (second to last period). We define $\lambda_{AB}^2(p) = 0.01\max\{1 - p/200, 0\}$ and $\lambda_{AC}^2(p) = 0.5\max\{1 - p/100, 0\}$ (note that the form of the demand function is the same, only the scaling factor is changed). Now for the case when hidden-city tickets are not used, according to (1), we solve the following optimization problem:

$$\max_{p_{AB}^2, p_{AC}^2} \quad \lambda_{AB}^2(p_{AB}^2)(p_{AB}^2 - 15) + \lambda_{AC}^2(p_{AC}^2)(p_{AC}^2 - 15)$$

and get the optimal prices $p_{AB}^2 = 107.5$ and $p_{AC}^2 = 57.5$ with optimal expected value $V^2((1,1)) = 24.4591$. For the case when the hidden-city opportunities are taken by the passengers, we solve the following optimization problem

$$\begin{array}{cc} \max_{\bar{p}_{AB}^2, \bar{p}_{AC}^2} & \lambda_{AB}^2(\bar{p}_{AB}^2)(\bar{p}_{AB}^2 - \frac{40}{3}) + \lambda_{AC}^2(\bar{p}_{AC}^2)(\bar{p}_{AC}^2 - \frac{40}{3}) \\ \text{s.t.} & \bar{p}_{AB}^2 \leq \bar{p}_{AC}^2 \end{array}$$

and get the optimal prices $\bar{p}_{AB}^2 = \bar{p}_{AC}^2 = 57.1617$ with optimal expected value $\bar{V}^2((1,1)) = 23.0269$ (this also verifies the relationship between V and $\bar{V}$ as stated in Theorem 2 and 3). Note that the optimal prices on both legs decrease in this case, which does not satisfy the conclusion in Theorem 4. This is because that the optimal prices of one period also changes with the value functions of future periods. In particular, when hidden-city tickets are used, the values of the seats are lower, and by Lemma 1, it may push down the price in the current period and result in a different price change direction as Theorem 4 predicts.

D Examples for Corollary 1 and 2

Example for linear demand function. Consider the network shown in Figure 1. There are two O-D pairs served by the airline, $A \to B$ and $A \to C$. And the demand functions are

$$\lambda_{AB}(p) = \max\{2 - p, 0\}, \quad \lambda_{AC}(p) = \max\{1 - p, 0\}.$$

It is easy to see that when passengers do not practice hidden-city ticketing, the optimal prices are $p_{AB} = 1$ and $p_{AC} = 0.5$. And the consumer surplus is

$$CS_1 = \int_1^2 (2 - p)dp + \int_{0.5}^1 (1 - p)dp = \frac{5}{8}.$$

Now if the passengers practice hidden-city ticketing, then the optimal prices for both O-D pairs are

$$\arg\max_p \quad p(\lambda_{AB}(p) + \lambda_{AC}(p)).$$

This yields $\tilde{p}_{AB} = \tilde{p}_{AC} = 0.75$. And the consumer surplus is

$$CS_2 = \int_{0.75}^2 (2 - p)dp + \int_{0.75}^1 (1 - p)dp = \frac{13}{16}$$

Therefore, $CS_2 \geq CS_1$ in this case. □

Example for logit demand function. Consider the same network, however, the demand functions are

$$\lambda_{AB}(p) = \frac{1}{1 + e^{p-1}}, \quad \lambda_{AC}(p) = \frac{1}{1 + e^p}.$$

One can compute that when passengers do not practice hidden-city ticketing, the optimal prices are $p_{AB} = 1.5671$ and $p_{AC} = 1.2785$. And the consumer surplus is

$$CS_1 = \int_{1.5671}^{\infty} \frac{1}{1 + e^{p-1}} dp + \int_{1.2785}^{\infty} \frac{1}{1 + e^p} dp = 0.6950.$$

Now if the passengers practice hidden-city ticketing, then the optimal prices for both O-D pairs are

$$\arg\max_p \quad p(\lambda_{AB}(p) + \lambda_{AC}(p)).$$

This yields $\tilde{p}_{AB} = \tilde{p}_{AC} = 1.4704$. And the consumer surplus is

$$CS_2 = \int_{1.4704}^{\infty} \frac{1}{1 + e^{p-1}} dp + \int_{1.4704}^{\infty} \frac{1}{1 + e^p} dp = 0.6923.$$

Therefore, $CS_1 \geq CS_2$ in this case. □

E When only a portion of passengers use hidden-city tickets

In Section 4, we assumed that all the passengers use the hidden-city tickets when available. However, in practice, as one can envision, only a small portion of passengers will take such opportunities. In this section, we extend our discussions to this case.

We keep using the notations introduced in Section 2. In addition, we assume that at time t, for each O-D pair $i \to j$ that the airline serves, an α proportion of the passengers will exploit the hidden-city opportunities when available, and the rest $1-\alpha$ proportion will simply respond to the price for that O-D pair. Similar to the dynamic programming problem (6), the airline's decision problem in this case can be written as follows:

$$\begin{aligned} V_\alpha^t(\boldsymbol{x}) &= \max_{k_{ij}^t, p_{ij}^t, \forall (i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \left((1-\alpha)\lambda_{ij}^t(p_{ij}^t) \left(p_{ij}^t + V_\alpha^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t}) \right) + \alpha\lambda_{ij}^t(\bar{p}_{ij}^t) \left(\bar{p}_{ij}^t + V_\alpha^{t-1}(\boldsymbol{x} - \bar{A}_{ij}^t) \right) \right) \right. \\ &\quad \left. + \left(1 - \sum_{(i,j)\in\mathcal{O}} \left((1-\alpha)\lambda_{ij}^t(p_{ij}^t) + \alpha\lambda_{ij}^t(\bar{p}_{ij}^t) \right) \right) V_\alpha^{t-1}(\boldsymbol{x}) \right\} \\ &= V_\alpha^{t-1}(\boldsymbol{x}) + \max_{k_{ij}^t, p_{ij}^t, \forall (i,j)\in\mathcal{O}} \left\{ \sum_{(i,j)\in\mathcal{O}} \left((1-\alpha)\lambda_{ij}^t(p_{ij}^t)(p_{ij}^t + V_\alpha^{t-1}(\boldsymbol{x} - A_{ij}^{k_{ij}^t}) - V_\alpha^{t-1}(\boldsymbol{x})) \right. \right. \\ &\quad \left. \left. + \alpha\lambda_{ij}^t(\bar{p}_{ij}^t)(\bar{p}_{ij}^t + V_\alpha^{t-1}(\boldsymbol{x} - \bar{A}_{ij}^t) - V_\alpha^{t-1}(\boldsymbol{x})) \right) \right\} \end{aligned}$$

where $\bar{p}_{ij}^t$ and $\bar{A}_{ij}^t$ are defined in Section 4.

In Theorem 1, we showed that if all the passengers use the hidden-city tickets when available, then the optimal pricing of the airline will not contain such opportunities. However, as the next example shows, this is not necessarily the case when only a portion of the passengers choose to do so.

Example 1 revisited. (Only a portion of passengers use hidden-city tickets) We consider the same problem as described in Example 1. The only difference is that there is an α proportion of the passengers that will use the hidden-city tickets when available. In this case, we can compute the optimal prices of the airline by considering two cases:

1. If the airline chooses to use $p_{AB} \le p_{AC}$ (i.e., the hidden-city ticket will be unavailable), then the optimization problem is

$$\begin{array}{cc} \max_{p_{AB}, p_{AC}} & p_{AB}\lambda_{AB}(p_{AB}) + p_{AC}\lambda_{AC}(p_{AC}) \\ \text{s.t.} & p_{AC} \ge p_{AB} \ge 0. \end{array}$$

   The optimal solution is $p_{AB} = p_{AC} = 3/4$ with expected revenue 9/8.

2. If the airline chooses to use $p_{AB} > p_{AC}$ (i.e., the hidden-city ticket will be available), then the optimization problem is

$$p_{AB}(1-\alpha)\lambda_{AB}(p_{AB}) + p_{AC}\alpha\lambda_{AB}(p_{AC}) + p_{AC}\lambda_{AC}(p_{AC}). \tag{15}$$

   By some computation, one can find that the optimal solution to (15) is $p_{AB} = 1$ and $p_{AC} = (2\alpha+1)/2(\alpha+1)$ with expected revenue of $1 + 1/(4\alpha+4)$.

It is easy to see that the second case always achieves higher revenue than the first one. That is, as long as some passengers do not use the hidden-city tickets, it might be of airlines interest to offer fares that contain such opportunities.

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