# Multi-Robot Manipulation with no Communication Using Only Local Measurements

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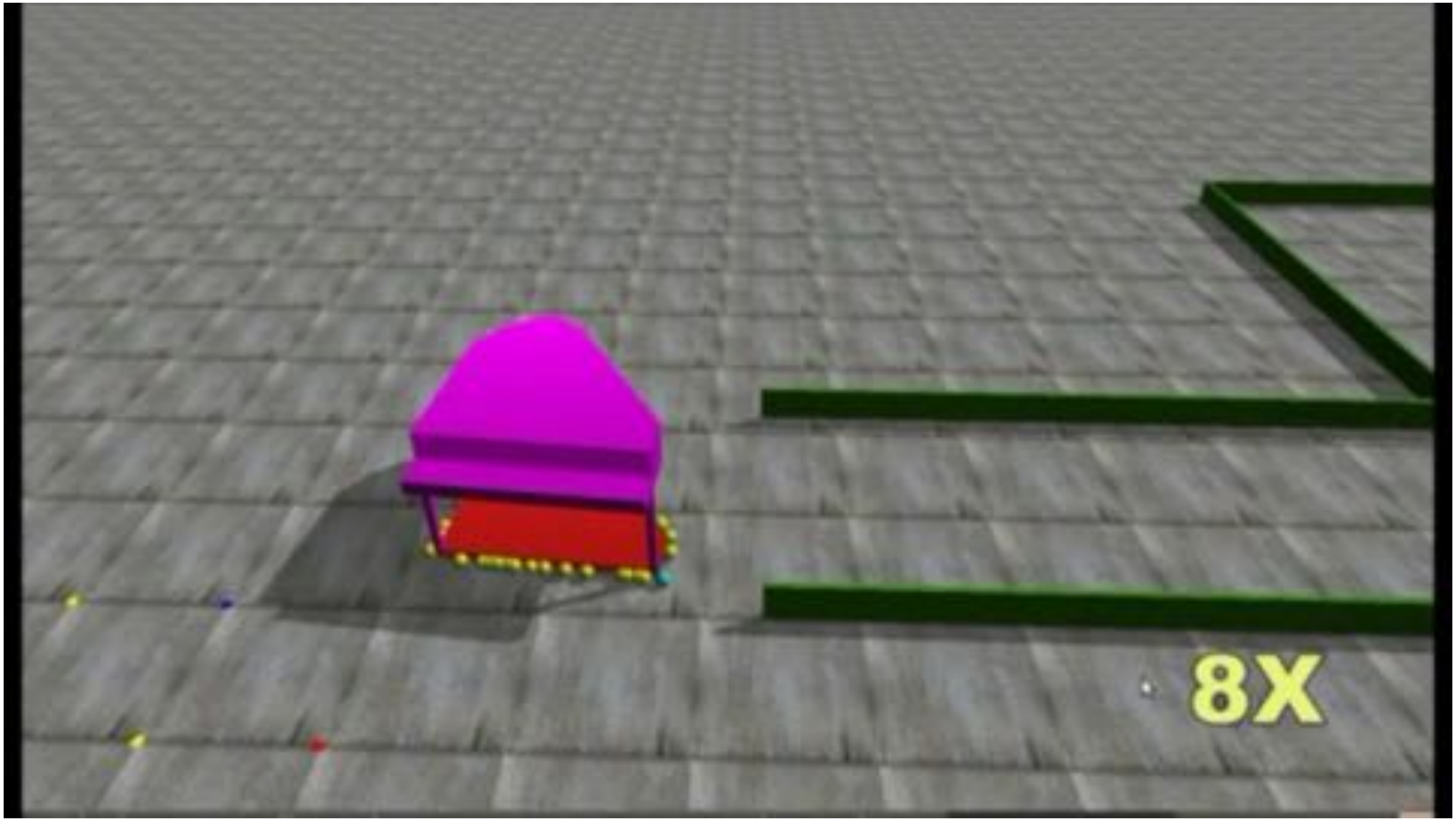
BOSTON UNIVERSITY

MSL

# 1001 Robots Moving a Piano



# Motivation

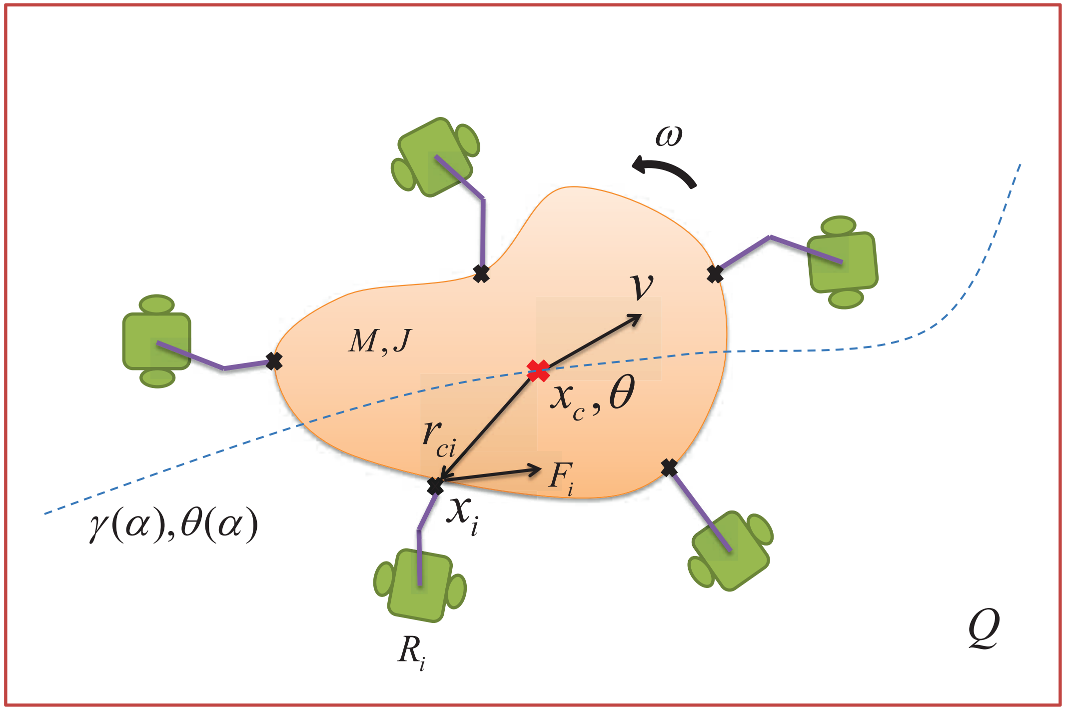
## Multi-Robot Manipulation and Transport

- Transport large objects
- Construction, manufacturing, disaster relief

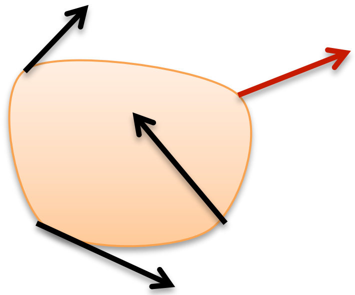
## Minimalist Approach

- Simple individual robots
- **No** explicit communication
- **No** global localization information
- Local measurements

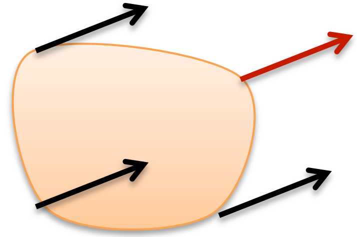
# Our Approach: Overview




Object itself "communicates" necessary information



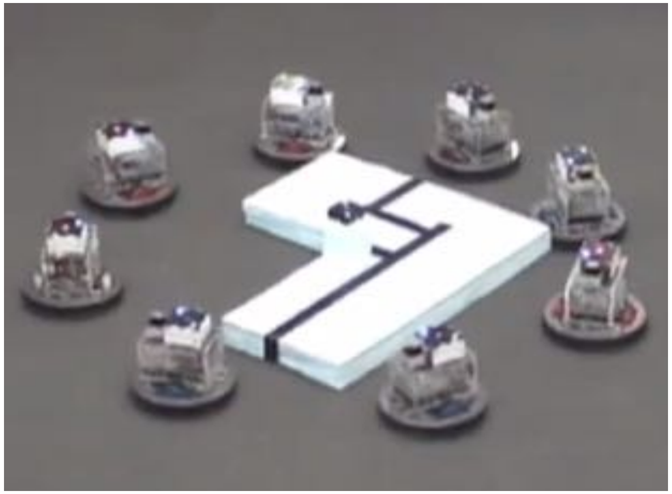
Uncoordinated forces



Coordinated Forces

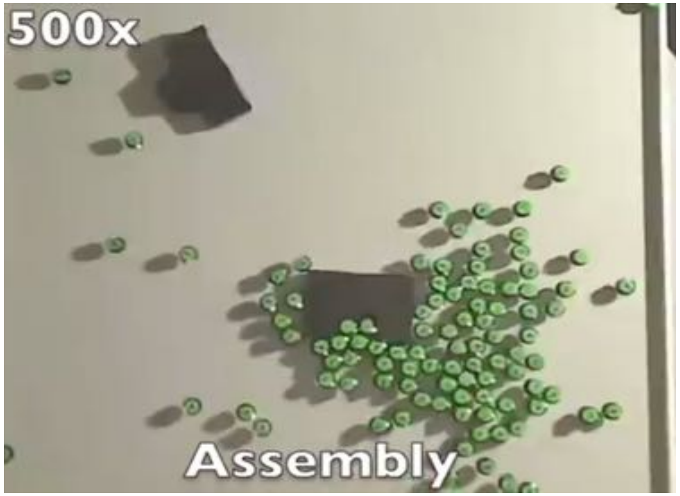
# Related Work

Caging/Force Closure



Fink, Michael, Kumar
ICRA 08

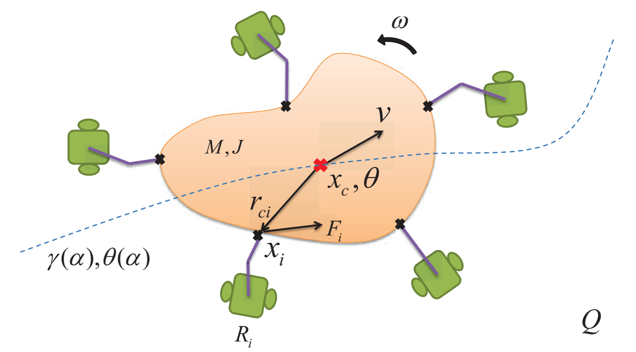
Ensemble Control




Becker, Habibi, Werfel,
Rubenstein, McLurkin
ICRA 13

# Problem Formulation

## Rigid-body Dynamics (Planar)




Translational

$$Ma_c = \sum_{i=1}^{N} F_i - \mu_v v_c$$

Rotational

$$J\dot{\omega} = \sum_{i=1}^{N} r_i \times F_i + T_1 - \frac{\mu_\nu}{M} J\omega$$

## Goal

Decentralized control law for $F_i$ $\longrightarrow$ $\sum_{i=1}^{N} F_i$ being controlled

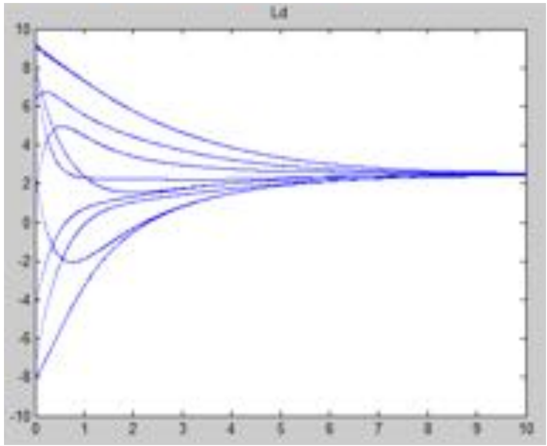
# Control Strategy

- Many follower robots, one leader (robot or human)
- Followers' forces will track leader's force using local measurements of the object's motion
- Leader uses feedback controller to steer the sum force, and then navigate the object through the desired trajectory

# Force Coordination via Consensus

- Linear consensus algorithm

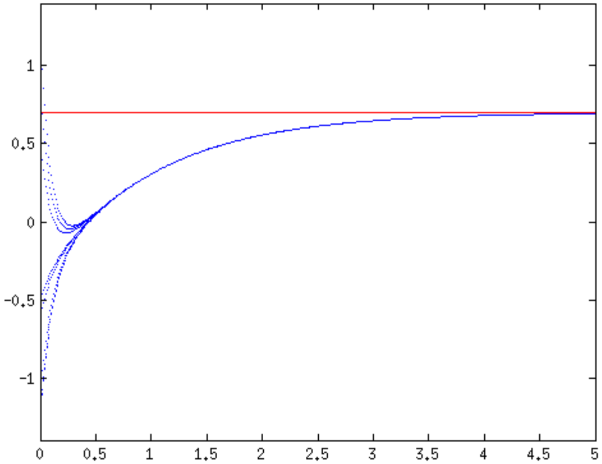
$$\dot{x}_i(t) = \sum_{v_j \in N_i} a_{ij}(x_j - x_i)$$



- Leader-following (steering)

Fix one robot

$$x_1(t) = x_1(0)$$




Olfati-Saber, Murray, TAC 2004
Jadbabaie, Lin, Morse, TAC 2003

# Prior Work: Follower Control Law

## Force Coordination without Communication

$$\dot{F}_i(t) = \sum_{j=1, j\neq i}^{N} \left(F_j(t) - F_i(t)\right)$$

$$= \sum_{j=1}^{N} F_j(t) - NF_i(t) = M a_c + \mu_v v_c - NF_i$$

Linear consensus law

Sum of forces

Newton's Law

Requires measurements at the C.O.M. of the object

Z. Wang and M. Schwager, Distributed Autonomous Robotic Systems (DARS), 2014

# Local Measurements

- Measurement at

  Local Attachment Points vs. Center of Mass

  (this paper) (prior work)

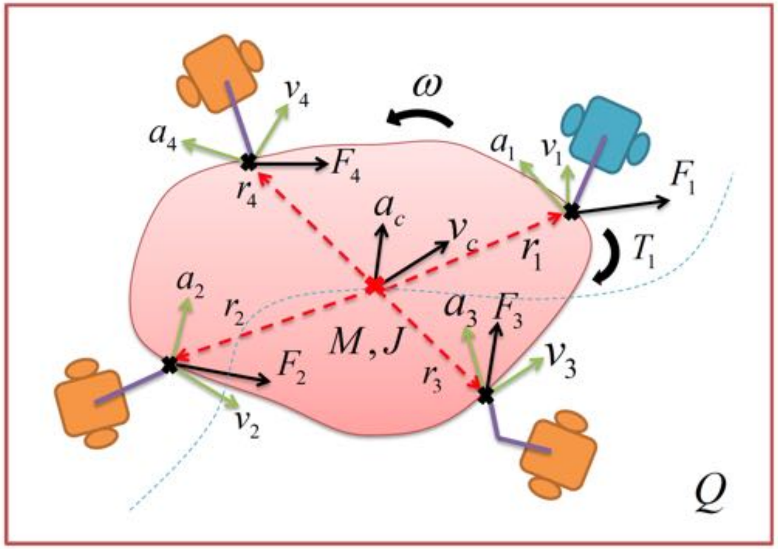
$$v_i = v_c + \omega \times r_i,$$

$$a_i = \dot{v}_i = a_c + \alpha \times r_i + \omega \times (\omega \times r_i)$$

New force coordination law:

$$\dot{F}_i = M a_i + \mu_v v_i - N F_i$$

Heterogenous Local Measurements



# Matrix Representation

$$\begin{aligned}\dot{F}_i &= M(a_c + \alpha \times r_i + \omega \times (\omega \times r_i)) + \\ &\quad \mu_v(v_c + \omega \times r_i) - NF_i \\ &= Ma_c + \mu_v v_c - NF_i + \\ &\quad M(\frac{1}{J}\sum_{i=1}^{N} r_i \times F_i - \frac{\mu_v}{M}\omega) \times r_i + \\ &\quad M\omega \times (\omega \times r_i) + \mu_v \omega \times r_i \\ &= (\sum_{j=1}^{N} F_j - NF_i) + \frac{M}{J}(\sum_{j=1}^{N} r_j \times F_j) \times r_i + \\ &\quad M\omega \times (\omega \times r_i),\end{aligned}$$

→ Centrosymmetric Assumption

↓

$$\dot{F}_i = (\sum_{j=1}^{N} F_j - NF_i) - \frac{M}{J} r_i \times (\sum_{j=1}^{N} r_j \times F_j)$$

↓

$$\dot{F} = \left(-L_a - \frac{M}{J} R_a(t)\right) F,$$

Time-varying

Force Consensus?

# Consensus Analysis 2

- Consensus without a leader

$$\dot{F} = \left(-L_a - \frac{M}{J} R_a(t)\right) F, \tag{11}$$

***Theorem 1:*** Under the centrosymmetric assumption (Assumption 1), (11) will reach a consensus on all forces if (18) is satisfied. The consensus value is the the average of all the initial forces.

$$\frac{M}{J} \sum_{i=1}^{N} \|r_i\|^2 < N. \tag{18}$$

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Proof: use Lyapunov Theorem & Barbalat's Lemma

# Consensus Analysis 1

- Time-independent Characterization

$$\dot{F} = \left(-L_a - \frac{M}{J}R_a(t)\right)F,$$

***Lemma** 1:* The rank of $R_a(t)$ is one, and the single nonzero eigenvalue of $R_a(t)$ is a constant $\lambda_{\min}(R_a(t)) = -\sum_{i=1}^{N}\|r_i\|^2$.

***Lemma** 2:* Under the centrosymmetric assumption, the eigenvalues of $(-L_a - \frac{M}{J}R_a(t))$ are less than or equal to zero if

$$\frac{M}{J}\sum_{i=1}^{N}\|r_i\|^2 < N. \tag{18}$$

***Lemma** 3*: Time-invariant equilibria

$$\mathbf{1}_x = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 1 \\ 0 \end{pmatrix}_{2N\times 1}, \quad \mathbf{1}_y = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ \vdots \\ 0 \\ 1 \end{pmatrix}_{2N\times 1}$$

# Consensus Analysis 2

MSL

- Consensus without a leader

$$\dot{F} = \left(-L_a - \frac{M}{J}R_a(t)\right)F,$$

Proof: use Lyapunov Theorem & Barbalat's Lemma

$$F = s_x\mathbf{1}_x + s_y\mathbf{1}_y + \delta,\ \ s_x, s_y \in \mathbb{R}$$

$$\dot{F} = \dot{\delta} = (-L_a - \frac{M}{J}R_a(t))(s\mathbf{1}_x + t\mathbf{1}_y + \delta)$$

$$= (-L_a - \frac{M}{J}R_a(t))\delta.$$

$$V = \frac{1}{2}\delta^T\delta$$

$$\dot{V} = \delta^T\dot{\delta} = \delta^T(-L_a - \frac{M}{J}R_a(t))\delta$$

$$\leqslant \lambda_{2N}(-L_a - \frac{M}{J}R_a(t))\|\delta\|^2 \leqslant 0$$

$$\ddot{V} = 2\delta^T\left(-L_a - \frac{M}{J}R_a(t)\right)^2\delta + \delta^T\left(-\frac{M}{J}\dot{R}_a(t)\right)\delta$$

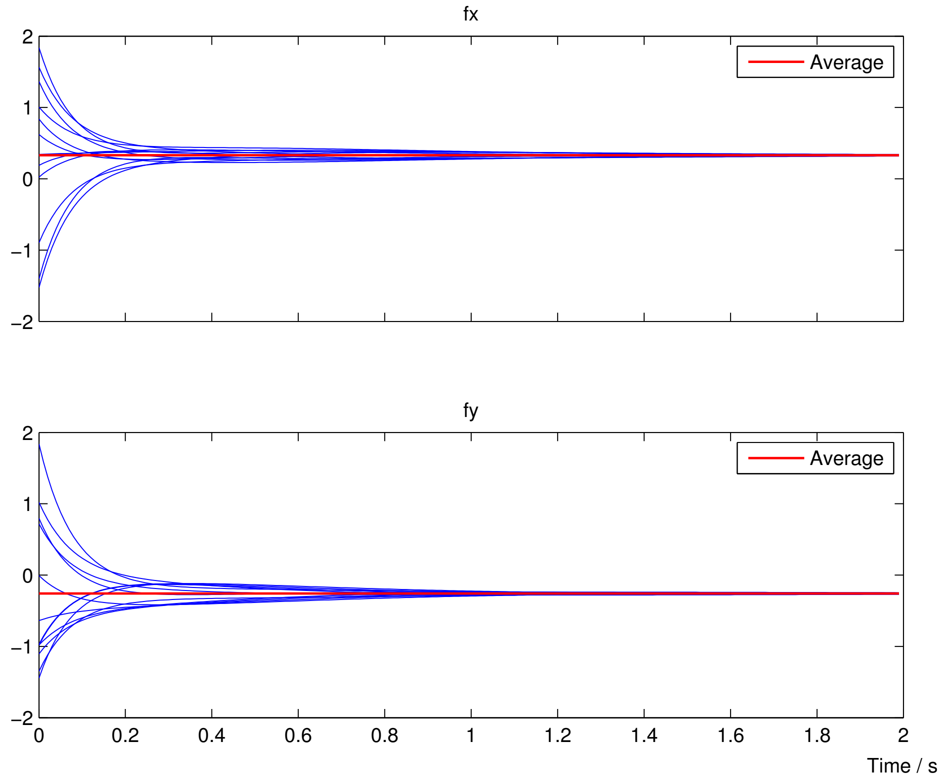
$\ddot{V}$ bounded

$\dot{V} \to 0$

$$\Omega = \{\delta \mid \delta = p_x\mathbf{1}_x + p_y\mathbf{1}_y,\ \ p_x, p_y \in \mathbb{R}\}$$

# Consensus Analysis 2

- Consensus without a leader



# Consensus Analysis 3

- Group force control via leader following

$$\dot{F} = \left(-L_a - \frac{M}{J}R_a(t)\right)F,$$

$$\left(-L_a - \frac{M}{J}R_a(t)\right) = \tilde{L}(t) = \begin{bmatrix} \tilde{L}_l(t) & \tilde{L}_{fl}^T(t) \\ \tilde{L}_{fl}(t) & \tilde{L}_f(t) \end{bmatrix}$$

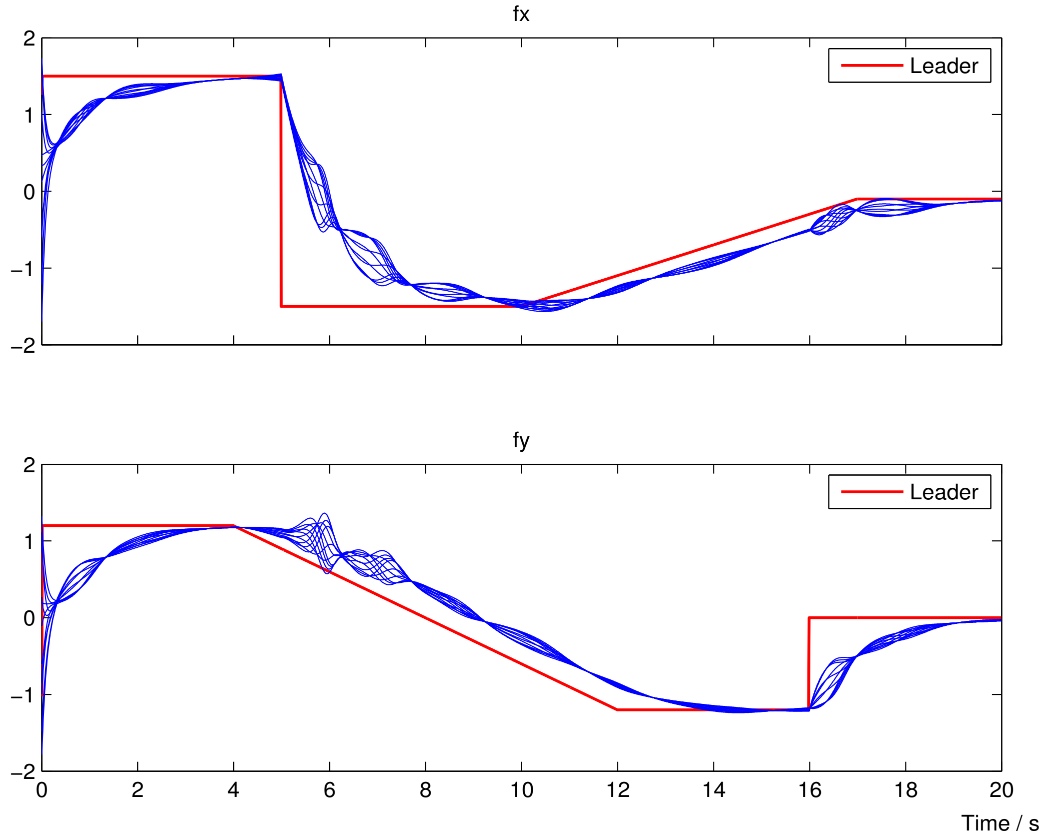
$$\dot{F}_f = -\tilde{L}_f(t)F_f - \tilde{L}_{fl}(t)F_1$$

Can show: equilibrium, eigenvalue (long proof)

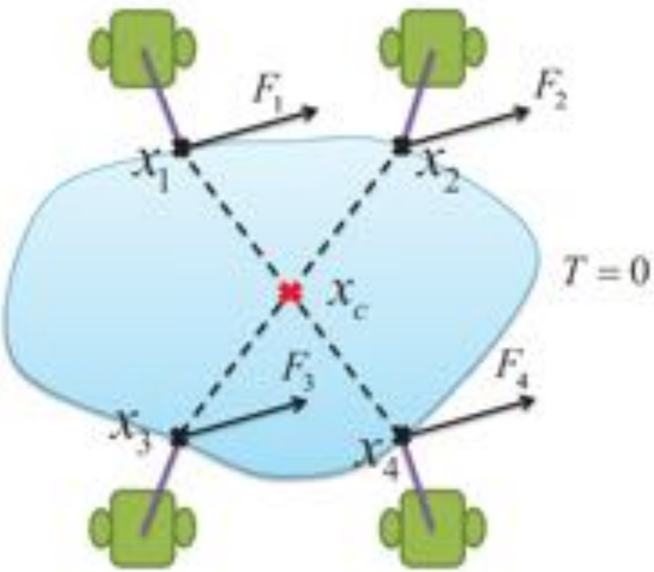
***Theorem** 2:* Under the centrosymmetric assumption (Assumption 1), all followers' forces in (20) will converge asymptotically to the leader's force $F_1$.

# Consensus Analysis 3

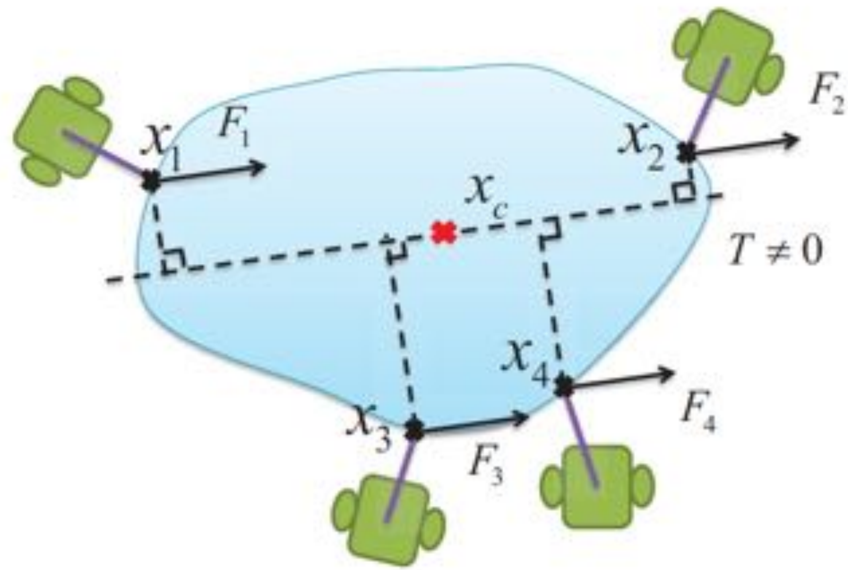
- Group force control via leader following



# Centrosymmetry




Centrosymmetric




Non-Centrosymmetric

# Relaxing The Centrosymmetric Assumption

- Problem induced

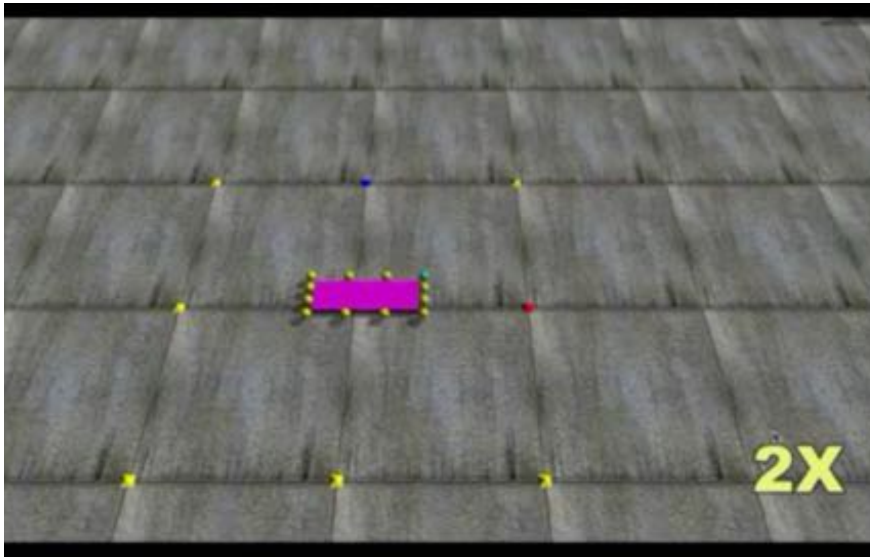
Change of equilibrium, eigenvalue.

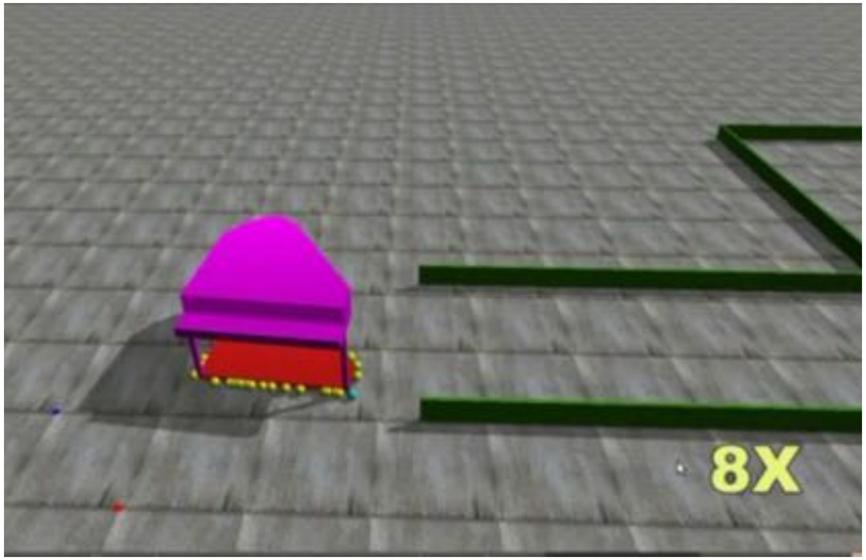
Introduce centrifugal force

- Model the asymmetry as a perturbation

$$\dot{F}_f = -\left(\tilde{L}_f(t) + \triangle_f\,(t)\right) F_f - \left(\tilde{L}_{fl}(t) + \triangle_{fl}\,(t)\right) F_1$$

# Simulations in Open Dynamic Engine (ODE)




12 Robots, 1 Leader
1kg Rectangular Plank
0.6m x 0.2m x 0.1m

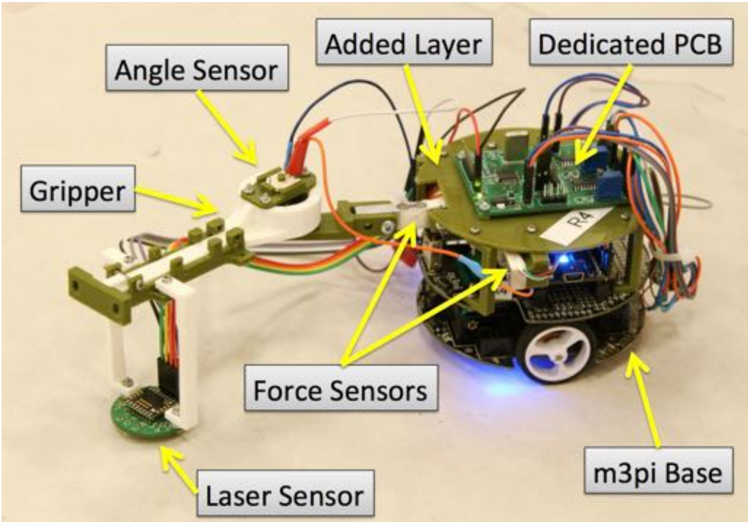
1001 Robots, 1 Leader
290kg Yamaha C1 Grand Piano
1.6m x 1.5m x 1.0m

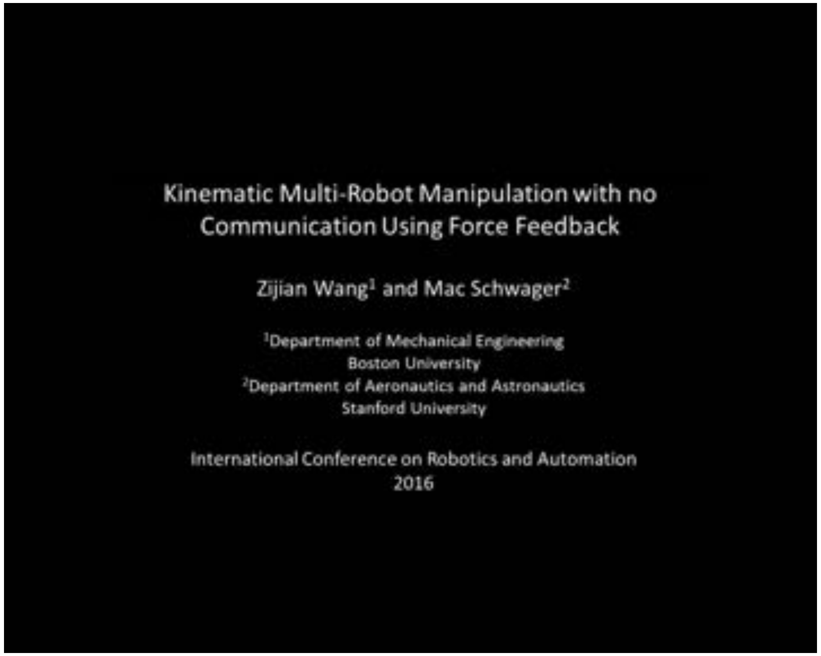
# Summary

- A decentralized, scalable multi-robot manipulation approach
- No explicit communication
- No global localization information for followers
- Guarantee followers‘ force coordination
- Use only local measurements
- Leader steers the group

# Future Work

- Rigorous analysis on the asymmetric case
- Physical experiment [Submitted to ICRA 2016]
- Human-swarm interaction [Submitted to ICRA 2016]
- Adaptive control



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Q & A

Thank you!

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