Multi-Robot Manipulation with no Communication Using Only Local Measurements

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1001 Robots Moving a Piano
Motivation

Multi-Robot Manipulation and Transport

- Transport large objects
- Construction, manufacturing, disaster relief

Minimalist Approach

- Simple individual robots
- **No** explicit communication
- **No** global localization information
- Local measurements
Our Approach: Overview

Object itself “communicates” necessary information

Uncoordinated forces

Coordinated Forces
Related Work

Caging/Force Closure

Fink, Michael, Kumar
ICRA 08

Ensemble Control

Becker, Habibi, Werfel, Rubenstein, McLurkin
ICRA 13
Problem Formulation

Rigid-body Dynamics (Planar)

Translational

\[ M \dot{a}_c = \sum_{i=1}^{N} F_i - \mu v v_c \]

Rotational

\[ J \ddot{\omega} = \sum_{i=1}^{N} r_i \times F_i + T_1 - \frac{\mu v}{M} J \omega \]

Goal

Decentralized control law for \( F_i \) being controlled
Control Strategy

• Many follower robots, one leader (robot or human)

• Followers’ forces will track leader’s force using local measurements of the object’s motion

• Leader uses feedback controller to steer the sum force, and then navigate the object through the desired trajectory
Force Coordination via Consensus

- **Linear consensus algorithm**

\[ \dot{x}_i(t) = \sum_{v_j \in N_i} a_{ij}(x_j - x_i) \]

- **Leader-following (steering)**

Fix one robot

\[ x_1(t) = x_1(0) \]

Olfati-Saber, Murray, TAC 2004
Jadbabaie, Lin, Morse, TAC 2003
Prior Work: Follower Control Law

Force Coordination without Communication

\[ \hat{F}_i(t) = \sum_{j=1, j \neq i}^{N} (F_j(t) - F_i(t)) \]

\[ = \sum_{j=1}^{N} F_j(t) - NF_i(t) = Ma_c + \mu v_c - NF_i \]

- Sum of forces
- Newton’s Law
- Linear consensus law
- Requires measurements at the C.O.M. of the object

Local Measurements

- Measurement at Local Attachment Points $\text{ vs. Center of Mass (this paper)}$

\[ v_i = v_c + \omega \times r_i, \]
\[ a_i = \dot{v}_i = a_c + \alpha \times r_i + \omega \times (\omega \times r_i) \]

New force coordination law:
\[ \dot{F}_i = M a_i + \mu_v v_i - N F_i \]

Heterogenous Local Measurements
Matrix Representation

\[ \dot{F}_i = M(a_c + \alpha \times r_i + \omega \times (\omega \times r_i)) + \mu_v (v_c + \omega \times r_i) - N F_i \]
\[ = M a_c + \mu_v v_c - N F_i + \]
\[ M \left( \frac{1}{J} \sum_{i=1}^{N} r_i \times F_i - \frac{\mu_v}{M} \omega \right) \times r_i + \]
\[ M \omega \times (\omega \times r_i) + \mu_v \omega \times r_i \]
\[ = \left( \sum_{j=1}^{N} F_j - N F_i \right) + \frac{M}{J} \left( \sum_{j=1}^{N} r_j \times F_j \right) \times r_i + \]
\[ M \omega \times (\omega \times r_i), \]

Centrosymmetric Assumption

\[ \dot{F}_i = \left( \sum_{j=1}^{N} F_j - N F_i \right) - \frac{M}{J} r_i \times \left( \sum_{j=1}^{N} r_j \times F_j \right) \]

Time-varying

\[ \dot{F} = \left( -L_\alpha - \frac{M}{J} R_\alpha(t) \right) F, \]

Force Consensus?
Consensus without a leader

\[
\dot{F} = \left( -L_a - \frac{M}{J} R_a(t) \right) F, \quad (11)
\]

**Theorem 1:** Under the centrosymmetric assumption (Assumption 1), (11) will reach a consensus on all forces if (18) is satisfied. The consensus value is the average of all the initial forces.

\[
\frac{M}{J} \sum_{i=1}^{N} \| r_i \|^2 < N. \quad (18)
\]

Proof: use Lyapunov Theorem & Barbalat’s Lemma
Consensus Analysis 1

➢ Time-independent Characterization

\[ \dot{F} = \left( -L_a - \frac{M}{J} R_a(t) \right) F, \]

**Lemma 1:** The rank of \( R_a(t) \) is one, and the single nonzero eigenvalue of \( R_a(t) \) is a constant \( \lambda_{\min}(R_a(t)) = -\sum_{i=1}^{N} \| r_i \|^2 \).

**Lemma 2:** Under the centrosymmetric assumption, the eigenvalues of \( (-L_a - \frac{M}{J} R_a(t)) \) are less than or equal to zero if

\[ \frac{M}{J} \sum_{i=1}^{N} \| r_i \|^2 < N. \]

(18)

**Lemma 3:** Time-invariant equilibria

\[ 1_x = \begin{pmatrix} 1 \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \end{pmatrix}_{2N \times 1}, \quad 1_y = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}_{2N \times 1} \]
Consensus Analysis 2

- Consensus without a leader

\[ \dot{F} = \left( -L_a - \frac{M}{J} R_a(t) \right) F, \]

Proof: use Lyapunov Theorem & Barbalat’s Lemma

\[ F = s_x \mathbf{1}_x + s_y \mathbf{1}_y + \delta, \quad s_x, s_y \in \mathbb{R} \]

\[ \dot{F} = \dot{\delta} = (-L_a - \frac{M}{J} R_a(t))(s \mathbf{1}_x + t \mathbf{1}_y + \delta) \]

\[ = (-L_a - \frac{M}{J} R_a(t))\delta. \]

\[ V = \frac{1}{2} \delta^T \delta \]

\[ \dot{V} = \delta^T \dot{\delta} = \delta^T (-L_a - \frac{M}{J} R_a(t))\delta \]

\[ \leq \lambda_{2N} (-L_a - \frac{M}{J} R_a(t))\|\delta\|^2 \leq 0 \]

\[ \dot{V} = 2\delta^T \left( -L_a - \frac{M}{J} R_a(t) \right)^2 \delta + \delta^T \left( -\frac{M}{J} \dot{R}_a(t) \right) \delta \]

\[ \dot{V} \text{ bounded} \]

\[ \dot{V} \to 0 \]

\[ \Omega = \{ \delta \mid \delta = p_x \mathbf{1}_x + p_y \mathbf{1}_y, \quad p_x, p_y \in \mathbb{R} \} \]
Consensus Analysis 2

- Consensus without a leader

![Graphs showing consensus analysis](image-url)
Consensus Analysis 3

Group force control via leader following

\[
\begin{align*}
\left( -L_a - \frac{M}{J} R_a(t) \right) &= \tilde{L}(t) = \\
&= \begin{bmatrix}
\tilde{L}_l(t) & \tilde{L}^T_{fl}(t) \\
\tilde{L}_{fl}(t) & \tilde{L}_f(t)
\end{bmatrix} \\
\hat{F}_f &= -\tilde{L}_f(t)F_f - \tilde{L}_{fl}(t)F_1
\end{align*}
\]

Can show: equilibrium, eigenvalue (long proof)

**Theorem 2**: Under the centrosymmetric assumption (Assumption 1), all followers’ forces in (20) will converge asymptotically to the leader’s force \( F_1 \).
Group force control via leader following

\[
\begin{align*}
\text{fx} & \quad \text{Leader} \\
\text{fy} & \quad \text{Leader}
\end{align*}
\]
Centrosymmetry

Centrosymmetric

Non-Centrosymmetric
Relaxing The Centrosymmetric Assumption

• Problem induced

Change of equilibrium, eigenvalue.

Introduce centrifugal force

• Model the asymmetry as a perturbation

\[ \dot{F}_f = - \left( \tilde{L}_f(t) + \Delta_f(t) \right) F_f - \left( \tilde{L}_{f1}(t) + \Delta_{f1}(t) \right) F_1 \]
Simulations in Open Dynamic Engine (ODE)

12 Robots, 1 Leader
1kg Rectangular Plank
0.6m x 0.2m x 0.1m

1001 Robots, 1 Leader
290kg Yamaha C1 Grand Piano
1.6m x 1.5m x 1.0m
Summary

- A decentralized, scalable multi-robot manipulation approach
- No explicit communication
- No global localization information for followers
- Guarantee followers’ force coordination
- Use only local measurements
- Leader steers the group
Future Work

• Rigorous analysis on the asymmetric case
• Physical experiment [Submitted to ICRA 2016]
• Human-swarm interaction [Submitted to ICRA 2016]
• Adaptive control
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Q & A

Thank you!

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