Discussion Problems 4

Problem One: Infinity Plus One
Let ★ be any value that isn't a natural number. Prove that |\mathbb{N} \cup \{★\}| = |\mathbb{N}|.

Problem Two: Propositional Equivalences
In lecture, we saw two different definitions of antisymmetry. If $R$ is a binary relation over a set $A$, we said that $R$ is antisymmetric iff

For any $x, y \in R$: if $xRy$ and $yRx$, then $x = y$

We also said $R$ is antisymmetric iff

For any $x, y \in R$: if $xRy$ and $x \neq y$, then the relation $yRx$ does not hold.

These statements are equivalent because the propositional formula $p \land q \rightarrow r$ is equivalent to the propositional formula $p \land \neg r \rightarrow \neg q$. Prove this by using a truth table.

Problem Three: Translating into Logic
i. Given the predicate $\text{Person}(x)$, which states that $x$ is a person, and $\text{Muggle}(x)$, which states that $x$ is a muggle, write a statement in first-order logic that says “some (but not all) people are muggles.”

ii. Given the predicate $\text{Person}(x)$, which states that $x$ is a person, and $\text{Commoner}(x)$, which states that $x$ is a commoner, write a statement in first-order logic that says “there are either zero or one people who are not commoners.”