Brief Preview of RF Systems (and architectures)

Direct Conversion Receiver:

- Antenna
- RF
- LO (Local Oscillator)
- Mixer
- Audio Amp
- Speaker (Audio Output)
- Unwanted Signal (I.e. station at “Image”)
- Final (desired) “Down Converted” Audio Signal

Solution: Use “Filter” to “Reject Image”
Super-Heterodyned Receiver
(one of Armstrong’s MANY contributions)

Antenna

RF

RF Filter

RF Mixer

IF Filter

VFO (Variable Frequency Oscillator)

LO (Local Oscillator)

Audio Amp

Speaker (Audio Output)

“Tune” both filter and VFO at the same time…giving output of mixer ALWAYS at Intermediate Freq.

Advantages of Super-Het:
- Tuning/Image Rej. Decoupled from detection
- IF filtering can be optimized and also reused (i.e. both AM and FM bands)
- MUCH easier to work at IF vs. directly at RF (especially at high frequencies)

IF at FIXED Freq. and therefore LO Freq. is also fixed in taking output of IF down to “basedband”
There are **LOTS of choices** and combinations here:

- Baseband-up-to-10’s MHz can be done with DSP!
- AM is really easy (it’s below 2MHz anyway)
- FM is also easy…BUT, frequency stability is not (so easy)
- Impedance matching and filtering at RF poses challenges as well...
A Quick Look at BW...(including preview of FCC perspective)

Direct Conversion:

Transmitted Carrier with Double Side-Bands (info.)

Local Oscillator Frequency used In multiplier

First-stage modulation (assuming NO fistortion)

Bandwidth is basically 2 x BW of baseband (i.e. voice)

Bandwidth can start to Look ugly (due to distortion)…
FCC may be “up close and in your face”…
Amplitude Modulation (AM) & Circuits

AM Modulation

\[ v(t) = v_c + v_m \cos(2\pi f_m t) \cos 2\pi f_c t \]

\[ \omega_m \]

\[ \omega_c \]

\[ v_c (1+m \cos \omega_m t) \]

\[ m \equiv \frac{V_m}{V_c} = \frac{A - B}{A + B} \]
\[ v(t) = v_c \cos \omega_c t + v_m \cos \omega_m t \cdot \cos \omega_c t \]

Using Law of Cosines

\[ v(t) = v_c \cos \omega_c t + \frac{v_m}{2} \cos(\omega_c - \omega_m) t + \frac{v_m}{2} \cos(\omega_c + \omega_m) t \]
This is the ideal situation...
How do we get modulation in reality and what are limitations?
Circuits to realize AM:
A few simple cautions (about AM) ---
   a) \( m \leq 1 \) otherwise information is lost
   b) law of Cosine => ideal multiplication…
   c) lets start here from circuits point of view
For high frequencies and at impedance levels often used, differential circuits with matched
impedance lines are very convenient…
Hence, lets revisit the Emitter Couples Pair (ECP) from 113

Basic BJT "Laws":
\[
I_c = I_s e^{V_{BE}/V_T}
\]
\[
V_T = \frac{kT}{q}
\]
\[
V_{BE} = \frac{kT}{q} \ln\left(\frac{I_c}{I_s}\right)
\]
\[ I_{c1} = I_s e^{\frac{V_{BE1}}{V_T}}; \quad I_{c2} = I_s e^{\frac{V_{BE2}}{V_T}} \]

\[ v_{id} = V_{BE1} - V_{BE2} = V_T \ln\left(\frac{I_{c1}}{I_s}\right) - V_T \ln\left(\frac{I_{c2}}{I_s}\right) \]

\[ = V_T \ln\left(\frac{I_{c1}}{I_{c2}}\right) \]

\[ I_{c1} = I_{c2} e^{\left(\frac{v_{id}}{V_T}\right)} \quad \text{or} \quad I_{c2} = I_{c1} e^{\left(-\frac{v_{id}}{V_T}\right)} \]

\[ I_{c1} + I_{c2} = I_{EE} \]
Now, for a bit of hacking to get the transfer function.

\[ I_{c1} = I_{EE} - I_{c2} = \frac{I_{EE}}{1 + e^{-\text{vid}/V_T}} \]

\[ I_{c2} = I_{EE} - I_{c1} = \frac{I_{EE}}{1 + e^{+\text{vid}/V_T}} \]

\[ \Delta I_c = I_{c1} - I_{c2} = I_{EE} \left( \frac{1}{1 + e^{-\text{vid}/V_T}} - \frac{1}{1 + e^{+\text{vid}/V_T}} \right) \]

\[ = I_{EE} \tanh \left( \frac{\text{vid}}{2V_T} \right) \]
Small Signal Regime

\[
\tanh \frac{v_{id}}{2V_T} \approx \frac{v_{id}}{2V_T}
\]

\[
\Delta I_c \approx I_{EE} \frac{v_{id}}{2V_T} \quad \text{and} \quad I_{c1} = I_{c2} = \frac{I_{EE}}{2} = I_c
\]

\[
gm \equiv \frac{I_c}{V_T}
\]

then \quad \Delta I_{c1} = g_m v_{id}
But, let's not stop here…  

(i.e. what we've done in 113)

$I_{EE}$ (Via Current Mirror) as 2$^{nd}$ Term in Mult.
\[ I_{EE} = \frac{v_{i2} - V_{BE}}{R} \quad \text{assume} \quad v_{i2} = v_{i2}(dc) + v_{i2}(ac) \]

\[ = \frac{v_{i2}(dc) - V_{BE}}{R} + \frac{v_{i2}(ac)}{R} \cos(\omega_m t) \]

\[ I_{EE} = \frac{v_c}{R} + \frac{v_m}{R} \cos(\omega_m t) \text{ where } v_c \equiv v_{i2}(dc) - V_{BE} \]

\[ v_m \equiv v_{i2}(ac) \]

Now, using this expression for \( I_{EE} \) back in \( \Delta I_c \) expression
\[
\Delta I_c = I_{EE} \tanh \left( \frac{v_{id}}{2V_T} \right) = \frac{1}{R} \left( v_c + v_m \cos \omega_m t \right) \tanh \left( \frac{v_{id}}{2V_T} \right)
\]

Adding Rc’s (see above figure) and looking at \( \Delta v_o \)

\[
\Delta v_o = -\Delta I_c R_c = -\frac{R_c}{R} \left( v_c + v_m \cos \omega_m t \right) \tanh \left( \frac{v_{id}}{2V_T} \right)
\]

Again, using small signal regime \((v_{id}<<V_T)\)

\[
\Delta v_o = -\frac{R_c}{2V_T R} \left( v_c + v_m \cos \omega_m t \right) \left( \bar{v}_{id} \cos \omega_c t \right)
\]

\textit{note: } v_{id} \equiv \bar{v}_{id} \cos \omega_c t

Although tanh formulation was used, with small signal approx. \((v_{id}<<V_T)\), one can slightly improve dynamic range by local feed back with \(R_E\)’s on the diff pair.
Limitation:

Basic EE113 Analysis gives:

\[ g_m(\text{eff}) = \frac{g_m}{1 + g_m R_E} \]

If \( g_m R_E \gg 1 \), then:

\[ g_m(\text{eff}) \approx \frac{1}{R_E} \]

\[ \Delta I_{c1} \approx \frac{2}{R_E} \quad \Delta I_{c2} \approx \frac{2}{R_E} \]

\[ \Delta I_c = \Delta I_{c1} - \Delta I_{c2} \approx \frac{v_{id}}{R_E} \]
But! There's no $I_{EE}$ left (ala $g_m = \frac{I_{EE}}{(2V_t)}) and hence multiplication is gone which is not what we wanted. Hence, $gmR_E \approx 1$ could increase dynamic range but not kill mult.

Looking at the other extreme, where $v_{id}$ drives $Q_1$ and $Q_2$ into a "switching mode" behavior, we find that a quite acceptable "modulated" signal is realized. Consider voltage waveforms, rather than $\Delta I_c$:

$$v_{o1} = V_{cc} - I_{c1}(t)R_c$$
$$v_{o2} = V_{cc} - I_{c2}(t)R_c$$

The following set of figures show how the "tail current" $I(V_2(t))$ gets switched between collectors C1 and C2 which gave the step-wise jumps downward for $V_{cc}$
\[ \nu_01 - \nu_02 = +V_T - V_T \]

\[ V_{cc} \]

\[ V_{cc} - \frac{I_o R_c}{2} \]

\[ t \]
A Quick Review (EE113 Final Exam 1991)

Consider the circuit shown below where Q1 and Q2 provide both biasing and an ac signal current to the differential pair. The transistor pair Q3 - Q4 provides a differential stage with a separate ac signal input (the circuit has two different ac signal inputs).
Assume that all $V_{BE}$'s for biasing are the same and given the following input to the current amplifier (mirror):

\[ V_{s1} = V_1 + \frac{V_1}{2} \sin \omega_1 t \]

(Q) Write an expression for $I_c(Q2)$ and sketch its value as a function of time.

\[ I_c(Q1) = \frac{V_{s1} - V_{BE}}{R_1} \]

Current mirror gives $I_c(Q1) = I_c(Q2)$

\[ I_c(Q2) = \frac{V_1 - V_{BE}}{R_1} + \frac{V_1 - V_{BE}}{2R_1} \sin \omega_1 t \]
(Q) Initially assuming that $V_{s2} = 0$ (inputs grounded), find the dc and ac signals at $V_{o1}$ and $V_{o2}$.

$$I_c(Q3) = I_c(Q4) = \frac{\alpha I_c(Q2)}{2}$$

$$V_{o1} = V_{o2} = V_{cc} - \frac{\alpha I_c(Q2)R_c}{2}$$

$$= V_{cc} - \frac{\alpha R_c}{2R_1} (V_1 - V_{BE}) - \frac{\alpha R_c}{4R_1} (V_1 - V_{BE}) \sin \omega_1 t$$

$$\underbrace{dc}_{\text{dc}} \quad \underbrace{ac}_{\text{ac}}$$
(Q) Now apply the following small signal input to the differential pair:

$$V_{s_2} = V_2 \sin \omega_2 t$$

Write an expression for the ac signal $V_{o1}$. HINT: Since the bias current is not constant, instead of using a constant $g_m$, use

$$g_m = \frac{kT}{q} I_c(t)$$

$$V_{o1} = -\frac{g_m R_c}{2} V_{s_2}$$

$$=-\frac{kT}{2q} R_c \left[ \frac{\alpha(V_1 - V_{BE})}{2R_i} \right] + \frac{\alpha(V_1 - V_{BE})}{4R_1} \sin \omega t \left( V_2 \sin \omega_2 t \right)$$

$$V_{o1} = \frac{-kT}{2q} I_c \left[ 1 + \frac{1}{2} \sin \omega t \right] v_2 \sin \omega_2 t$$

Assuming $\omega_2 = \frac{\omega}{10}$ sketch the Fourier Spectrum and suggest what such a circuit might be useful for. (this is for you to do now…)