2X2 Quad==4Quad Multiplier

The following are qualitative and conceptual points about 2Q & 4Q Multipliers. Basically Prelab 2 and Lab 2 moves fairly quickly into useful chips that do multiplying. At one level you won't have to struggle with biasing etc. Nonetheless, it is really nice to have a grasp of how things fit together.

2Quad:
+\(v_{i2}\) only

4Quad:
+\&- \(v_{i2}\)

\(\Delta v_{o}\)

\(v_{i2}\) must be positive and greater than \(V_{BE}\)

\(v_{i2}\) can be both +/- (due to diff par) but needs to be “small signal” (unless using Gilbert trick)

\(v_{i2}\)
Fig 3 (prelab 2) can be redrawn as follows, primarily to emphasize the symmetry and bias vs signal points:

Bisa point for bases set by “head room” needed relative to +4.5V and outputs

Output (dc) is: \( V_{cc} - I_{EE}R_c \)

Resistive bias network used to balance currents to diff. pair.

See next page discussion\(^2\)
Symmetry point

Equivalence of these two circuits...both give same input-output per base & collectors
Now, assuming all that made sense, look at Lab circuit ala 602:
Where are bias points (i.e. What voltages?) Start from nodes 4 & 5

\[
\frac{V_{cc} - V_4}{R_c} = I_1 ; \quad \frac{V_{cc} - V_5}{R_c} = I_2
\]

\[
I_{EE} = I_1 + I_2
\]

If we know \( R_E \Rightarrow V_x = I_{EE} R_E \)

\[
\therefore V_{BIAS2} = V_x + V_{BE}
\]

\[
V_{BIAS \, 1 \, \text{min}} = V_{BIAS \, 2} + V_{BE} + V_{CE\, (sat)}
\]

\[
V_{BIAS \, 1 \, \text{max}} = V_4 = V_5
\]

Basically, \( V_{BIAS1} \) needs to be somewhere between these limits.
Final comments on differences between Fig 3 (Prelab 2; "1496" circuit) and 602:

Comment: the bottom diff pair limits how much swing at $v_m$. To understand, look at earlier discussion of differential pair.

**Note:** having the $R_E$ included in the 1496 diff. pairs results in a larger input voltage swing vs. the 602 (without $R_E$'s).
Now, time for the down-and-detailed…Four Quad Multiplier:

Even with the beauty and simplicity of the multiplier with diff. Pair and current source, there are limits (2 quad, linearity, dynamic range…) The following is a fully symmetric 4 Quad "core"
By exact analogy to previous derivation for single diff pair

\[ I_{c3} - I_{c4} = \frac{I_{c1}}{1 + e^{-v_1/V_T}} - \frac{I_{c1}}{1 + e^{+v_1/V_T}} \]

\[ = \tanh\left(\frac{v_1}{2V_T}\right) \cdot I_{c1} \]

where

\[ \frac{I_{EE}}{1 + e^{-v_2/V_T}} \]

and for \( Q5 - Q6 \)

\[ I_{c5} - I_{c6} = \tanh\left(\frac{v_1}{2V_T}\right) \cdot I_{c2} \]

\[ \frac{I_{EE}}{1 + e^{+v_2/V_T}} \]
Now taking \((I_{c3} - I_{c4}) + (I_{c5} - I_{c6})\), the exponential terms involving 
\(\exp\left(\pm \frac{v_2}{V_T}\right)\) also reduce to \(\tanh\): 

\[
(I_{c3} - I_{c4}) + (I_{c5} - I_{c6}) = I_{EE} \tanh\left(\frac{v_1}{2V_T}\right) \cdot \tanh\left(\frac{v_2}{2V_T}\right)
\]
The circuit connectivity to actually realize this \( \tanh(x) \cdot \tanh(y) \) function is cross-coupled as shown and voltage output \( R(I_{c3}+I_{c5})-R(I_{c4}+I_{c6}) \)

As noted in previous diff pair is equal to (2 Quad) discussions, there are both : limitations due to \( \tanh \) functions and ways to "fix" and get around them:

1) Use small \( v_1 \) and \( v_2 \) (not always useful)
2) "linearize" lower diff pair by adding \( R_E \) to each emitter (quite useful)
3) "prescale" both \( x \) and \( y \) arguments (i.e. \( x=\tanh^{-1}v_1 \) and \( y=\tanh^{-1}v_2 \))

This last approach is the very elegant (and useful) Gilbert multiplier “with all the trimmings”).

There will be a handout on the fully prescaled Gilbert multiplier (but it will not be discussed further)
Distortion

We will consider two main kinds: harmonic (HD) and intermodulation (IM), lets start with HD and then come back to IM.

\[ a \cos(\omega_1 t) \rightarrow A \rightarrow b_1 \cos(\omega_1 t + \phi_1) + b_2 \cos(2\omega_1 t + \phi_2) + b_3 ... \]

\[ HD_2 \equiv \frac{|b_2|}{|b_1|} ; \quad HD_3 \equiv \frac{|b_3|}{|b_1|} \]

\[ THD \equiv \frac{\left[ b_2^2 + b_3^2 + ... \right]^{1/2}}{|b_1|} \]
Using power series expansion on collector current, expressions for the diff pair:

\[
\frac{I_{c2}}{I_{EE}} = \frac{1}{1 + \exp(d)} = \frac{1}{2} - \frac{d}{4} + \frac{1}{48}d^3 - \frac{1}{480}d^5 ...
\]

↑

note: \(d^2\) term is not here!

This is different for the expression (and harmonic terms) we noted in Ch. 4 Krauss for a single CE amplifier)
Using $d = \cos \omega_1 t$ and doing a bit of math/algebra
(see Mayaram & Pederson, Kluwer '91 p.17)

\[
\frac{I_{e^2}}{I_{EE}} = b_o^1 + b_1 \cos \omega_1 t + b_2 \cos 2\omega_1 t + b_3 \cos 3\omega_1 t \ldots
\]

\[
b_o^1 = \frac{1}{2}
\]

\[
b_3 = \frac{1}{192} \left( \frac{a}{V_T} \right)^3
\]

\[
b_1 = \frac{1}{4} \left( \frac{a}{V_T} - \frac{1}{16} \left( \frac{a}{V_T} \right)^3 \right) \quad \vdots
\]

\[
b_2 = 0
\]
Figure 1.16 (Mayaram & Pederson) shows how these harmonics vary vs $v_1/V_T$ (input signal in normalized units of $V_T$):

From this figure we can easily estimate $HD_3$… and THD. As noted above, $b_2 = 0$ $\therefore HD_2 = 0$ (good news for using diff. Pair). Analytically, $HD_3$ can be expressed:
$HD_3 \approx \frac{1}{48} \left( \frac{a}{V_T} \right)^2$

$\uparrow \frac{kT}{q}$

Following a similar procedure for MOS device using the simple current expression:

$I_D = k' \frac{W}{L} \left[ (V_{GS} - V_{TH})V_{DS} - \frac{1}{2} V_{DS}^2 \right]$

$HD_3(MOS) = \frac{1}{32} \left( \frac{a}{V_{GG} - V_{TH}(MOS)} \right)^2$
If we use a single transistor (common emitter) amplifier, things: 1) get more complex and 2) performance is worse.

\[ I_c = I_{CA} e^{V_{VT}} \quad \text{where} \quad I_{CA} = I_s e^{\frac{V_{BE}}{V_T}} \]

\[ \approx I_{CA} \left[ 1 + \frac{v_1}{V_T} + \frac{1}{2} \left( \frac{v_1}{V_T} \right)^2 + \frac{1}{6} \left( \frac{v_1}{V_T} \right)^3 \ldots \right] \]

Considering the case where \( \frac{v_1}{V_T} = d \)

\[ e^{(d \cos \omega_t)} = I_o(d) + 2I_1(d) \cos \omega_t + 2I_2(d) \cos 2\omega_t + \ldots + 2I_n(d) \cos n\omega_t \ldots \]
Where the $I_n$'s are modified Bessel function. (order n)

$$I_c = I_{CA} I_o(d) \left[ 1 + \frac{2I_1(d)}{I_o(d)} \cos \omega t + \frac{2I_n(d)}{I_o(d)} \cos n \omega t \ldots \right]$$

$$b_1 = I_{dc} \left[ \frac{2I_1(d)}{I_o(d)} \right] \quad HD_2 = \frac{I_2(d)}{I_1(d)} \left( \approx \frac{1}{4} d, \quad \text{for} \quad d \ll 1 \right)$$

$$b_2 = I_{dc} \left[ \frac{2I_2(d)}{I_o(d)} \right] \quad HD_3 = \frac{I_3(d)}{I_1(d)}$$
Plotting the respective ratios $I_n(d)/I_0(d)$ we see something qualitatively like Fig 1.16 (Mayaram & Pederson) but much more severe quantitatively in terms of HD

Figure 2.6 (Mayaram & Pederson)
Intermodulation (IM) distortion arises when multiple frequencies are present

\[ v_i = v_{1a} \cos \omega_1 t + v_{2a} \cos \omega_2 t \]

\[ v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots \]

Using \( v_i \) in \( v_o \) expression gives lots of terms. The primary ones of concern come from 3\(^{rd}\) order as follows:

\[ a_3 v_i^3 = \ldots a_{32} \cos(\omega_1 t \pm 2\omega_2 t) + a_{33} \cos(2\omega_1 t \pm \omega_2 t) \]

\[ a_{32} = \frac{3a_3}{4} V_{1a} V_{2a}^2 \quad a_{33} = \frac{3a_3}{4} V_{1a}^2 V_{2a} \]
Assuming $V_{1a} = V_{2a}$, then approximately

$$IM_3 = \frac{V_o [IM_3]}{a_1 V_{1a}} = \frac{3}{4} \frac{a_3 V_{1a}^3}{a_1 V_{1a}} = \frac{3}{4} \frac{a_3}{a_1} V_{1a}^2$$

There are several basic points to note as well as practical considerations:

- the IM3 grows super linearly
- this means it overtakes the linear gain term (a big problem)
- we need to control output power to avoid such IM distortion problems (see figure and physical interpretation)