Gray level histograms

![Brain image]

Histogram

#pixels

gray level

$4 \times 10^4$

0

50

100

150

200

250

0.5

1

1.5

2

2.5

3

3.5

4
Gray level histograms

Bay image
Gray level histogram in viewfinder
Gray level histograms

- To measure a histogram:
  - For B-bit image, initialize $2^B$ counters with 0
  - Loop over all pixels $x, y$
  - When encountering gray level $f[x, y] = i$, increment counter $\#i$

- Normalized histogram can be thought of as an estimate of the probability distribution of the continuous signal amplitude

- Use fewer, larger bins to trade off amplitude resolution against sample size.
Histogram equalization

Idea:

Find a non-linear transformation

\[ g = T(f) \]

that is applied to each pixel of the input image \( f[x,y] \), such that a uniform distribution of gray levels results for the output image \( g[x,y] \).
Histogram equalization

Analyse ideal, continuous case first ...

Assume

- Normalized input values $0 \leq f \leq 1$ and output values $0 \leq g \leq 1$
- $T(f)$ is differentiable, increasing, and invertible, i.e., there exists

$$f = T^{-1}(g) \quad 0 \leq g \leq 1$$

**Goal:** pdf $p_g(g) = 1$ over the entire range $0 \leq g \leq 1$
Histogram equalization for continuous case

- From basic probability theory

\[ p_f(f) \xrightarrow{f} T(f) \xrightarrow{g} p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} \]

- Consider the transformation function

\[ g = T(f) = \int_{0}^{f} p_f(\alpha) \, d\alpha \quad 0 \leq f \leq 1 \]

- Then...

\[ \frac{dg}{df} = p_f(f) \]

\[ p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} = \left[ p_f(f) \frac{1}{p_f(f)} \right]_{f=T^{-1}(g)} = 1 \quad 0 \leq g \leq 1 \]
Histogram equalization for discrete case

- Now, $f$ only assumes discrete amplitude values $f_0, f_1, \ldots, f_{L-1}$ with empirical probabilities

$$P_0 = \frac{n_0}{n} \quad P_1 = \frac{n_1}{n} \quad \ldots \quad P_{L-1} = \frac{n_{L-1}}{n}$$

where $n$ is total number of pixels

- Discrete approximation of $g = T(f) = \int_0^f p_f(\alpha) d\alpha$

$$g_k = T[f_k] = \sum_{i=0}^{k} P_i \quad \text{for} \quad k = 0, 1, \ldots, L - 1$$

- The resulting values $g_k$ are in the range $[0, 1]$ and might have to be scaled and rounded appropriately.
Histogram equalization example

Original image *Bay*  
... after histogram equalization
Histogram equalization example

Original image Bay ... after histogram equalization
Histogram equalization example

Original image *Brain*  

... after histogram equalization
Histogram equalization example

Original image *Brain* ... after histogram equalization
Histogram equalization example

Original image *Moon* ... after histogram equalization
Histogram equalization example

Original image *Moon* . . . after histogram equalization

![Histogram equalization example](image)
Contrast-limited histogram equalization

![Graph showing contrast-limited histogram equalization](image)
Adaptive histogram equalization

- Histogram equalization based on a histogram obtained from a portion of the image

- Limit contrast expansion in flat regions of the image, e.g., by clipping histogram values. ("Contrast-limited adaptive histogram equalization")

[Pizer, Amburn et al. 1987]
Adaptive histogram equalization

Original image
Parrot

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles

Global histogram equalization
Adaptive histogram equalization

Original image
*Dental Xray*

Adaptive histogram equalization, 8x8 tiles

Global histogram equalization

Adaptive histogram equalization, 16x16 tiles
Adaptive histogram equalization

Original image
*Skull Xray*

Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles
Ansel Adam’s Zone System (1939)

- Full tonal gradation
- 11-step gradation

<table>
<thead>
<tr>
<th>Zone</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Pure black</td>
</tr>
<tr>
<td>I</td>
<td>Near black, with slight tonality but no texture</td>
</tr>
<tr>
<td>II</td>
<td>Textured black; the darkest part of the image in which slight detail is recorded</td>
</tr>
<tr>
<td>III</td>
<td>Average dark materials and low values showing adequate texture</td>
</tr>
<tr>
<td>IV</td>
<td>Average dark foliage, dark stone, or landscape shadows</td>
</tr>
<tr>
<td>V</td>
<td>Middle gray: clear north sky; dark skin, average weathered wood</td>
</tr>
<tr>
<td>VI</td>
<td>Average Caucasian skin; light stone; shadows on snow in sunlit landscapes</td>
</tr>
<tr>
<td>VII</td>
<td>Very light skin; shadows in snow with acute side lighting</td>
</tr>
<tr>
<td>VIII</td>
<td>Lightest tone with texture: textured snow</td>
</tr>
<tr>
<td>IX</td>
<td>Slight tone without texture; glaring snow</td>
</tr>
<tr>
<td>X</td>
<td>Pure white: light sources and specular reflections</td>
</tr>
</tbody>
</table>

Histogram of 11 zones → not quite flat
Photoshop Demo