Lecture 5:
Kinesthetic haptic devices:
Control

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important stability concepts
instability / limit cycle oscillation

video credit: Jake Abbott
review stability in the context of the s-plane

common second-order system: \[ m\ddot{x} + b\dot{x} + kx = f \]

take the Laplace transform of both sides:

\[
\mathcal{L}[m\ddot{x} + b\dot{x} + kx] = \mathcal{L}[f]
\]

\[
ms^2X(s) + bsX(s) + kX(s) = F(s)
\]

\[
(ms^2 + bs + k)X(s) = F(s)
\]

transfer function/characteristic equation:

\[
\frac{F(s)}{X(s)} = ms^2 + bs + k
\]
review stability in the context of the s-plane

roots of the characteristic equation:

$$s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

plot roots on the s-plane for any values of $m$, $b$, and $k$:

examples on the board...
discussion

why wouldn’t this approach work well for haptic devices?
why do instabilities occur?

fundamentally, instability has the potential to occur because real-world interactions are only approximated in the virtual world

although these approximation errors are small, their potentially non-passive nature can have profound effects, notably:

- instability
- limit cycle oscillations (which can be just as bad as instability)
passivity

a useful tool for studying the stability and performance of haptic systems

a one-port is passive if the integral of power extracted over time does not exceed the initial energy stored in the system.

\[ \int_0^t f(\tau) \dot{x}(\tau) d\tau \geq 0, \quad \forall t \geq 0 \]
"Z-width" is the dynamic range of impedances that can be rendered with a haptic display while maintaining passivity.

We want a large z-width, in particular:

• zero impedance in free space
• large impedance during interactions with highly massive/viscous/stiff objects

\[
\frac{F(s)}{X(s)} = ms^2 + bs + k = Z(s) \quad F(s) = Z(s)X(s)
\]
Z-width

\[ Z_{\text{to, stiff}} \]

\[ Z_{\text{to, free}} \]

Frequency [rad/s]

Impedance (F/V) [dB]

Christiansson et al. 2008
Z-width (experimental)

![Graph showing Z-width with different damping conditions.](image)

- **Physical damper engaged (1 KHz)**
- **Physical damper engaged (100 Hz)**
- **No physical damping (1 KHz)**
- **No physical damping (100 Hz)**

Colgate and Brown 1994

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how do you improve Z-width?

lower bound depends primarily on mechanical design (can be modified through control)

upper bound depends on sensor quantization, sampled data effects, time delay (in teleoperators), and noise (can be modified through control)

in a different category are methods that seek to create a perceptual effect (e.g., event-based rendering)
stability of the virtual wall
sampled-data system example

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Colgate and Schenkel 1997
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sampled-data system example

A necessary and sufficient condition for passivity of a sampled-data system is:

\[
\begin{align*}
b &> \frac{T}{2} \frac{1}{1 - \cos(\omega T)} \mathcal{R}\{(1 - e^{-j\omega T})H(e^{j\omega T})\} \quad \text{for} \quad 0 \leq \omega \leq \omega_N
\end{align*}
\]

where:

- \( b \) is the physical damping in the mechanism
- \( T \) is the sampling period
- \( H(z) \) is a transfer function representing the virtual environment
- \( \omega_N = \pi/T \) is the Nyquist frequency
for a virtual wall with stiffness and damping:

\[ H(z) = K + B \frac{z - 1}{Tz} \]

assumes a velocity estimate from a backward difference differentiation

this result can be simplified into a simple analytical expression:

\[ b > \frac{KT}{2} + |B| \]

where:
- \( K > 0 \) is the virtual stiffness
- \( B \) is the virtual damping
- \( T \) is the sampling period

Weir and Colgate 2008
effects of sampling

first detected extra-wall position which toggles off control law

manipulandum displacement \( y(t) \)

commanded force \( f_k(t) \)

\[
f_k = t \cdot y_k
\]

\( \Delta t_a \)

\( \Delta t_b \)

Gillespie and Cutkosky 1996
detail of energy leak due to sampling

in order to maintain passivity, the energy dissipated must be greater than the energy introduced by the energy leak

Weir and Colgate 2008
position quantization

\[ K \leq \min \left( \frac{2b}{T}, \frac{2f_c}{\Delta} \right) \]
The conceptual derivation of the passivity limit of virtual stiffness involves compressing a virtual spring, where the force is given by $F = kx$. Imaging this process, the energy stored after compressing a distance $\Delta x = x_{k+1} - x_k = vT$ during one sample period is:

$$E = \frac{1}{2} k (\Delta x)^2$$

Due to damping and sampling, the energy stored is:

$$E_{leak} = \frac{1}{2} K (vT)^2$$
$$E_{dissip} = bv^2 T$$
$$E_{leak} \leq E_{dissip}$$
$$\frac{1}{2} Kv^2 T^2 \leq Tbv^2$$

And due to Coulomb friction and position quantization, the energy stored is:

$$E_{leak} = \frac{1}{2} K \Delta^2$$
$$E_{dissip} = f_c \Delta$$
$$E_{leak} \leq E_{dissip}$$
$$\frac{1}{2} K \Delta^2 \leq f_c \left( \frac{\Delta}{T} \right) T$$

$$K \leq \frac{2b}{T}$$

$$K \leq \frac{2f_c}{\Delta}$$

References:

- Abbott & Okamura 2005
- Diolaiti et al. 2006
- Weir & Colgate 2008
dimensionless stability plane

\[ K \leq \min \left( \frac{2b}{T}, \frac{2f_c}{\Delta} \right) \]

can be nondimensionalized by dividing by \( K \)

\[ \beta := \frac{b}{KT} \quad \sigma := \frac{f_c}{K\Delta} \]

\[ \dot{\xi}(\tau) = \frac{\dot{x}T}{\Delta} \]

\( \dot{\xi}_{\text{max}} \) is the maximum allowable velocity

Diolaiti et al. 2006
where do commercial haptic devices stand?
how to make your system stable/passive?
approach #1:

lower your expectations
(i.e., just respect the existing Z-width)
approach #2:

change your hardware
(add damping, increase sampling rate, increase encoder resolution, etc.)
approach #3:

passivity observer/
passivity controller
and/or prediction/
compensation
Passivity Observer (PO) measures energy flow in and out of one or more subsystems in real-time software.

Active behavior is indicated by a negative value of the PO at any time.

Passivity Controller (PC) is an adaptive dissipative element which, at each time sample, absorbs exactly the net energy output (if any) measured by the PO.

Hannaford and Ryu 2002
prediction and compensation

bouncing ball simulation

original
prediction and compensation

bouncing ball simulation

prediction
prediction and compensation

bouncing ball simulation
correction
approach #4:

virtual coupling
virtual coupling

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Adams and Hannaford 1998

Passive Tool Simulation

"Virtual Coupling"
summary: design for passivity

• haptic instability arises from a lack of passivity when rendering virtual environments

• in order to maintain passivity, virtual environment impedance can be reduced to acceptable levels for passivity

• given a perfect model of a haptic device, we can compute the requirements for passivity
summary: control for passivity

- passivity observers and controllers can effectively damp out any oscillations occurring due to non-passivity
- virtual couplings can be used to modulate the impedance transmitted between the haptic display and the virtual environment to ensure passivity
- perceptual methods of improving performance take advantage of the limits of human perception to create the illusion of higher performance rendering on existing haptic display hardware
key point

in order to make the system passive (guaranteed stable), you are likely to lose some accuracy/fidelity of your virtual environment in the process
Hapkit check!