MS&E 247s International Investments

MWF 3:15-4:30     Gates B01

Final Exam MS&E 247S

Fri Aug 15 2008 12:15PM-3:15PM Gates B01
Or Saturday Aug 16 2008 12:15PM-3:15PM Gates B01

Remote SCPD participants will also take the exam on Friday, 8/15.

Please Submit Exam Proctor’s Name, Contact info as SCPD requires. C.c. the above info to yeetienfu@yahoo.com, preferably a week before the exam.

Local SCPD students please come to Stanford to take the exam.

Light refreshments will be served.

Handout #13 as of 0725 2008

International Asset Portfolios
Bond Portfolios

Reading Assignments for this Week

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International Asset Portfolios
Bond Portfolios

MS&E 247S International Investments
Yee-Tien Fu
Return and Risk in National Bond Markets

• We assume that our investor treats the US$ as his base currency (the numeraire currency used by the investor for measuring performance).

• In general, the return on a foreign bond, as measured in US$ terms, has 3 components:
  1. Interest income earned or accrued.
  2. The capital gain or loss on the bond, resulting from the inverse relationship between interest rates and bond prices.
  3. The foreign exchange gain or loss, applied to the above two items.
Calculating Unhedged Returns in US$ Terms

$B_t$ - the initial purchase price of the bond in foreign currency (FC) terms

$S_t$ - the spot exchange rate, in $/FC terms, on the purchase date

After one month:

$\tilde{B}_{t+1}$ - the value of the bond after one month, representing the initial bond price plus the price change over the month ($\tilde{A}_{t+1}$) plus accrued interest ($C_{t+1}$).
Calculating Unhedged Returns in US$ Terms

Therefore:

$B_t S_t$ - the US$ purchase price of the foreign bond

$\tilde{B}_{t+1} \tilde{S}_{t+1}$ - the value of the bond after one month in US$ terms, where

$$\tilde{B}_{t+1} \equiv B_t + \tilde{\Delta}_{t+1} + C_{t+1}$$

Note that:

If interest rates rise, bond prices fall ($\tilde{\Delta}_{t+1} < 0$).
If interest rates fall, bond prices rise ($\tilde{\Delta}_{t+1} > 0$).
Yield to Maturity

- Interest rate that makes the present value of the bond’s payments equal to its price

Solve the bond formula for $r$

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{ParValue}{(1+r)^T}$$
Calculating Unhedged Returns in US$ Terms

The continuous rate of return on the foreign bond measured in US$ and on an unhedged basis is:

\[
\tilde{R}_{S,U} = \ln\left(\frac{\tilde{B}_{t+1}\tilde{S}_{t+1}}{B_tS_t}\right) = \ln\left(\frac{\tilde{B}_{t+1}}{B_t}\right) + \ln\left(\frac{\tilde{S}_{t+1}}{S_t}\right) = \tilde{B}_{FC} + \tilde{S}_{US$,FC} \quad (14.1)
\]

Therefore the unhedged US$ return on the foreign bond has two pieces:

1. the return on the bond in foreign currency terms ($\tilde{B}_{FC}$);
   and
2. the return on the foreign currency used to buy the bond ($\tilde{S}_{US$,FC}$).
Calculating Unhedged Returns in US$ Terms

Note that the return on the bond in foreign currency terms is uncertain because of the possible capital gain or loss on the bond. But the return measured in US$ has an additional source of uncertainty, the foreign exchange gain or loss.
Calculating Unhedged Returns in US$ Terms

The variance of the returns in equation (14.1) reflects the variance of each term and the covariance between the returns on the foreign bond and the returns on spot foreign exchange, or:

\[
\sigma^2(\tilde{R}_{S,U}) = \sigma^2(\tilde{B}_{FC}) + \sigma^2(\tilde{S}_{US\$,FC}) + 2\text{Cov}(\tilde{B}_{FC};\tilde{S}_{US\$,FC}) \quad (14.2)
\]

Note that the covariance term can be either positive or negative, as shown in Table 14.2.
Currency Market Returns and Bond Market Return Combinations

<table>
<thead>
<tr>
<th>Bond Market Returns</th>
<th>Currency Market Returns</th>
<th>Spot FX Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Negative</strong></td>
<td>FC interest rates ↑</td>
<td>$/€, measures value of €</td>
</tr>
<tr>
<td>Spot FX ↓</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Positive</strong></td>
<td>FC interest rates ↓</td>
<td></td>
</tr>
<tr>
<td>Spot FX ↓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Spot FX has the dimension of $/€, hence measures the value of €.
Currency and Bond Market Returns Combinations

Cases A and B have a Fisherian foundation.

For Case A, a forecast of future inflation may raise interest rates (bond market losses),

\[
i_{DM} = r_{DM} + E(\Delta P_{Germany})
\]

and depress the foreign exchange rate (currency market losses).

International Fisher Effect (Fisher Open)

\[
i_{\$} - i_{DM} = E(\Delta \text{ Spot})
\]
Currency and Bond Market Returns Combinations

Cases C and D result in negative covariance between currency and bond market returns.

Case C corresponds to a tight monetary policy that raises interest rates (bond market losses) but attracts foreign capital and appreciates the exchange rate (currency market gains).

Case D suggests a low interest rate environment (bond market gains) that encourages an outflow of funds and a weaker currency market (currency market losses).
Currency and Bond Market Returns Combinations

If foreign interest rates are headed down, the manager may want to buy foreign bonds. But if lower interest rates imply a weaker currency (case D), the manager must weigh this possibility and consider a hedge to limit currency losses.
Calculating Currency-Hedged Returns in US$ Terms

After buying the foreign bond at a price $B_t$, one possible strategy is to sell all future coupon payments forward in exchange for US$ as well as sell the final return of principal forward.

This strategy is much like a currency swap that eliminates all currency risks and transforms this foreign bond into a US$ bond.
The Basic Cash Flows of a Currency Swap

- Firms A and B can each issue a 7-year bond in either the US$ or SFr market.
- Firm A has a comparative advantage in borrowing US$ while firm B has a comparative advantage in borrowing SFr.
- By borrowing in their comparative advantage currencies and then swapping, lower cost financing is possible.

<table>
<thead>
<tr>
<th></th>
<th>Firm A</th>
<th>Firm B</th>
<th>Difference (A-B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US$ finance</td>
<td>10%</td>
<td>11.5%</td>
<td>-1.5%</td>
</tr>
<tr>
<td>SFr finance</td>
<td>5%</td>
<td>6%</td>
<td>-1.0%</td>
</tr>
</tbody>
</table>

Figure 13.2 Pg 453
The Basic Cash Flows of a Currency Swap

- Together, A and B save 0.5 percent. Note that if a bank or swap dealer intermediates the transaction and charges a fee, the aggregate interest savings will be reduced.
Suppose our investor sells a one-month forward currency contract (priced at $F_t$) for an amount equal to next month’s estimated value of the bond with accrued interest, $\hat{B}_{t+1}$, where:

$$\hat{B}_{t+1} \equiv B_t + \Delta_{t+1} + C_{t+1}$$

If our investor guesses right, and $\hat{B}_{t+1} = \hat{B}_{t+1}$, then he has made a perfect hedge. The US$ value of his foreign bond is $\hat{B}_{t+1} F_t$ and the continuous rate of return measured in US$ is:

$$\tilde{R}_{\$,H^*} = \ln\left(\frac{\hat{B}_{t+1} F_t}{B_t S_t}\right) = \ln\left(\frac{\hat{B}_{t+1}}{B_t}\right) + \ln\left(\frac{F_t}{S_t}\right) = \hat{B}_{FC} + F_{US\$,FC} \quad (14.3)$$

where the $H^*$ subscript indicates a perfect hedge.
The return $R_{\$,H^*}$ also has two pieces:

The return on the bond in foreign currency terms ($\hat{B}_{FC}$) plus the one-month forward premium ($F_{US\$,FC}$). The variance of returns in equation (14.3) is:

$$\sigma^2(\tilde{R}_{\$,H^*}) = \sigma^2(\hat{B}_{FC}) + \sigma^2(F_{US\$,FC}) + 2\text{Cov}(\hat{B}_{FC};F_{US\$,FC}) \quad (14.4)$$

As an empirical matter, $\sigma^2(\tilde{R}_{\$,H^*})$ should be less than $\sigma^2(\tilde{R}_{\$,U})$ because the volatility of the forward premium is far smaller than the volatility of exchange rate changes.
Forward Premium and Spot Rate Change

Figure 5.8A  Forward Premium and Spot Rate Change
$/DM One-Month Forward Rates: January 1979–December 1998

Source: Bank of Montreal/Harris Bank database and author’s calculations.
Forward Premium and Spot Rate Change

Figure 5.8B  Forward Premium and Spot Rate Change

Source: Bank of Montreal/Harris Bank database and author’s calculations.

Levich Figure 5.8B
Most likely, our investor cannot perfectly predict the future price of the foreign bond.

We define the prediction error as the actual minus the expected bond price, or:

$$\tilde{\varepsilon}_{t+1} \equiv \tilde{B}_{t+1} - \hat{B}_{t+1} = \tilde{\Delta}_{t+1} - \hat{\Delta}_{t+1}$$

The term $\tilde{\varepsilon}_{t+1}$ and its volatility represent the interest rate risk in the foreign bond market.

If $\tilde{\varepsilon}_{t+1} > 0$, our hedge amount was too small and the unexpected (positive) excess value of the bond is valued at $S_{t+1}$ and we need to sell the unexpected additional funds in the market at $S_{t+1}$. 
Conversely, if $\tilde{\varepsilon}_{t+1} < 0$, our hedged amount was too large (we oversold foreign currency forward) and we need to buy unexpected additional funds in the market at $S_{t+1}$.

In general, once the value of the future exchange rate is known, we measure the continuous rate of return on the foreign bond measured in US$ and on a currency-hedged basis as:

$$\tilde{R}_{S,H} = \ln\left(\frac{\hat{B}_{t+1} F_t}{B_t S_t}\right) + \ln\left(\frac{\tilde{\varepsilon}_{t+1} S_{t+1}}{B_t S_t}\right)$$

$$= \ln\left(\frac{\hat{B}_{t+1}}{B_t}\right) + \ln\left(\frac{F_t}{S_t}\right) + \ln\left(\tilde{\varepsilon}_{t+1} S_{t+1}\right)$$

$$= \hat{B}_{FC} + F_{US\$;FC} + \ln\left(\frac{\tilde{\varepsilon}_{t+1} S_{t+1}}{B_t S_t}\right)$$
\[ \tilde{R}_{\$,H} = \ln\left(\frac{\hat{B}_{t+1}F_t}{B_tS_t}\right) + \ln\left(\frac{\tilde{\epsilon}_{t+1}S_{t+1}}{B_tS_t}\right) \]

\[ = \ln\left(\frac{\hat{B}_{t+1}}{B_t}\right) + \ln\left(\frac{F_t}{S_t}\right) + \ln\left(\frac{\tilde{\epsilon}_{t+1}S_{t+1}}{B_tS_t}\right) \quad (14.5) \]

\[ = \hat{B}_{FC} + F_{US\$,FC} + \ln\left(\frac{\tilde{\epsilon}_{t+1}S_{t+1}}{B_tS_t}\right) \]

In order to allow for the possibility that \( \tilde{\epsilon}_{t+1} \) may be negative, we need to modify the definition of the hedged US$ return in equation (14.5) on page 497.
\[ \tilde{R}_{s,H} = \ln \left( \frac{\hat{B}_{t+1} F_t}{B_t S_t} + \frac{\tilde{\varepsilon}_{t+1} \tilde{S}_{t+1}}{B_t S_t} \right) \] (14.5a)

\[ = \ln \left[ \frac{\hat{B}_{t+1} F_t}{B_t S_t} \left( 1 + \frac{\tilde{\varepsilon}_{t+1} \tilde{S}_{t+1}}{\hat{B}_{t+1} F_t} \right) \right] \] (14.5b)

\[ = \ln \left[ \frac{\hat{B}_{t+1} F_t}{B_t S_t} \right] + \ln \left[ \left( 1 + \frac{\tilde{\varepsilon}_{t+1} \tilde{S}_{t+1}}{\hat{B}_{t+1} F_t} \right) \right] \] (14.5c)
Using equation (14.5c), we can allow for cases where $\tilde{\varepsilon}_{t+1} < 0$. The formula produces sensible answers, and it is still valid to think of the hedged return as the sum of three pieces: the predicted price change of the bond, the forward premium, and the residual unpredicted price change of the bond.

With $\tilde{\varepsilon}_{t+1} = 0$, we still have the result that the perfectly hedged portfolio earns a constant (known return) and no error variance.
Equation (14.5) shows the return on a *currency-hedged foreign bond*.

This return has three pieces:

1. the return from the predicted price change on the bond in foreign currency terms,
2. the forward premium (or discount) on the foreign currency used to buy the bonds, and
3. a residual term representing the unpredicted price change in the *foreign* bond that is valued at the future uncertain spot exchange rate.
Notice that the US$ returns on the first two pieces are certain, because the predicted end-of-month value of the bond has been sold forward at a price $F_t$. The primary source of uncertainty ($\tilde{\varepsilon}_{t+1}$) stems from our inability to predict $\tilde{B}_{t+1}$ and $\tilde{\Delta}_{t+1}$ with certainty because of interest rate risk in the foreign bond market.

The variance of returns for the currency-hedged bond in equation (14.5) is now:

$$\sigma^2(R_{\$,H}) = \left[ \frac{S_{t+1}}{B_tS_t} \right]^2 \sigma^2(\tilde{\varepsilon}_{t+1}) \tag{14.6}$$
Equation (14.6) is a conditional variance, conditional on $\tilde{S}_{t+1} = S_{t+1}$. In general, $\sigma^2(R_{s,H})$ depends on the combined effects of $\sigma^2(\tilde{\varepsilon}_{t+1})$ and $\sigma^2(\tilde{S}_{t+1})$.

$$\sigma^2(\tilde{R}_{s,H}) = \left[ \frac{S_{t+1}}{(B_t S_t)} \right]^2 \sigma^2(\tilde{\varepsilon}_{t+1}) \quad (14.6)$$

If there were no interest rate risk, then the prediction of $\tilde{B}_{t+1}$ is perfect, and $\tilde{\varepsilon}_{t+1} = 0$. In this special case, there is no residual element associated with interest rate risk. Thus, the US$ returns on the currency-hedged bond are given with certainty. Variance of returns in this case is zero.
Calculation of Prices and Returns for a Five-Year German Bund on an Unhedged Investment

The 5-year German Bund is priced at par with a 4.00% coupon paid annually.

The initial spot exchange rate $S_0 = $0.65/DM, so the purchase of a DM1 million bond requires an outlay of $1,000,000 \times 0.65 = $650,000.

At the end of year 1:
Suppose $i_{DM}$ falls to 3.75%, spot DM weakens to $0.625.
Each coupon payment is $1,000,000 \times 0.04 = DM40,000.

DM bond price

\[
\begin{align*}
    &= \frac{40,000}{(1 + .0375)} + \frac{40,000}{(1 + .0375)^2} + \frac{40,000}{(1 + .0375)^3} + \frac{40,000 + 1,000,000}{(1 + .0375)^4} \\
    &= 1,009,128.46
\end{align*}
\]

Box 14.1
So, in DM terms, the return for the first year

\[
= \ln \left( \frac{1,009,128.46 + 40,000}{1,000,000} \right) \times 100 = 4.80\%
\]

Since the spot DM has weakened to $0.625, the bond’s US$ value is \(1,009,128.46 \times 0.625 = $630,705.29\), and the coupon value is \(40,000 \times 0.625 = $25,000\).

So, in US$ terms, the first year return that reflects the coupon and exchange rate loss

\[
= \ln \left( \frac{630,705.29 + 25,000}{650,000} \right) \times 100 = 0.87\%
\]
Calculation of Prices and Returns for a Five-Year German Bund on a Currency-Hedged Investment

The 5-year German Bund is priced at par with a 4.00% coupon paid annually.

The DM is at roughly a 1% forward premium, and slightly larger when DM interest rates fall.

Estimate of the expected future bond price = previous value of the bond plus the coupon payment.
Calculation of Prices and Returns for a Five-Year German Bund on a Currency-Hedged Investment

<table>
<thead>
<tr>
<th>Year</th>
<th>German Interest</th>
<th>Spot Rate</th>
<th>Forward Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.00%</td>
<td>0.6500</td>
<td>0.6560</td>
</tr>
<tr>
<td>1</td>
<td>3.75%</td>
<td>0.6250</td>
<td>0.6325</td>
</tr>
<tr>
<td>2</td>
<td>3.50%</td>
<td>0.6350</td>
<td>0.6440</td>
</tr>
<tr>
<td>3</td>
<td>3.75%</td>
<td>0.6600</td>
<td>0.6675</td>
</tr>
</tbody>
</table>

For the first year:
The initial bond value = DM1,000,000.

With DM40,000 of interest expected, the investor hedges by selling DM1,040,000 forward at $0.6560/DM, resulting in 1,040,000x0.6560 = $682,240.
Calculation of Prices and Returns for a Five-Year German Bund on a Currency-Hedged Investment

Because DM interest rates dropped, the bond price rose to DM1,009,128 (underhedged by the amount of DM9,128). The additional DM9,128 must be marked to market at the current spot rate, which gives a US$ value of 9,128x0.625 = $5,705.

The total rate of return over the first period is thus

\[
= \ln \left[ \frac{682,240 + 5705}{650,000} \right] \times 100 = 5.67\%
\]
Central Bank Risk vs Interest Rate Risk

Interest rate risk refers to uncertainty about future interest rates that introduces the possibility for capital gains and losses on long-term bonds.

Central bank risk refers to uncertainty about the national monetary authority to deliver monetary policy that results in a particular level of interest rate risk.
Active Hedging vs Passive Hedging Strategies

What are the their advantages?

In a passive hedging strategy, the investor follows the same hedging plan over time independent of market conditions. For example, rules whereby the investor always hedges 100%, or always hedges 10%.

With active hedging, the amount hedged fluctuates.

A passive strategy is a low-cost means of reducing exposure to risks. The investor is sure to be protected against large negative shocks, but he also forgoes the opportunities of large gains from positive shocks.

In an active strategy, the investor retains risks during certain periods. This offers the possibility of higher returns if the investor has expertise in judging when to hedge and when not to hedge.
Three Types of Active Strategies

“Currency-driven” investment strategy places the focus on finding good performing currencies and buy safe assets (e.g., government bonds) denominated in that currency.

The obvious way to speculate on a currency view is to take a position in foreign exchange spot, forward, or futures contracts.

If the currency forecast is correct, the speculation will be profitable.
Three Types of Active Strategies

A second strategy is to “ignore currency” when making international investments, based on the premise that the currency effects cancel out over the long run. If the investor has expertise in picking good stocks or bonds, this expertise will be awarded.

What if the stocks or bonds are denominated in a foreign currency? If the forward rate and future spot rate are equal, on average, the investor can hedge this currency risk without sacrificing long-term returns.
Three Types of Active Strategies

A final strategy is the “ultraselective” approach, in which the investor picks only those situations where both positive foreign bond and currency returns are expected.

This calls for either short positions in unhedged bonds (Case A in Table 14.2), when both bonds and currency are expected to weaken, or long positions in unhedged bonds (Case B), when both bonds and currency are expected to strengthen.
## Currency Market Returns and Bond Market Return Combinations

<table>
<thead>
<tr>
<th>Bond Market Returns</th>
<th>Currency Market Returns</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>FC interest rates ↑</td>
<td>(A)</td>
</tr>
<tr>
<td></td>
<td>Spot FX ↓</td>
<td></td>
</tr>
<tr>
<td>Positive</td>
<td>FC interest rates ↓</td>
<td>(D)</td>
</tr>
<tr>
<td></td>
<td>Spot FX ↓</td>
<td></td>
</tr>
</tbody>
</table>

- **Negative**: FC interest rates increase, Spot FX decreases.
- **Positive**: FC interest rates decrease, Spot FX increases.

Table 14.2 Pg 496
While this approach may be successful, it is far too limiting. The investor passes up favorable bond markets when the currency is expected to weaken (Case D), and passes up favorable currency plays when profitable foreign stocks or bonds cannot be identified (Case C).

Our analysis demonstrates that the currency and interest rate risk dimensions of an international bond portfolio are separable investments. We show this in Table 14.5 for a world with three countries and three currencies.

An investor who wanted to invest in U.S. bonds but hold an exposure to ¥ currency risk would buy U.S. Treasuries and currency hedge them into ¥ (cell A) by selling US$ forward for ¥.
The Expanded Opportunity Set of National Bond Markets Currency Risk and Interest Rate Risk Dimensions

<table>
<thead>
<tr>
<th>Interest Rate Risk</th>
<th>Currency Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>US$</td>
</tr>
<tr>
<td></td>
<td>U.S. Treasury Bond</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>Germany Bund:</td>
<td>German Bund: Currency hedged to $</td>
</tr>
<tr>
<td>Japan</td>
<td>JGB:</td>
</tr>
<tr>
<td></td>
<td>Currency</td>
</tr>
<tr>
<td></td>
<td>hedged to $</td>
</tr>
</tbody>
</table>

Table 14.5 Pg 508
What are the key elements to take into consideration when investing in an international bond portfolio?

The first criterion is selecting a market with no controls on capital outflows. Other institutional considerations - market size, liquidity, taxation - play a role. Active portfolio decisions are made on the basis of estimated risk and return. The investor can hedge a large portion of the currency risk in foreign bonds if liquid short-term currency forward markets are available.

Average Return (% per annum)

Risk: Standard Deviation of Returns

- Global Portfolio
- Unhedged Portfolios
- Hedged Portfolios
- Dollar Portfolio

Figure 14.4 Pg 503
About Figure 14.4 --

- The sample period is January 1977 - December 1990, monthly data.
- Securities are 10-year government bonds issued by the United States, Canada, Germany, Japan, and the United Kingdom.
- The unhedged global portfolio is an equally-weighted portfolio of non-U.S. securities.
- The hedged global portfolio is based on one-month forward currency contract, rolled over monthly.
- The end points of the frontier represent 100 percent in U.S. bonds or 100% in global bonds.
- Interior points on the frontier represent 90/10, 80/20, 70/30, etc. combinations.
Efficient Portfolio Frontiers with Active and Passive Hedges 1977-1990

Figure 14.5 Pg 510
About Figure 14.5 --

• The sample period is Jan. 1977 - Dec. 1990, monthly data.

• Securities are 10-year government bonds issued by U.S., Canada, Germany, Japan, and U.K..

• The *unhedged* global portfolio is an equally-weighted portfolio of non-U.S. securities.

• The *hedged* global portfolio is based on one-month forward currency contract, rolled over monthly.

• The *tactical hedge* portfolio actively hedges a percentage of the global portfolio based on the signals from 10 technical trading rules.

• The *overlay* portfolio reflects the performance of the hedged global portfolio combined with a currency fund actively managed based on the signals from 10 technical trading rules.
A tactical hedging strategy is one where the percentage of currency futures to sell for currency I ($P_{T,I}$) based on the 10 technical rules is determined by the formula:

$$P_{T,I} = \left[10 - (N_{L,I} - N_{S,I})\right] \times 10\%, \quad \text{for } N_{L,I} \geq 5$$

$$= 100\% \quad \text{for } N_{L,I} \leq 4$$

where $N_{L,I}$ and $N_{S,I}$ are the number of technical rules advocating long and short currency positions respectively.

The currency overlay strategy is actually a combination of two separate investments: (1) a foreign currency bond position that is always hedged against currency risk, and (2) a currency position governed by the trading rule $P_I = \left[(N_{L,I} - N_{S,I})\right] \times 10\%$. If all trading rules recommend a long (short) position the currency overlay strategy will be 100% unhedged (overhedged to become net 100% short in the foreign currency).
Assignment from Chapter 14

Exercises 1, 2.
Fixed Income Securities: Analytics and Derivatives

Individual Bond Strategies:

Barbells Strategy

Ladders Strategy

Bullets Strategy
Ladders Strategy

Ladders are a popular strategy for staggering the maturity of your bond investments and for setting up a schedule for reinvesting them as they mature. A ladder can help you reap the typically higher coupon rates of longer-term investments, while allowing you to reinvest a portion of your funds every few years.

Example: Ladder strategy
You buy three bonds with different maturity dates: two years, four years, and six years. As each bond matures, you have the option of buying another bond to keep the ladder going. In this example, you buy 10-year bonds. Longer-term bonds typically offer higher interest rates.
Ladders Strategy

Ladders are popular among investors who want bonds as part of a long-term investment objective, such as saving for college tuition, or seeking additional predictable income for retirement planning.

Ladders have several potential advantages:

1. The periodic return of principal provides the investor with additional income beyond the set interest payments.

2. The income derived from principal and interest payments can either be directed back into the ladder if interest rates are relatively high or invested elsewhere if they are relatively low.

3. Interest rate volatility is reduced because the investor now determines the best investment option every few years, as each bond matures.

4. Investors should be aware that laddering can require commitment of assets over time, and return of principal at time of redemption is not guaranteed.
Barbell Strategies

Barbells

Barbells are a strategy for buying short-term and long-term bonds, but not intermediate-term bonds. The long-term end of the barbell allows you to lock into attractive long-term interest rates, while the short-term end insures that you will have the opportunity to invest elsewhere if the bond market takes a downturn.

Example: Barbell strategy
You see appealing long-term interest rates, so you buy two long-term bonds. You also buy two short-term bonds. When the short-term bonds mature, you receive the principal and have the opportunity to reinvest it.
Bullets Strategies

Bullets

Bullets are a strategy for having several bonds mature at the same time and minimizing the interest rate risk by staggering when you buy the bonds. This is useful when you know that you will need the proceeds from the bonds at a specific time, such as when a child begins college.

Example: Bullet strategy
You want all bonds to mature in 10 years, but want to stagger the investment to reduce the interest rate risk. You buy the bonds over four years.

* Numbers represent years
Constructing the Theoretical Spot Rate Curve for Treasuries

To determine the value of each zero-coupon instrument, it is necessary to know the yield on a zero-coupon Treasury with that same maturity. This yield is called the spot rate, and the graphical depiction of the relationship between the spot rate and maturity is called the spot rate curve. Because there are no zero-coupon Treasury debt issues with a maturity greater than one year, it is not possible to construct such a curve solely from observations of market activity on Treasury securities. Rather, it is necessary to derive this curve from theoretical considerations as applied to the yields of the actually traded Treasury debt securities. Such a curve is called a theoretical spot rate curve and is the graphical depiction of the term structure of interest rate.
Constructing the Theoretical Spot Rate Curve for Treasuries

A default-free theoretical spot rate curve can be constructed from the yield on Treasury securities. The Treasury issues that are candidates for inclusion are (i) on-the-run Treasury issues, (ii) on-the-run Treasury issues and selected off-the-run Treasury issues, (iii) all Treasury coupon securities, and bills, and (iv) Treasury coupon strips.

After the securities that are to be included in the construction of the theoretical spot rate curve are selected, the methodology for constructing the curve must be determined. If Treasury coupon strips are used, the procedure is simple, because the observed yields are the spot rates. If the on-the-run Treasury issues with or without selected off-the-run Treasury issues are used, a methodology called bootstrapping is used.
Figure 5-4. Maturity and Yield to Maturity for 20

<table>
<thead>
<tr>
<th>Period</th>
<th>Years</th>
<th>Yield to Maturity/Coupon Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>5.25</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>5.50</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>5.75</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>6.00</td>
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<tr>
<td>5</td>
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<td>6.50</td>
</tr>
<tr>
<td>7</td>
<td>3.5</td>
<td>6.75</td>
</tr>
<tr>
<td>8</td>
<td>4.0</td>
<td>6.80</td>
</tr>
<tr>
<td>9</td>
<td>4.5</td>
<td>7.00</td>
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<td>10</td>
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<td>7.10</td>
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<tr>
<td>13</td>
<td>6.5</td>
<td>7.30</td>
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<tr>
<td>14</td>
<td>7.0</td>
<td>7.35</td>
</tr>
<tr>
<td>15</td>
<td>7.5</td>
<td>7.40</td>
</tr>
<tr>
<td>16</td>
<td>8.0</td>
<td>7.50</td>
</tr>
<tr>
<td>17</td>
<td>8.5</td>
<td>7.60</td>
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<tr>
<td>18</td>
<td>9.0</td>
<td>7.60</td>
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<tr>
<td>19</td>
<td>9.5</td>
<td>7.70</td>
</tr>
<tr>
<td>20</td>
<td>10.0</td>
<td>7.80</td>
</tr>
</tbody>
</table>

$Z_1 = 5.25/2$

$Z_2 = 5.50/2$
0.5 year: \(0.0575 \times 100 \times 0.5 = \$2.875\)
1.0 year: \(0.0575 \times 100 \times 0.5 = 2.875\)
1.5 years: \(0.0575 \times 100 \times 0.5 + 100 = 102.875\)

The present value of the cash flow is then

\[
\frac{2.875}{1+z_1} + \frac{2.875}{(1+z_2)^2} + \frac{102.875}{(1+z_3)^3}
\]

where:

\[z_1 = \text{one-half the annualized 6-month theoretical spot rate}\]
\[z_2 = \text{one-half the 1-year theoretical spot rate}\]
\[z_3 = \text{one-half the annual value of the 1.5-year theoretical spot rate}\]

Because the six-month spot rate and one-year spot rate are 5.25% and 5.50%, respectively, we know these facts:
\[z_1 = 0.02625\text{ and } z_2 = 0.0275\]

We can compute the present value of the 1.5-year coupon Treasury security as

\[
\frac{2.875}{1.02625} + \frac{2.875}{(1.0275)^2} + \frac{102.875}{(1+z_3)^3}
\]

\[Z_1 = 5.25/2\]
\[Z_2 = 5.50/2\]
Because the price of the 1.5-year coupon Treasury security is $100, the following relationship must hold:

\[ 100 = \frac{2.875}{1.02625} + \frac{2.875}{(1.0275)^2} + \frac{102.875}{(1 + z_3)^3} \]

We can solve for the theoretical 1.5-year spot rate as follows:

\[ 100 = 2.801461 + 2.723166 + \frac{102.875}{(1 + z_3)^3} \]
\[ 94.47537 = \frac{102.875}{(1 + z_3)^3} \]
\[ (1 + z_3)^3 = 1.028798 \]
\[ z_3 = 0.028798 \]
Doubling this yield, we obtain the bond-equivalent yield of 0.0576 or 5.76%, which is the theoretical 1.5-year spot rate. That rate is the rate that the market would apply to a 1.5-year zero-coupon Treasury security if, in fact, such a security existed.

Given the theoretical 1.5-year spot rate, we can obtain the theoretical 2-year spot rate. The cash flow for the two-year coupon Treasury in Exhibit 5-4 is

\[
\begin{align*}
0.5 \text{ year: } & 0.060 \times $100 \times 0.5 = 3.00 \\
1.0 \text{ year: } & 0.060 \times $100 \times 0.5 = 3.00 \\
1.5 \text{ years: } & 0.060 \times $100 \times 0.5 = 3.00 \\
2.0 \text{ years: } & 0.060 \times $100 \times 0.5 + $100 = 103.00
\end{align*}
\]
The present value of the cash flow is then

\[
\frac{3.00}{1+z_1} + \frac{3.00}{(1+z_2)^2} + \frac{3.00}{(1+z_3)^3} + \frac{103.00}{(1+z_4)^4}
\]

where \(z_4\) is one-half the two-year theoretical spot rate. Because the 6-month spot rate, 1-year spot rate, and 1.5-year spot rate are 5.25%, 5.50%, and 5.76%, respectively, then

\[z_1 = 0.02625 \quad z_2 = 0.0275 \quad z_3 = 0.028798\]

Therefore, the present value of the two-year coupon Treasury security is

\[
\frac{3.00}{1.002625^2} + \frac{3.00}{(1.0275)^2} + \frac{3.00}{(1.028798)^3} + \frac{103.00}{(1+z_4)^4}
\]

Because the price of the two-year coupon Treasury security is $100, the following relationship must hold:

\[
100 = \frac{3.00}{1.002625^2} + \frac{3.00}{(1.0275)^2} + \frac{3.00}{(1.028798)^3} + \frac{103.00}{(1+z_4)^4}
\]
We can solve for the theoretical two-year spot rate as follows:

\[ 100 = 2.92326 + 2.84156 + 2.75506 + \frac{103.00}{(1 + z_4)^4} \]

\[ 91.48011 = \frac{103.00}{(1 + z_4)^4} \]

\[ (1 + z_4)^4 = 1.125927 \]

\[ z_4 = 0.030095 \]

Doubling this yield, we obtain the theoretical two-year spot rate bond-equivalent yield of 6.02%.
Theoretical Spot Rate Curve for Treasuries
Source: Fabozzi, Pages 103-8

<table>
<thead>
<tr>
<th>Period</th>
<th>Years</th>
<th>Spot Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>5.25</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>5.50</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>5.75</td>
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<td>4</td>
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<td>7</td>
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<td>15</td>
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</tr>
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<td>16</td>
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<td>17</td>
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<td>19</td>
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</tr>
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<td>8.07</td>
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</table>
One can follow this approach sequentially to derive the theoretical 2.5-year spot rate from the calculated values of $z_1$, $z_2$, $z_3$, $z_4$ (the 6-month, 1-year, 1.5-year, and 2-year rates), and the price and coupon of the bond with a maturity of 2.5 years. Further, one could derive theoretical spot rates for the remaining 15 half-yearly rates.

The spot rates using this process are shown in Exhibit 5-5. They represent the term structure of interest rates for maturities up to 10 years at the particular time to which the bond price quotations refer.
Try questions 13 and 14 in Chapter 5 of Fabozzi.
Exhibit 22-4. Types of Yield Curve Shifts

(a) Parallel Shifts

- Upward Parallel Shift
- Downward Parallel Shift

(b) Twists

- Flattening Twist
- Steepening Twist

(c) Butterfly Shifts

- Positive Butterfly
- Negative Butterfly
Exhibit 22-6. Yield Curve Strategies: Bullet, Barbell, and Ladder

Bullet Strategy
Spikes indicate maturing principal
Comment: bullet concentrated around year 10

Barbell Strategy
Spikes indicate maturing principal
Comment: barbell below and above 10 years

Ladder Strategy
Spikes indicate maturing principal
Comment: laddered up to year 20
In a **bullet strategy**, the portfolio is constructed so that the maturity of the securities in the portfolio are highly concentrated at one point on the yield curve. In a **barbell strategy**, the maturity of the securities included in the portfolio are concentrated at two extreme maturities. Actually, in practice when managers refer to a barbell strategy it is relative to a bullet strategy. For example, a bullet strategy might be to create a portfolio with maturities concentrated around 10 years, whereas a corresponding barbell strategy might be a portfolio with 5- and 20-year maturities. In a **ladder strategy** the portfolio is constructed to have approximately equal amounts of each maturity. So, for example, a portfolio might have equal amounts of securities with one year to maturity, two years to maturity, and so on.
### Exhibit 22-7. Three Hypothetical Treasury Securities

<table>
<thead>
<tr>
<th>Bond</th>
<th>Coupon (%)</th>
<th>Maturity (years)</th>
<th>Price Plus Accrued</th>
<th>Yield to Maturity (%)</th>
<th>Dollar Duration</th>
<th>Dollar Convexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8.50</td>
<td>5</td>
<td>100</td>
<td>8.50</td>
<td>4.005</td>
<td>19.8164</td>
</tr>
<tr>
<td>B</td>
<td>9.50</td>
<td>20</td>
<td>100</td>
<td>9.50</td>
<td>8.882</td>
<td>124.1702</td>
</tr>
<tr>
<td>C</td>
<td>9.25</td>
<td>10</td>
<td>100</td>
<td>9.25</td>
<td>6.434</td>
<td>55.4506</td>
</tr>
</tbody>
</table>

Consider the following two yield curve strategies: a bullet strategy and a barbell strategy. We will label the portfolios created based on these two strategies as the “bullet portfolio” and the “barbell portfolio” and they comprise the following:

- **Bullet portfolio**: 100% bond C
- **Barbell portfolio**: 50.2% bond A and 49.8% bond B
The bullet portfolio consists of only bond C, the 10-year bond. In our hypothetical portfolio, all the principal is received when bond C matures in 10 years. The barbell portfolio consists of almost an equal amount of the short- and long-term securities. It is the result of a barbell strategy because principal will be received at two ends of the maturity spectrum. Specifically, relative to the bullet portfolio, which in our illustration has all its principal being returned in 10 years, for the barbell portfolio the principal is being returned at shorter (5 years) and longer (20 years) dates.

As we explained in Chapter 4, dollar duration is a measure of the dollar price sensitivity of a bond or a portfolio. As indicated in Exhibit 22-7, the dollar duration for the bullet portfolio per 100-basis-point change in yield is 6.434. For the barbell portfolio, the dollar duration is just the weighted average of the dollar duration of the two bonds. Therefore,

dollar duration of barbell portfolio = 0.502(4.005) + 0.498(8.882) = 6.434

The dollar duration of the barbell portfolio is the same as that of the bullet portfolio. (In fact, the barbell portfolio was designed to produce this result.)
As we explained in Chapter 4, duration is just a first approximation of the change in price resulting from a change in interest rates. Convexity provides a second approximation. Although we did not discuss dollar convexity, it has a meaning similar to convexity, in that it provides a second approximation to the dollar price change. For two portfolios with the same dollar duration, the greater the convexity, the better the performance of a bond or a portfolio when yields change. What is necessary to understand for this illustration is that the larger the dollar convexity, the greater the dollar price change due to a portfolio’s convexity. As shown in Exhibit 22-7, the dollar convexity of the bullet portfolio is 55.4506. The dollar convexity for the barbell portfolio is a weighted average of the dollar convexity of the two bonds. That is,
dollar convexity of barbell portfolio = 0.502(19.8164) + 0.498(124.1702) = 71.7846

Therefore, the dollar convexity of the barbell portfolio is greater than that of the barbell portfolio.

Similarly, the yield for the two portfolios is not the same. The yield for the bullet portfolio is simply the yield to maturity of bond C, 9.25%. The traditional yield calculation for the barbell portfolio, which is found by taking a weighted average of the yield to maturity of the two bonds included in the portfolio, is 8.998%:

portfolio yield for barbell portfolio = 0.502(8.50%) + 0.498(9.50%) = 8.998%

This approach suggests that the yield of the bullet portfolio is 25.2 basis points greater than that of the barbell portfolio (9.25% - 8.998%). Although both portfolios have the same dollar duration, the yield of the bullet portfolio is greater than the yield of the barbell portfolio. However, the dollar convexity of the barbell portfolio is greater than that of the bullet portfolio. The difference in the two yields is sometimes referred to as the cost of convexity (i.e., giving up yield to get better convexity).
Try questions 15 and 18 in Chapter 22 of Fabozzi.
TIPS work as follows. The coupon rate on an issue is set at a fixed rate. That rate is determined via the auction process described later in this section. The coupon rate is called the “real rate” since it is the rate that the investor ultimately earns above the inflation rate. The inflation index that the government has decided to use for the inflation adjustment is the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for All Urban Consumers (CPI-U).

Source: Fabozzi Chapter 6, Pages 128-9
The adjustment for inflation is as follows. The principal that the Treasury Department will base both the dollar amount of the coupon payment and the maturity value on is adjusted semiannually. This is called the **inflation-adjusted principal**. For example, suppose that the coupon rate for a TIPS is 3.5% and the annual inflation rate is 3%. Suppose further that an investor purchases on January 1 $100,000 par value (principal) of this issue. The semiannual inflation rate is 1.5% (3% divided by 2). The inflation-adjusted principal at the end of the first six-month period is found by multiplying the original par value by one plus the semiannual inflation rate. In our example, the inflation-adjusted principal at the end of the first six-month period is $101,500. It is this inflation-adjusted principal that is the basis for computing the coupon interest for the first six-month period. The coupon payment is then 1.75% (one-half the real rate of 3.5%) multiplied by the inflation-adjusted principal at the coupon payment date ($101,500). The coupon payment is therefore $1,776.25.
Let's look at the next six months. The inflation-adjusted principal at the beginning of the period is $101,500. Suppose that the semiannual inflation rate for the second six-month period is 1%. Then the inflation-adjusted principal at the end of the second six-month period is the inflation-adjusted principal at the beginning of the six-month period ($101,500) increased by the semiannual inflation rate (1%). The adjustment to the principal is $1,015 (1% times $101,500). So, the inflation-adjusted principal at the end of the second six-month period (December 31 in our example) is $102,515 ($101,500 + $1,015). The coupon interest that will be paid to the investor at the second coupon payment date is found by multiplying the inflation-adjusted principal on the coupon payment date ($102,515) by one-half the real rate (i.e., one-half of 3.5%). That is, the coupon payment will be $1,794.01.
Try question 2 in Chapter 6 of Fabozzi.
Exhibit 11-1. Creation of a Pass-Through and Its Cash Flow

Monthly Cash Flow

Loan #1
- Interest
- Scheduled principal repayment
- Prepayments

Loan #2
- Interest
- Scheduled principal repayment
- Prepayments

Loan #3
- Interest
- Scheduled principal repayment
- Prepayments

Loan #4
- Interest
- Scheduled principal repayment
- Prepayments

Loan #5
- Interest
- Scheduled principal repayment
- Prepayments

Loan #6
- Interest
- Scheduled principal repayment
- Prepayments

Loan #7
- Interest
- Scheduled principal repayment
- Prepayments

Loan #8
- Interest
- Scheduled principal repayment
- Prepayments

Loan #9
- Interest
- Scheduled principal repayment
- Prepayments

Loan #10
- Interest
- Scheduled principal repayment
- Prepayments

Each loan is for $100,000
Total loans: $1 million

Pass-Through: $1 million par
Pooled mortgage loans

Pooled Monthly Cash Flow:
- Interest
- Scheduled principal repayment
- Prepayments

Rule for distribution of cash flow
Pro rata basis
Conditional Prepayment Rate

Another benchmark for projecting prepayments and the cash flow of a pass-through requires assuming that some fraction of the remaining principal in the pool is prepaid each month for the remaining term of the mortgage. The prepayment rate assumed for a pool, called the **conditional prepayment rate (CPR)**, is based on the characteristics of the pool (including its historical prepayment experience) and the current and expected future economic environment. It is referred to as a conditional rate because it is conditional on the remaining mortgage balance.

**Single-Monthly Mortality Rate**

The CPR is an annual prepayment rate. To estimate monthly prepayments, the CPR must be converted into a monthly prepayment rate, commonly referred to as the **single-monthly mortality rate (SMM)**. A formula can be used to determine the SMM for a given CPR:

\[
SMM = 1 - (1 - CPR)^{1/12}
\]
Suppose that the CPR used to estimate prepayments is 6%. The corresponding SMM is

\[ SMM = 1 - (1 - 0.06)^{1/12} \]
\[ = 1 - (0.94)^{0.08333} = 0.005143 \]

**SMM Rate and Monthly Prepayment**

An SMM of \( w \)% means that approximately \( w \)% of the remaining mortgage balance at the beginning of the month, less the scheduled principal payment, will prepay that month. That is,

prepayment for month \( t \)

\[ \text{prepayment for month } t = SMM \times (\text{beginning mortgage balance for month } t - \text{scheduled principal payment for month } t) \]

For example, suppose that an investor owns a pass-through in which the remaining mortgage balance at the beginning of some month is $290 million. Assuming that the SMM is 0.5143% and the scheduled principal payment is $3 million, the estimated prepayment for the month is

\[ 0.005143(290,000,000 - 3,000,000) = 1,476,041 \]
Collateralized mortgage obligations (CMOs) are bond classes created by redirecting the cash flows of mortgage-related products so as to mitigate prepayment risk. The mere creation of a CMO cannot eliminate prepayment risk; it can only transfer the various forms of this risk among different classes of bondholders. The technique of redistributing the coupon interest and principal from the underlying mortgage-related products to different classes, so that a CMO class has a different coupon rate from that for the underlying collateral, results in instruments that have varying risk-return characteristics that may be more suitable to the needs and expectations of investors, thereby broadening the appeal of mortgage-backed products to various traditional fixed-income investors.

CMO Structure
A CMO is a security backed by a pool of pass-throughs, whole loans, or stripped mortgage-backed securities (explained later in the chapter). CMOs are structured so that there are several classes of bondholders with varying stated maturities. When there is more than one class of bondholders with the same level of credit priority, the structure is called a pay-through structure, as opposed to a pass-through structure in which there is only one class of bondholders at a given level of credit priority.
Exhibit 12-1. FJF-01: Hypothetical Four-Tranche Sequential-Pay Structure

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Par Amount</th>
<th>Coupon Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$194,500,000</td>
<td>7.5</td>
</tr>
<tr>
<td>B</td>
<td>36,000,000</td>
<td>7.5</td>
</tr>
<tr>
<td>C</td>
<td>96,500,000</td>
<td>7.5</td>
</tr>
<tr>
<td>D</td>
<td>73,000,000</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>$400,000,000</td>
<td></td>
</tr>
</tbody>
</table>

1. For payment of periodic coupon interest: Disburse periodic coupon interest to each tranche on the basis of the amount of principal outstanding at the beginning of the period.

2. For disbursement of principal payments: Disburse principal payments to tranche A until it is paid off completely. After tranche A is paid off completely, disburse principal payments to tranche B until it is paid off completely. After tranche B is paid off completely, disburse principal payments to tranche C until it is paid off completely. After tranche C is paid off completely, disburse principal payments to tranche D until it is paid off completely.
Exhibit 12-4. FJF-02: Hypothetical Four-Tranche Sequential-Pay Structure with an Accrual Bond Class

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Par Amount</th>
<th>Coupon Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$194,500,000</td>
<td>7.5</td>
</tr>
<tr>
<td>B</td>
<td>36,000,000</td>
<td>7.5</td>
</tr>
<tr>
<td>C</td>
<td>96,500,000</td>
<td>7.5</td>
</tr>
<tr>
<td>Z (accrual)</td>
<td>73,000,000</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>$400,000,000</td>
<td></td>
</tr>
</tbody>
</table>

1. For payment of periodic coupon interest: Disburse periodic coupon interest to tranches A, B, and C on the basis of the amount of principal outstanding at the beginning of the period. For tranche Z, accrue the interest based on the principal plus accrued interest in the preceding period. The interest for tranche Z is to be paid to the earlier tranches as a principal pay down.

2. For disbursement of principal payments: Disburse principal payments to tranche A until it is completely paid off. After tranche A is paid off completely, disburse principal payments to tranche B until it is paid off completely. After tranche B is paid off completely, disburse principal payments to tranche C until it is paid off completely. After tranche C is paid off completely, disburse principal payments to tranche Z, until the original principal balance plus accrued interest is paid off completely.
Accrual Bonds

In FJF-01, the payment rules for interest provide for all tranches to be paid interest each month. In many sequential-pay CMO structures, at least one tranche does not receive current interest. Instead, the interest for that tranche would accrue and be added to the principal balance. Such a bond class is commonly referred to as an **accrual tranche**, or a **Z bond** (because the bond is similar to a zero-coupon bond). The interest that would have been paid to the accrual bond class is then used to speed up the pay down of the principal balance of earlier bond classes.
Exhibit 12-6. FJF-03: Hypothetical Five-Tranche Sequential-Pay Structure with Floater, Inverse Floater, and Accrual Bond Classes[a]

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Par Amount</th>
<th>Coupon Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$194,500,000</td>
<td>7.50</td>
</tr>
<tr>
<td>B</td>
<td>36,000,000</td>
<td>7.50</td>
</tr>
<tr>
<td>FL</td>
<td>72,375,000</td>
<td>1-month LIBOR + 0.50</td>
</tr>
<tr>
<td>IFL</td>
<td>24,125,000</td>
<td>28.50 - 3 * (1-month LIBOR)</td>
</tr>
<tr>
<td>Z (accrual)</td>
<td>73,000,000</td>
<td>7.50</td>
</tr>
<tr>
<td></td>
<td>$400,000,000</td>
<td></td>
</tr>
</tbody>
</table>

1. For payment of periodic coupon interest: Disburse periodic coupon interest to tranches A, B, FL, and IFL on the basis of the amount of principal outstanding at the beginning of the period. For tranche Z, accrue the interest based on the principal plus accrued interest in the preceding period. The interest for tranche Z is to be paid to the earlier tranches as a principal pay down. The maximum coupon rate for FL is 10%; the minimum coupon rate for IFL is 0%.

2. For disbursement of principal payments: Disburse principal payments to tranche A until it is paid off completely. After tranche A is paid off completely, disburse principal payments to tranche B until it is paid off completely. After tranche B is paid off completely, disburse principal payments to tranches FL and IFL until they are paid off completely. The principal payments between tranches FL and IFL should be made in the following way: 75% to tranche FL and 25% to tranche IFL. After tranches FL and IFL are paid off completely, disburse principal payments to tranche Z until the original principal balance plus accrued interest is paid off completely.
## Exhibit 12-8. FJF-04: CMO Structure with One PAC Bond and One Support Bond

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Par Amount</th>
<th>Coupon Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (PAC)</td>
<td>$243,800,000</td>
<td>7.5</td>
</tr>
<tr>
<td>S (Support)</td>
<td>156,200,000</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>$400,000,000</td>
<td></td>
</tr>
</tbody>
</table>

1. **For payment of periodic coupon interest** Disburse periodic coupon interest to each tranche on the basis of the amount of principal outstanding at the beginning of the period.

2. **For disbursement of principal payments** Disburse principal payments to tranche P based on its schedule of principal repayments. Tranche P has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule for tranche P are paid to tranche S. When tranche S is paid off completely, all principal payments are to be made to tranche P regardless of the schedule.
### Exhibit 12-15. FJF-06: Hypothetical Five Tranche Sequential Pay with an Accrual Tranche and an Interest-Only Tranche

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Par Amount</th>
<th>Notional Amount</th>
<th>Coupon Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$194,500,000</td>
<td></td>
<td>6.00</td>
</tr>
<tr>
<td>B</td>
<td>36,000,000</td>
<td></td>
<td>6.50</td>
</tr>
<tr>
<td>C</td>
<td>96,500,000</td>
<td></td>
<td>7.00</td>
</tr>
<tr>
<td>Z</td>
<td>73,000,000</td>
<td></td>
<td>7.25</td>
</tr>
<tr>
<td>IO</td>
<td></td>
<td>52,566,667</td>
<td>7.50</td>
</tr>
<tr>
<td></td>
<td>$400,000,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. For payment of periodic coupon interest: Disburse periodic coupon interest to tranches A, B, and C on the basis of the amount of principal outstanding at the beginning of the period. For tranche Z, accrue the interest based on the principal plus accrued interest in the preceding period. The interest for tranche Z is to be paid to the earlier tranches as a principal pay down. Disburse periodic interest to the IO tranche based on the notional amount at the beginning of the period.

2. For disbursement of principal payments: Disburse principal payments to tranche A until it is paid off completely. After tranche A is paid off completely, disburse principal payments to tranche B until it is paid off completely. After tranche B is paid off completely, disburse principal payments to tranche C until it is paid off completely. After tranche C is paid off completely, disburse principal payments to tranche Z until the original principal balance plus accrued interest is paid off completely.

Source: Fabozzi Chapter 12, Pages 273-303
Try question 20 in Chapter 12 of Fabozzi.