Introduction to Swaps

• A swap is a contract calling for an exchange of payments, on one or more dates, determined by the difference in two prices

• A swap provides a means to hedge a stream of risky payments

• A single-payment swap is the same thing as a cash-settled forward contract
An Example of a Commodity Swap

• An industrial producer, IP Inc., needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.

• The forward prices for deliver in 1 year and 2 years are $20 and $21/barrel.

• The 1- and 2-year zero-coupon bond yields are 6% and 6.5%.
An Example of a Commodity Swap (cont’d)

- IP can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years. The PV of this cost per barrel is

\[
\frac{20}{1.06} + \frac{21}{1.065^2} = \$37.383
\]

- Thus, IP could pay an oil supplier $37.383, and the supplier would commit to delivering one barrel in each of the next two years.

- A **prepaid swap** is a single payment today for multiple deliveries of oil in the future.
An Example of a Commodity Swap (cont’d)

- With a prepaid swap, the buyer might worry about the resulting credit risk. Therefore, a better solution is to defer payments until the oil is delivered, while still fixing the total price.

- Any payment stream with a PV of $37.383 is acceptable. Typically, a swap will call for equal payments in each year.
  - For example, the payment per year per barrel, $x$, will have to be $20.483 to satisfy the following equation:

$$\frac{x}{1.06} + \frac{x}{1.06^2} = $37.383$$

- We then say that the 2-year swap price is $20.483.
Physical Versus Financial Settlement

- **Physical settlement** of the swap

**Figure 8.1** Illustration of a swap where the oil buyer pays $20.483/year and receives 1 barrel of oil each year.
Physical Versus Financial Settlement (cont’d)

• Financial settlement of the swap
  – The oil buyer, IP, pays the swap counterparty the difference between $20.483 and the spot price, and the oil buyer then buys oil at the spot price
  – If the difference between $20.483 and the spot price is negative, then the swap counterparty pays the buyer
Physical Versus Financial Settlement (cont’d)

• Whatever the market price of oil, the net cost to the buyer is the swap price, $20.483

\[
\text{Spot price} - \text{Swap price} = \text{Swap Payment} - \text{Spot Purchase of Oil}
\]

• Note that 100,000 is the **notional amount** of the swap, meaning that 100,000 barrels is used to determine the magnitude of the payments when the swap is settled financially.
Physical Versus Financial Settlement (cont’d)

- The results for the buyer are the same whether the swap is settled physically or financially. In both cases, the net cost to the oil buyer is $20.483

Figure 8.2 Cash flows from a transaction where the oil buyer enters into a financially settled 2-year swap. Each year the buyer pays the spot price for oil and receives spot price – $20.483. The buyer’s net cost of oil is $20.483/barrel.
Swaps are nothing more than forward contracts coupled with borrowing and lending money

- Consider the swap price of $20.483/barrel. Relative to the forward curve price of $20 in 1 year and $21 in 2 years, we are overpaying by $0.483 in the first year, and we are underpaying by $0.517 in the second year.
- Thus, by entering into the swap, we are lending the counterparty money for 1 year. The interest rate on this loan is

\[
\frac{0.517}{0.483} - 1 = 7\%
\]

- Given 1- and 2-year zero-coupon bond yields of 6% and 6.5%, 7% is the 1-year implied forward yield from year 1 to year 2.

- If the deal is priced fairly, the interest rate on this loan should be the implied forward interest rate.
The Swap Counterparty

• The swap counterparty is a dealer, who is, in effect, a broker between buyer and seller

• The fixed price paid by the buyer, usually, exceeds the fixed price received by the seller. This price difference is a bid-ask spread, and is the dealer’s fee

• The dealer bears the credit risk of both parties, but is not exposed to price risk
The Swap Counterparty (cont’d)

- The situation where the dealer matches the buyer and seller is called a **back-to-back transaction** or “matched book” transaction.

**Figure 8.4** Cash flows from a transaction where an oil buyer and seller each enters into a financially settled 2-year swap. The buyer pays the spot price for oil and receives spot price – $20.483 each year as a swap payment. The oil seller receives the spot price for oil and receives $20.483 – spot price as a swap payment.
The Swap Counterparty (cont’d)

• Alternatively, the dealer can serve as counterparty and hedge the transaction by entering into long forward or futures contracts

Table 8.1 Positions and cash flows for a dealer who has an obligation to receive the fixed price in an oil swap and who hedges the exposure by going long year 1 and year 2 oil forwards.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment from Oil Buyer</th>
<th>Long Forward</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20.483 – year 1 spot price</td>
<td>Year 1 spot price – $20</td>
<td>$0.483</td>
</tr>
<tr>
<td>2</td>
<td>$20.483 – year 2 spot price</td>
<td>Year 2 spot price – $21</td>
<td>$-0.517</td>
</tr>
</tbody>
</table>

– Note that the net cash flow for the hedged dealer is a loan, where the dealer receives cash in year 1 and repays it in year 2
– Thus, the dealer also has interest rate exposure (which can be hedged by using Eurodollar contracts or forward rate agreements)
The Market Value of a Swap

- The market value of a swap is zero at interception.
- Once the swap is struck, its market value will generally no longer be zero because:
  - the forward prices for oil and interest rates will change over time.
  - even if prices do not change, the market value of swaps will change over time due to the implicit borrowing and lending.
- A buyer wishing to exit the swap could enter into an offsetting swap with the original counterparty or whomever offers the best price.
- The market value of the swap is the difference in the PV of payments between the original and new swap rates.
Box 8.1  Enron’s Hidden Debt

When energy giant Enron collapsed in the fall of 2001, there were charges that other companies had helped Enron mislead investors. In July 2003, the Securities and Exchange Commission announced that J. P. Morgan Chase and Citigroup had each agreed to pay more than $100 million to settle allegations that they had helped Enron commit fraud. Specifically, the SEC alleged that both banks had helped Enron characterize loan proceeds as operating income.

The basic outline of the transaction with J. P. Morgan Chase is as follows. Enron entered into “prepaid forward sales contracts” (essentially a prepaid swap) with an entity called Mahonia; Enron received a lump-sum payment and agreed to deliver natural gas in the future. Mahonia in turn received a lump-sum payment from Chase and agreed to deliver natural gas in the future. Chase, which controlled Mahonia, then hedged its Mahonia transaction with Enron. With all transactions netted out, Enron had no commodity exposure, and received its lump-sum initial payment from Mahonia in exchange for making future fixed installment payments to Chase. In other words, Enron in effect had a loan with Chase. Not only did Enron not record debt from these transactions, but the company reported operating income. The transaction is illustrated in Figure 8.5.

The SEC complaint included a revealing excerpt from internal Chase e-mail:

WE ARE MAKING DISGUISED LOANS, USUALLY BURIED IN COMMODITIES OR EQUITIES DERIVATIVES (AND I’M SURE IN OTHER AREAS). WITH A FEW [sic] EXCEPTIONS, THEY ARE UNDERSTOOD TO BE DISGUISED LOANS AND APPROVED AS SUCH. (Capitalization in the original.)
Figure 8.5
Enron’s swaps with Mahonia and Chase.

Source: Securities and Exchange Commission.
Interest Rate Swaps

• The notional principle of the swap is the amount on which the interest payments are based

• The life of the swap is the swap term or swap tenor

• If swap payments are made at the end of the period (when interest is due), the swap is said to be settled in arrears
An Example of an Interest Rate Swap

- XYZ Corp. has $200M of floating-rate debt at LIBOR, i.e., every year it pays that year’s current LIBOR

- XYZ would prefer to have fixed-rate debt with 3 years to maturity

- XYZ could enter a swap, in which they receive a floating rate and pay the fixed rate, which is 6.9548%
An Example of an Interest Rate Swap (cont’d)

Figure 8.6 Illustration of cash flows for a company that borrows at LIBOR and swaps to fixed-rate exposure at 6.9548%.

- On net, XYZ pays 6.9548%

\[
XYZ \text{ net payment} = -\text{LIBOR} + \text{LIBOR} - 6.9548\% = -6.9548\%
\]

Floating Payment \hspace{1cm} Swap Payment
Computing the Swap Rate

- Suppose there are $n$ swap settlements, occurring on dates $t_i$, $i = 1, \ldots, n$
- The implied forward interest rate from date $t_{i-1}$ to date $t_i$, known at date 0, is $r_0(t_{i-1}, t_i)$
- The price of a zero-coupon bond maturing on date $t_i$ is $P(0, t_i)$
- The fixed swap rate is $R$

* The market-maker is a counterparty to the swap in order to earn fees, not to take on interest rate risk. Therefore, the market-maker will hedge the floating rate payments by using, for example, forward rate agreements
Computing the Swap Rate

• The requirement that the hedged swap have zero net PV is

\[ \sum_{i=1}^{n} P(0,t_i) [R - r_0(t_{i-1},t_i)] = 0 \]  \hspace{2cm} (8.1)

• Equation (8.1) can be rewritten as

\[ R = \frac{\sum_{i=1}^{n} P(0,t_i)r(t_{i-1},t_i)}{\sum_{i=1}^{n} P(0,t_i)} \]  \hspace{2cm} (8.2)

where \( \sum_{i=1}^{n} P(0,t) r(t_{i-1},t_i) \) is the PV of interest payments implied by the strip of forward rates, and \( \sum_{i=1}^{n} P(0,t_i) \) is the PV of a $1 annuity when interest rates vary over time.
Computing the Swap Rate (cont’d)

- We can rewrite equation (8.2) to make it easier to interpret

\[ R = \sum_{i=1}^{n} \left[ \frac{P(0,t_i)}{\sum_{j=1}^{n} P(0,t_j)} \right] r(t_{i-1}, t_i) \]

- Thus, the fixed swap rate is as a weighted average of the implied forward rates, where zero-coupon bond prices are used to determine the weights
Computing the Swap Rate (cont’d)

- Alternative way to express the swap rate is
  \[ R = \frac{1 - P_0(0,t_n)}{\sum_{i=1}^{n} P_0(0,t_i)} \] (8.3)
  
- This equation is equivalent to the formula for the coupon on a par coupon bond

- Thus, the swap rate is the coupon rate on a par coupon bond
Deferred Swap

• A **deferred swap** is a swap that begins at some date in the future, but its swap rate is agreed upon today.

• The fixed rate on a deferred swap beginning in $k$ periods is computed as

\[
R = \frac{\sum_{i=k}^{T} P_0(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=k}^{T} P(0, t_i)} \quad (8.4)
\]

• Equation (8.4) is equal to equation (8.2), when $k = 1$.
The Swap Curve

- A set of swap rates at different maturities is called the *swap curve*.

- The swap curve should be consistent with the interest rate curve implied by the Eurodollar futures contract, which is used to hedge swaps.

- Recall that the Eurodollar futures contract provides a set of 3-month forward LIBOR rates. In turn, zero-coupon bond prices can be constructed from implied forward rates. Therefore, we can use this information to compute swap rates.
The Swap Curve (cont’d)

Table 8.4 Three-month LIBOR forward rates and swap rates implied by Eurodollar futures prices with maturity dates given in the first column. Prices are from November 8, 2007. Source: Wall Street Journal online.

<table>
<thead>
<tr>
<th>Maturity Date of Eurodollar Futures Contract</th>
<th>Price of Eurodollar Futures</th>
<th>3-Month Forward Rate Implied by Eurodollar Futures Price</th>
<th>Implied Dec 2008 Price of $1 Paid 3 Months after Futures Mat. Date</th>
<th>Swap Rate (%) for Loan Made Dec 2008, Ending 3 Months after Futures Mat. Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 2007</td>
<td>95.250</td>
<td>0.01201</td>
<td>0.98814</td>
<td>4.8028</td>
</tr>
<tr>
<td>Mar 2008</td>
<td>95.720</td>
<td>0.01082</td>
<td>0.97756</td>
<td>4.5664</td>
</tr>
<tr>
<td>Jun 2008</td>
<td>95.965</td>
<td>0.01020</td>
<td>0.96769</td>
<td>4.4059</td>
</tr>
<tr>
<td>Sep 2008</td>
<td>96.075</td>
<td>0.00992</td>
<td>0.95818</td>
<td>4.2982</td>
</tr>
<tr>
<td>Dec 2008</td>
<td>96.080</td>
<td>0.00991</td>
<td>0.94878</td>
<td>4.2326</td>
</tr>
<tr>
<td>Mar 2009</td>
<td>96.000</td>
<td>0.01011</td>
<td>0.93928</td>
<td>4.2021</td>
</tr>
<tr>
<td>Jun 2009</td>
<td>95.865</td>
<td>0.01045</td>
<td>0.92957</td>
<td>4.1991</td>
</tr>
<tr>
<td>Sep 2009</td>
<td>95.735</td>
<td>0.01078</td>
<td>0.91965</td>
<td>4.2128</td>
</tr>
<tr>
<td>Dec 2009</td>
<td>95.605</td>
<td>0.01111</td>
<td>0.90955</td>
<td>4.2374</td>
</tr>
</tbody>
</table>
Amortizing and Accreting Swaps

- An **amortizing swap** is a swap where the notional value is *declining* over time (e.g., floating rate mortgage).

- An **accreting swap** is a swap where the notional value is *growing* over time.

- The fixed swap rate is still a weighted average of implied forward rates, but now the weights also involve changing notional principle, $Q_t$

\[
R = \frac{\sum_{i=1}^{n} Q_t P(0,t_i) r(t_{i-1},t_i)}{\sum_{i=1}^{n} Q_t P(0,t_i)} \quad (8.7)
\]
Currency Swaps

- A currency swap entails an exchange of payments in different currencies

- A currency swap is equivalent to borrowing in one currency and lending in another
An Example of a Currency Swap

- Suppose a dollar-based firm enters into a swap where it pays dollars and receives euros.
- The position of the market-maker is summarized below

Table 8.5 Unhedged and hedged cash flows for a dollar-based firm with euro-denominated debt.

<table>
<thead>
<tr>
<th>Year</th>
<th>Unhedged Euro Cash Flow</th>
<th>Forward Exchange Rate</th>
<th>Hedged Dollar Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>€3.5</td>
<td>0.922</td>
<td>$3.226</td>
</tr>
<tr>
<td>2</td>
<td>€3.5</td>
<td>0.944</td>
<td>$3.304</td>
</tr>
<tr>
<td>3</td>
<td>€103.5</td>
<td>0.967</td>
<td>$100.064</td>
</tr>
</tbody>
</table>

- The PV of the market-maker’s net cash flows is $(\frac{2.174}{1.06}) + (\frac{2.096}{1.06^2}) - (\frac{4.664}{1.06^3}) = 0$
Currency Swap Formulas

- Consider a swap in which a dollar annuity, $R$, is exchanged for an annuity in another currency, $R^*$
- There are $n$ payments
- The time-0 forward price for a unit of foreign currency delivered at time $t_i$ is $F_{0,t_i}$
- The dollar-denominated zero-coupon bond price is $P_{0,t_i}$
Currency Swap Formulas (cont’d)

- Given $R^*$, what is $R$?

$$R = \frac{\sum_{i=1}^{n} P_{0,t_i} R^* F_{0,t_i}}{\sum_{i=1}^{n} P_{0,t_i}}$$

(8.8)

- This equation is equivalent to equation (8.2), with the implied forward rate, $r_0(t_{i-1}, t_i)$, replaced by the foreign-currency-denominated annuity payment translated into dollars, $R^* F_{0,t_i}$.
Currency Swap Formulas (cont’d)

– When coupon bonds are swapped, one has to account for the difference in maturity value as well as the coupon payment
– If the dollar bond has a par value of $1, the foreign bond will have a par value of $1/x_0$, where $x_0$ is the current exchange rate expressed as dollar per unit of the foreign currency

• The coupon rate on the dollar bond, $R$, in this case is

$$ R = \frac{\sum_{i=1}^{n} P_{0,t_i} R \ast F_{0,t_i} / x_0 + P_{0,t_n} (F_{0,t_o} / x_0 - 1)}{\sum_{i=1}^{n} P_{0,t_i}} $$ (8.9)
Other Currency Swaps

- A **diff swap**, short for differential swap, is a swap where payments are made based on the difference in floating interest rates in two different currencies, with the notional amount in a single currency.

- Standard currency forward contracts cannot be used to hedge a diff swap.
  - We can’t easily hedge the exchange rate at which the value of the interest rate change is converted because we don’t know in advance how much currency will need to be converted.
Commodity Swaps

• The fixed payment on a commodity swap is

\[
\bar{F} = \frac{\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} P(0,t_i)} \quad (8.11)
\]

• The commodity swap price is a weighted average of commodity forward prices
Swaptions

- A swaption is an option to enter into a swap with specified terms. This contract will have a premium

- A swaption is analogous to an ordinary option, with the PV of the swap obligations (the price of the prepaid swap) as the underlying asset

- Swaptions can be American or European
Swaptions (cont’d)

• A payer swaption gives its holder the right, but not the obligation, to pay the fixed price and receive the floating price
  – The holder of a receiver swaption would exercise when the fixed swap price is above the strike

• A receiver swaption gives its holder the right to pay the floating price and receive the fixed strike price.
  – The holder of a receiver swaption would exercise when the fixed swap price is below the strike
Total Return Swaps

- A **total return swap** is a swap, in which one party pays the realized total return (dividends plus capital gains) on a reference asset, and the other party pays a floating return such as LIBOR.

- The two parties exchange only the difference between these rates.

- The party paying the return on the reference asset is the **total return payer**.
Total Return Swaps (cont’d)

• Some uses of total return swaps are
  – avoiding withholding taxes on foreign stocks
  – management of credit risk

• A default swap is a swap, in which the seller makes a payment to the buyer if the reference asset experiences a “credit event” (e.g., a failure to make a scheduled payment on a bond)
  – A default swap allows the buyer to eliminate bankruptcy risk, while retaining interest rate risk
  – The buyer pays a premium, usually amortized over a series of payments
Summary

- The swap formulas in different cases all take the same general form.

- Let $f_0(t_i)$ denote the forward price for the floating payment in the swap. Then the fixed swap payment is

$$R = \frac{\sum_{i=1}^{n} P(0,t_i) f_0(t_i)}{\sum_{i=1}^{n} P(0,t_i)}$$

(8.13)
Summary

- The following table summarizes the substitutions to make in equation (8.10) to get various swap formulas.

<table>
<thead>
<tr>
<th></th>
<th>First Leg</th>
<th>Second Leg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payer</td>
<td>Broker-dealer</td>
<td>Counterparty</td>
</tr>
<tr>
<td>Equity</td>
<td>S&amp;P Total Return Index</td>
<td>Floating Rate 1-month USD LIBOR</td>
</tr>
<tr>
<td>Notional</td>
<td>USD 100,000,000</td>
<td>Notional Equity notional</td>
</tr>
<tr>
<td>Number of Index Units</td>
<td>TBD</td>
<td>Spread(bps) + 12bps</td>
</tr>
<tr>
<td>Initial Price</td>
<td>TBD</td>
<td>Initial Rate TBD</td>
</tr>
<tr>
<td>Currency</td>
<td>USD</td>
<td>Currency USD</td>
</tr>
<tr>
<td>Valuation Dates</td>
<td>TBD</td>
<td>Day Count Actual/360</td>
</tr>
<tr>
<td>Payment Dates</td>
<td>3 business days following valuation dates</td>
<td>Payment Dates Equity</td>
</tr>
<tr>
<td>Pay Frequency</td>
<td>Monthly</td>
<td>Frequency Monthly</td>
</tr>
<tr>
<td>Return</td>
<td>Total Return</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8.8 Example of a term sheet for an equity swap based on the S&P 500 total return index. TBD means “To be determined.”
Chapter 8

Additional Art
Figure 8.3  Illustrative example of the terms for an oil swap based on West Texas Intermediate (WTI) crude oil.

Sample Commodity Swap Term Sheet

Fixed-rate payer:  Broker-dealer
Floating-rate payer: Counterparty
Notional Amount:  100,000 barrels
Trade Date:  November 8, 2007
Effective Date:  January 1, 2008
Termination Date:  March 31, 2008
Period End Date:  Final Pricing Date of each Calculation Period as defined in the description of the Floating Price.
Fixed Price:  93.87 USD per barrel
Commodity Reference Price:  OIL-WTI-NYMEX
Floating price:  The average of the first nearby NYMEX WTI Crude Oil Futures settlement prices for each successive day of the Calculation Period during which such prices are quoted
Calculation Period:  Each calendar month during the transaction
Method of averaging:  Unweighted
Settlement and payment:  If the Fixed Amount exceeds the Floating Amount for such Calculation Period, the Fixed Price Payor shall pay the Floating Price Payor an amount equal to such excess. If the Floating Amount exceeds the Fixed Amount for such Calculation Period, the Floating Price Payor shall pay the Fixed Price Payor an amount equal to such excess.
Payment Date:  5 business days following each Period End Date.
Figure 8.5 Enron’s swaps with Mahonia and Chase. Source: Securities and Exchange Commission.
Table 8.2  Cash flows faced by a market-maker who receives fixed and pays floating and hedges the resulting exposure using forward rate agreements.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment on Forward</th>
<th>Net Swap Payment</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>—</td>
<td>$R - 6%$</td>
<td>$R - 6%$</td>
</tr>
<tr>
<td>2</td>
<td>$\tilde{r}_2 - 7.0024%$</td>
<td>$R - \tilde{r}_2$</td>
<td>$R - 7.0024%$</td>
</tr>
<tr>
<td>3</td>
<td>$\tilde{r}_3 - 8.0071%$</td>
<td>$R - \tilde{r}_3$</td>
<td>$R - 8.0071%$</td>
</tr>
</tbody>
</table>
Table 8.3  Cash flows faced by a floating-rate borrower who enters into a 3-year swap with a fixed rate of 6.9548%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Floating-Rate Debt Payment</th>
<th>Net Swap Payment</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−6%</td>
<td>6%−6.9548%</td>
<td>−6.9548%</td>
</tr>
<tr>
<td>2</td>
<td>−(\tilde{r}_2)</td>
<td>(\tilde{r}_2−6.9548%)</td>
<td>−6.9548%</td>
</tr>
<tr>
<td>3</td>
<td>−(\tilde{r}_3)</td>
<td>(\tilde{r}_3−6.9548%)</td>
<td>−6.9548%</td>
</tr>
</tbody>
</table>
Figure 8.7  Example of a term sheet for an interest rate swap.

<table>
<thead>
<tr>
<th><strong>Sample Interest Rate Swap Term Sheet</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed-Rate Payer</strong></td>
</tr>
<tr>
<td>Payer: Broker-dealer</td>
</tr>
<tr>
<td>Fixed Rate: 6.59%</td>
</tr>
<tr>
<td>Notional Principal: USD 100,000,000</td>
</tr>
<tr>
<td>Currency: USD</td>
</tr>
<tr>
<td>Day Count: 30/360</td>
</tr>
<tr>
<td>Payment Dates: Semiannually on</td>
</tr>
<tr>
<td>the 15th day of each</td>
</tr>
<tr>
<td>April and October</td>
</tr>
</tbody>
</table>
Equation 8.1

\[ \sum_{i=1}^{n} P(0, t_i)[R - r_0(t_{i-1}, t_i)] = 0 \]
Equation 8.2

\[ R = \frac{\sum_{i=1}^{n} P(0, t_i) r(t_{i-1}, t_i)}{\sum_{i=1}^{n} P(0, t_i)} \]
Equation 8.3

\[ R = \frac{1 - P_0(0, t_n)}{\sum_{i=1}^{n} P_0(0, t_i)} \]
Equation 8.4

\[ R = \frac{\sum_{i=k}^{n} P_0(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=k}^{n} P(0, t_i)} \]
Equation 8.5

\[ R = \frac{P(0, t_{k-1}) - P(0, t_n)}{\sum_{i=k}^{n} P(0, t_i)} \]
\[
\sum_{i=1}^{n} Q_{t_i} P(0, t_i)[R - r(t_{i-1}, t_i)] = 0
\]
Equation 8.7

\[ R = \frac{\sum_{i=1}^{n} Q_{t_i} P(0, t_i)r(t_{i-1}, t_i)}{\sum_{i=1}^{n} Q_{t_i} P(0, t_i)} \]
Table 8.6  Unhedged and hedged cash flows for a dollar-based firm with euro-denominated debt. The effective annual dollar-denominated interest rate is 6%, and the effective annual euro-denominated interest rate is 3.5%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Forward Exchange Rate ($/€)</th>
<th>Receive Dollar Interest</th>
<th>Pay Hedged Euro Interest</th>
<th>Net Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9217</td>
<td>$5.40</td>
<td>–€3.5 × 0.9217</td>
<td>$2.174</td>
</tr>
<tr>
<td>2</td>
<td>0.9440</td>
<td>$5.40</td>
<td>–€3.5 × 0.9440</td>
<td>$2.096</td>
</tr>
<tr>
<td>3</td>
<td>0.9668</td>
<td>$95.40</td>
<td>–€103.5 × 0.9668</td>
<td>–$4.664</td>
</tr>
</tbody>
</table>
Equation 8.8

\[ R = \frac{\sum_{i=1}^{n} P_{0,t_i} R^* F_{0,t_i}}{\sum_{i=1}^{n} P_{0,t_i}} \]
Equation 8.9

\[
R = \frac{\sum_{i=1}^{n} P_{0,t_i} R^* F_{0,t_i} / x_0 + P_{0,t_n} (F_{0,t_n} / x_0 - 1)}{\sum_{i=1}^{n} P_{0,t_i}}
\]
**Figure 8.9** Cash flows for a total return swap. The total return payer pays the per-period total return on the reference asset, receiving the floating rate from the counterparty.
**Table 8.7** Illustration of cash flows on a total return swap with annual settlement for 3 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>S&amp;P Capital Gain</th>
<th>S&amp;P Dividend</th>
<th>Floating Rate</th>
<th>Net Payment to Total Return Payer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5%</td>
<td>1.5%</td>
<td>7.2%</td>
<td>0.7%</td>
</tr>
<tr>
<td>2</td>
<td>−12%</td>
<td>1.5%</td>
<td>7.5%</td>
<td>18.0%</td>
</tr>
<tr>
<td>3</td>
<td>22%</td>
<td>1.5%</td>
<td>7.0%</td>
<td>−16.5%</td>
</tr>
</tbody>
</table>
Equation 8.10

\[ R = \frac{\sum_{i=1}^{n} P(0, t_i) f_0(t_i)}{\sum_{i=1}^{n} P(0, t_i)} \]
<table>
<thead>
<tr>
<th>Quarter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil forward price</td>
<td>21</td>
<td>21.1</td>
<td>20.8</td>
<td>20.5</td>
<td>20.2</td>
<td>20</td>
<td>19.9</td>
<td>19.8</td>
</tr>
<tr>
<td>Gas swap price</td>
<td>2.2500</td>
<td>2.4236</td>
<td>2.3503</td>
<td>2.2404</td>
<td>2.2326</td>
<td>2.2753</td>
<td>2.2583</td>
<td>2.2044</td>
</tr>
<tr>
<td>Zero-coupon bond price</td>
<td>0.9852</td>
<td>0.9701</td>
<td>0.9546</td>
<td>0.9388</td>
<td>0.9231</td>
<td>0.9075</td>
<td>0.8919</td>
<td>0.8763</td>
</tr>
<tr>
<td>Euro-denominated zero-coupon bond price</td>
<td>0.9913</td>
<td>0.9825</td>
<td>0.9735</td>
<td>0.9643</td>
<td>0.9551</td>
<td>0.9459</td>
<td>0.9367</td>
<td>0.9274</td>
</tr>
<tr>
<td>Euro forward price ($/€)</td>
<td>0.9056</td>
<td>0.9115</td>
<td>0.9178</td>
<td>0.9244</td>
<td>0.9312</td>
<td>0.9381</td>
<td>0.9452</td>
<td>0.9524</td>
</tr>
</tbody>
</table>