FACTORS AFFECTING BOND YIELDS AND THE TERM STRUCTURE OF INTEREST RATES

CHAPTER SUMMARY

In this chapter we look at the factors that affect the yield offered in the bond market. We begin with the minimum interest rate that an investor wants from investing in a bond, the yield on U.S. Treasury securities. Then we describe why the yield on a non-U.S. Treasury security will differ from that of a U.S. Treasury security. Finally, we focus on one particular factor that affects the yield offered on a security: maturity. The pattern of interest rates on securities of the same issuer but with different maturities is called the term structure of interest rates.

BASE INTEREST RATE

The securities issued by the U.S. Department of the Treasury are the key interest rates in the U.S. economy as well as in international capital markets. The lack of credit risk and the large size of any single issue have contributed to making the Treasury market the most active and hence the most liquid market in the world.

The minimum interest rate that investors want is referred to as the base interest rate or benchmark interest rate that investors will demand for investing in a non-Treasury security. This rate is the yield to maturity (hereafter referred to as simply yield) offered on a comparable maturity Treasury security that was most recently issued (“on the run”).

RISK PREMIUM

Market participants talk of interest rates on non-Treasury securities as trading at a spread to a particular on-the-run Treasury security. The yield spread, called a risk premium, reflects the additional risks the investor faces by acquiring a security that is not issued by the U.S. government. Thus we can express the interest rate offered on a non-Treasury security as base interest rate + spread or, equivalently, base interest rate + risk premium.

Yield spreads can be measured in terms of the difference between the yields on two bonds. The difference is measured in basis points. Unless otherwise stated, yield spreads are typically measured in this way. Yield spreads can also be measured on a relative basis by taking the ratio of the yield spread to the yield level. This is called a relative yield spread. Sometimes bonds are compared in terms of a yield ratio, the quotient of two bond yields.

The factors that affect the yield spread include the type of issuer, the issuer’s perceived credit worthiness, the term or maturity of the instrument, provisions that grant either the issuer or the investor the option to do something, the taxability of the interest received by investors, and the expected liquidity of the security.
Types of Issuers

The bond market is classified by the type of issuer, including the U.S. government, U.S. government agencies, municipal governments, credit (domestic and foreign corporations), and foreign governments. These classifications are referred to as market sectors. Different sectors are generally perceived to represent different risks and rewards.

Some market sectors are further subdivided into categories intended to reflect common economic characteristics. For example, within the credit market sector, issuers are classified as follows: industrial, utility, finance, and noncorporate. The spread between the interest rate offered in two sectors of the bond market with the same maturity is referred to as an intermarket sector spread. The spread between two issues within a market sector is called an intramarket sector spread.

Perceived Credit Worthiness of Issuer

Default risk or credit risk refers to the risk that the issuer of a bond may be unable to make timely principal and/or interest payments. Most market participants rely primarily on commercial rating companies to assess the default risk of an issuer. The spread between Treasury securities and non-Treasury securities that are identical in all respects except for quality is referred to as a quality spread or credit spread.

Inclusion of Options

It is not uncommon for a bond issue to include a provision that gives either the bondholder and/or the issuer an option to take some action against the other party. The presence of an embedded option has an effect on the spread of an issue relative to a Treasury security and the spread relative to otherwise comparable issues that do not have an embedded option.

Taxability of Interest

Because of the tax-exempt feature of municipal bonds, the yield on municipal bonds is less than that on Treasuries with the same maturity. The yield on a taxable bond issue after federal income taxes are paid is called the after-tax yield. Alternatively, we can determine the yield that must be offered on a taxable bond issue to give the same after-tax yield as a tax-exempt issue. This yield, called the equivalent taxable yield, is the tax-exempt yield divided by (1 – marginal tax rate).

The municipal bond market is divided into two major sectors: general obligations and revenue bonds. State and local governments may tax interest income on bond issues that are exempt from federal income taxes. Some municipalities exempt interest income from all municipal issues from taxation; others do not. Some states exempt interest income from bonds issued by municipalities within the state but tax the interest income from bonds issued by municipalities outside the state.

Municipalities are not permitted to tax the interest income from securities issued by the U.S. Treasury. Thus part of the spread between Treasury securities and taxable non-Treasury securities of the same maturity reflects the value of the exemption from state and local taxes.
Expected Liquidity of an Issue

Bonds trade with different degrees of liquidity. Bonds with greater expected liquidity will have lower yields that investors would require. The lower yield offered on Treasury securities relative to non-Treasury securities reflects the difference in liquidity.

Financeability of an Issue

A portfolio manager can use an issue as collateral for borrowing funds. By borrowing funds, a portfolio manager can create leverage. The typical market used by portfolio managers to borrow funds using a security as collateral for a loan is the repurchase agreement market or “repo” market. When a portfolio manager wants to borrow funds via a repo agreement, a dealer provides the funds. The interest rate charged by the dealer is called the repo rate.

There are times when dealers need particular issues to cover a short position. If so, the dealer will be willing to offer to lend funds at a lower repo rate than the market repo rate. The dealer is willing to offer attractive financing because it can use the collateral (i.e., the particular issue it needs) to cover a short position for the life of the repo agreement. The spread between the yield on such issues and issues that do not offer a below-market repo rate reflects the financing advantage. This spread is commonly observed in the Treasury market between on-the-run and off-the-run issues. The spread between on-the-run and off-the-run issues of approximately the same maturity reflects not only differences in liquidity but any financing advantage.

Term to Maturity

The time remaining on a bond’s life is referred to as its term to maturity or simply maturity. The volatility of a bond’s price is dependent on its term to maturity. More specifically, with all other factors constant, the longer the term to maturity of a bond, the greater the price volatility resulting from a change in market yields. Generally, bonds are classified into three maturity sectors: Bonds with a term to maturity of between 1 to 5 years are considered short term; bonds with a term to maturity between 5 and 12 years are viewed as intermediate term; and long-term bonds are those with a term to maturity greater than 12 years. The spread between any two maturity sectors of the market is called a maturity spread. The relationship between the yields on otherwise comparable securities with different maturities is called the term structure of interest rates.

TERM STRUCTURE OF INTEREST RATES

The term structure of interest rates plays a key role in the valuation of bonds.

Yield Curve

The graphical depiction of the relationship between the yield on bonds of the same credit quality but different maturities is known as the yield curve. In the past, most investors have constructed yield curves from observations of prices and yields in the Treasury market. Two factors account for this tendency. First, Treasury securities are free of default risk, and differences in credit worthiness do not affect yields. Therefore, these instruments are directly comparable. Second, as
the largest and most active bond market, the Treasury market offers the fewest problems of illiquidity or infrequent trading. The disadvantage, as noted previously, is that the yields may be biased downward because they reflect favorable financing opportunities.

Market participants are coming to realize that the traditionally constructed Treasury yield curve is an unsatisfactory measure of the relation between required yield and maturity. The key reason is that securities with the same maturity may carry different yields. This phenomenon reflects the role and impact of differences in the bonds’ coupon rates.

Why the Yield Curve Should Not Be Used to Price a Bond

The price of a bond is the present value of its cash flow. The bond pricing formula assumes that one interest rate should be used to discount all the bond’s cash flows. Because of the different cash flow patterns, it is not appropriate to use the same interest rate to discount all cash flows. Instead, each cash flow should be discounted at a unique interest rate appropriate for the time period in which the cash flow will be received.

The correct way to think about bonds is that they are packages of zero-coupon instruments. Each zero-coupon instrument in the package has a maturity equal to its coupon payment date or, in the case of the principal, the maturity date. The value of the bond should equal the value of all the component zero-coupon instruments. If this does not hold, it is possible for a market participant to generate riskless profits by stripping off the coupon payments and creating stripped securities.

To determine the value of each zero-coupon instrument, it is necessary to know the yield on a zero-coupon Treasury with that same maturity. This yield is called the **spot rate**, and the graphical depiction of the relationship between the spot rate and maturity is called the **spot rate curve**. Because there are no zero-coupon Treasury debt issues with a maturity greater than one year, it is not possible to construct such a curve solely from observations of market activity on Treasury securities. Rather, it is necessary to derive this curve from theoretical considerations as applied to the yields of the actually traded Treasury debt securities. Such a curve is called a **theoretical spot rate curve** and is the graphical depiction of the **term structure of interest rate**.

Constructing the Theoretical Spot Rate Curve for Treasuries

A default-free theoretical spot rate curve can be constructed from the yield on Treasury securities. The Treasury issues that are candidates for inclusion are (i) on-the-run Treasury issues, (ii) on-the-run Treasury issues and selected off-the-run Treasury issues, (iii) all Treasury coupon securities, and bills, and (iv) Treasury coupon strips.

After the securities that are to be included in the construction of the theoretical spot rate curve are selected, the methodology for constructing the curve must be determined. If Treasury coupon strips are used, the procedure is simple, because the observed yields are the spot rates. If the on-the-run Treasury issues with or without selected off-the-run Treasury issues are used, a methodology called **bootstrapping** is used.

The **on-the-run Treasury issues** are the most recently auctioned issue of a given maturity. These
issues include the 3-month, 6-month, and 1-year Treasury bills; the 2-year, 5-year, and 10-year Treasury notes; and the 30-year Treasury bond. Treasury bills are zero-coupon instruments; the notes and the bond are coupon securities.

There is an observed yield for each of the on-the-run issues. For the coupon issues, these yields are not the yields used in the analysis when the issue is not trading at par. Instead, for each on-the-run coupon issue, the estimated yield necessary to make the issue trade at par is used. The resulting on-the-run yield curve is called the **par coupon curve**.

The goal is to construct a theoretical spot rate curve with 60 semiannual spot rates: 6 month rate to 30-year rate. Excluding the three-month bill, there are only six maturity points available when only on-the-run issues are used. The 54 missing maturity points are extrapolated from the surrounding maturity points on the par yield curve. Then, the yield for all intermediate semiannual maturity points is found by adding to the yield at the lower maturity the amount computed.

For example, suppose that the yield from the par yield curve for the two-year and five-year on-the-run issues is 6% and 6.6%, respectively. There are six semiannual periods between these two maturity points. The extrapolated yield for the 2.0, 2.5, 3.0, 3.5, 4.0, and 4.5 maturity points can then be found.

There are two problems with using just the on-the-run issues. First, there is a large gap between some of the maturities points, which may result in misleading yields for those maturity points when estimated using the linear interpolation method. The second problem is that the yields for the on-the-run issues may be misleading because most offer the favorable financing opportunity in the repo market.

To overcome these problems, we convert the par yield curve into the theoretical spot rate curve using bootstrapping. To explain the process of estimating the theoretical spot rate curve from observed yields on Treasury securities, consider (i) a six-month Treasury bill where its annualized yield is the six-month spot rate and (ii) a one-year Treasury where its annualized yield is the one-year spot rate. Given these two spot rates, we can compute the spot rate for a theoretical 1.5-year zero-coupon Treasury. The price of a theoretical 1.5-year zero-coupon Treasury should equal the present value of three cash flows from an actual 1.5-year coupon Treasury, where the yield used for discounting is the spot rate corresponding to the cash flow. We can solve for the theoretical 1.5-year spot rate. Doubling this rate, we can obtain the bond-equivalent yield, which is the theoretical 1.5-year spot rate. This rate is the rate that the market would apply to a 1.5-year zero-coupon Treasury security if, in fact, such a security existed. Given the theoretical 1.5-year spot rate, we can obtain the theoretical 2-year spot rate and so forth until we derive theoretical spot rates for the remaining 15 half-yearly rates. The spot rates using this process represent the term structure of interest rates.

It would seem logical that the observed yield on strips could be used to construct an actual spot rate curve rather than go through the tedious computation procedure to get yields. There are three problems with using the observed rates on strips. First, the liquidity of the strips market is not as great as that of the Treasury coupon market. Thus, the observed rates on strips reflect a premium for liquidity. Second, the tax treatment of strips is different from that of Treasury coupon securities.
Specifically, the accrued interest on strips is taxed even though no cash is received by the investor. Finally, there are maturity sectors in which non-U.S. investors find it advantageous to trade off yield for tax advantages associated with a strip.

Using the Theoretical Spot Rate Curve

Arbitrage forces a Treasury to be priced based on spot rates and not the yield curve. The ability of dealers to purchase securities and create value by stripping forces Treasury securities to be priced based on the theoretical spot rates.

Spot Rates and the Base Interest Rate

The base interest rate for a given maturity should not be seen as simply the yield on the on-the-run Treasury security for that maturity, but the theoretical Treasury spot rate for that maturity. To value a non-Treasury security, we should add a risk premium to the theoretical Treasury spot rates.

Forward Rates

From the yield curve we can extrapolate the theoretical spot rates. In addition, we can extrapolate what some market participants refer to as the market’s consensus of future interest rates. To illustrate, buying either a one-year instrument or a six-month instrument and when it matures in six months, buy another six-month instrument. Given the one-year spot rate, there is some rate on a six-month instrument six months from now that will make the investor indifferent between the two alternatives. We denote that rate by \( f \) which can be readily determined given the theoretical one-year spot rate and the six-month spot rate. Doubling \( f \) gives the bond-equivalent yield for the six-month rate six months from now in which we are interested.

The market prices its expectations of future interest rates into the rates offered on investments with different maturities. This is why knowing the market’s consensus of future interest rates is critical. The rate that we determined for \( f \) is the market’s consensus for the six-month rate six months from now. A future interest rate calculated from either the spot rates or the yield curve is called a forward rate.

In general, the relationship between a \( t \)-period spot rate, the current six-month spot rate, and the six month forward rates is

\[
z_t = [(1 + z_1)(1 + f_1)(1 + f_2) \cdots (1 + f_{t-1})]^{1/t} - 1 \text{ where } f_i \text{ is the six-month forward rate beginning } t \text{ six-month periods from now.}
\]

The forward rate may never be realized but is important is what it tells investors about his expectation relative to what the market consensus expects. Some market participants prefer not to talk about forward rates as being market consensus rates. Instead, they refer to forward rates as being hedgeable rates. For example, by buying the one-year security, the investor can hedge the six-month rate six months from now.

Determinants of the Shape of the Term Structure

If we plot the term structure—the yield to maturity, or the spot rate, at successive maturities
against maturity—we find three typically shapes: an upward-sloping yield curve; a downward-sloping or inverted yield curve, or a flat yield curve.

Two major theories have evolved to account for these observed shapes of the yield curve: expectations theories and market segmentation theory.

There are several forms of the expectations theory: pure expectations theory, liquidity theory, and preferred habitat theory. Expectations theories share a hypothesis about the behavior of short-term forward rates and also assume that the forward rates in current long-term bonds are closely related to the market’s expectations about future short-term rates. These three theories differ, however, as to whether other factors also affect forward rates, and how. The pure expectations theory postulates that no systematic factors other than expected future short-term rates affect forward rates; the liquidity theory and the preferred habitat theory assert that there are other factors. Accordingly, the last two forms of the expectations theory are sometimes referred to as biased expectations theories.

The liquidity theory states that investors will hold longer-term maturities if they are offered a long-term rate higher than the average of expected future rates by a risk premium that is positively related to the term to maturity. Put differently, the forward rates should reflect both interest-rate expectations and a “liquidity” premium (really a risk premium), and the premium should be higher for longer maturities. The preferred habitat theory also adopts the view that the term structure reflects the expectation of the future path of interest rates as well as a risk premium. However, the preferred habitat theory rejects the assertion that the risk premium must rise uniformly with maturity. The market segmentation theory also recognizes that investors have preferred habitats dictated by the nature of their liabilities. However, the market segmentation theory differs from the preferred habitat theory in that it assumes that neither investors nor borrowers are willing to shift from one maturity sector to another to take advantage of opportunities arising from differences between expectations and forward rates.

There are two risks that cause uncertainty about the return over some investment horizon: price risk and reinvestment risk. The first is the uncertainty about the price of the bond at the end of the investment horizon. The second risk has to do with the uncertainty about the rate at which the proceeds from a bond can be reinvested until the expected maturity date: that is, reinvestment risk.

The Main Influences of the Shape of the Yield Curve

Empirical evidence suggests that the three main influences on the shape of the Treasury yield curve are (1) the market’s expectations of future rate changes, (2) bond risk premiums, and (3) convexity bias. The convexity bias influence is the least well known of the three influences. The longer the maturity, the more convexity the security has. That is, longer term Treasury securities have a more attractive feature due to convexity than shorter term Treasury securities. As a result, investors are willing to pay more for longer term Treasury securities and therefore accept lower returns. This influence on the shape of the Treasury yield curve is what is referred to as the convexity bias.
1. In the September 13, 1996, *Weekly Market Update* published by Goldman, Sachs & Co., the following information was reported in various exhibits for the Treasury market as of the close of business Thursday, September 12, 1996:

<table>
<thead>
<tr>
<th>On-the-Run Treasuries</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity</td>
<td></td>
<td>Yield (%)</td>
</tr>
<tr>
<td>3 months</td>
<td></td>
<td>5.29%</td>
</tr>
<tr>
<td>6 months</td>
<td></td>
<td>5.49%</td>
</tr>
<tr>
<td>1 year</td>
<td></td>
<td>5.90%</td>
</tr>
<tr>
<td>2 years</td>
<td></td>
<td>6.27%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td>6.45%</td>
</tr>
<tr>
<td>5 years</td>
<td></td>
<td>6.63%</td>
</tr>
<tr>
<td>10 years</td>
<td></td>
<td>6.87%</td>
</tr>
<tr>
<td>30 years</td>
<td></td>
<td>7.08%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key Off-the-Run Treasuries</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Issue</td>
<td></td>
<td>Yield (%)</td>
</tr>
<tr>
<td>Old 5-year</td>
<td></td>
<td>6.65%</td>
</tr>
<tr>
<td>Old 10-year</td>
<td></td>
<td>6.89%</td>
</tr>
<tr>
<td>Old 30-year</td>
<td></td>
<td>7.12%</td>
</tr>
</tbody>
</table>

Answer the following questions.

(a) What is the credit risk associated with a Treasury security?

Credit risk or default risk refers to the risk associated with the possibility that the issuer of a bond may be unable to make timely principal and/or interest payments. Market participants throughout the world view a Treasury security as having no credit risk because it is issued by the U.S. Department of the Treasury and backed by the full faith and credit of the U.S. government. Thus, the payments are viewed as virtually default free.

(b) Why is the Treasury yield considered the base interest rate?

The base interest rate (or benchmark interest rate) is the minimum interest rate that investors demand for investing in a non-Treasury security. The base interest rate is typically viewed as the yield to maturity (or simply the yield) offered on a comparable maturity Treasury security that was most recently issued. We can express the interest rate offered on a non-Treasury security as: base interest rate + spread where spread can be viewed as the required risk premium. However, to be precise, the base interest rate for a given maturity is not simply the yield for a recently auctioned Treasury security for that maturity, but the theoretical Treasury spot rate for that maturity. Thus, the base interest rate is the theoretical Treasury spot rates that a risk premium must be added to if we are to more properly value a non-Treasury security.
(c) What is meant by on-the-run Treasuries?

The on-the-run Treasury issues are the most recently auctioned issue of a given maturity. These issues include the 3-month, 6-month, and 1-year Treasury bills; the 2-year, 5-year, and 10-year Treasury notes; and the 30-year Treasury bond. Treasury bills are zero-coupon instruments; the notes and the bond are coupon securities.

There is an observed yield for each on-the-run issue. For the coupon issues, these yields are not the yields used in the analysis when the issue is not trading at par. Instead, for each on-the-run coupon issue, the estimated yield necessary to make the issue trade at par is used. The resulting on-the-run yield curve is called the par coupon curve.

Market participants talk of interest rates on non-Treasury securities as trading at a spread to a particular on-the-run Treasury security.

(d) What is meant by off-the-run Treasuries?

Off-the-run Treasuries refers to Treasuries that are not recently auctioned at designated maturities. If using just the on-the-run issues to form a yield curve, there are large gaps between maturities, particularly after five years. To mitigate this problem, some dealers and vendors use selected off-the-run Treasury issues. Typically, the issues used are the 20-year issue and 25-year issue. Given the par coupon curve including any off-the-run selected issues, the linear extrapolation method is used to fill in the gaps for the other maturities. The bootstrapping method is then used to construct the theoretical spot rate curve. The theoretical spot rate curve comes close to the coupon strips curve when the on-the-run issues are supplemented with the 20-year and 25-year off-the-run issues.

(e) What is the yield spread between (1) the off-the-run 10-year Treasury issue and the on-the-run 10-year Treasury issue, and (2) the off-the-run 30-year Treasury issue and the on-the-run 30-year Treasury issue?

Yield spreads are typically measured in terms of the difference between the yields on two bonds. The difference is measured in basis points. Inserting in our numbers, we have:

\[
\text{yield spread} = \text{off-the-run 10-year Treasury issues} - \text{on-the-run 10-year Treasury issue} = 6.89\% - 6.87\% = 0.02\% = 0.0002 \text{ which is } 2 \text{ basis points.}
\]

\[
\text{yield spread} = \text{off-the-run 30-year Treasury issues} - \text{on-the-run 30-year Treasury issue} = 7.12\% - 7.08\% = 0.04\% = 0.0004 \text{ which is } 4 \text{ basis points.}
\]

(f) What does the yield spread between the off-the-run Treasury issue and the on-the-run Treasury issue reflect?

The spread between on-the-run and off-the-run issues of approximately the same maturity reflect differences in liquidity and financing advantages.
2. In the May 29, 1992, *Weekly Market Update* published by Goldman, Sachs & Co., the following information was reported in various exhibits for certain corporate bonds as of the close of business Thursday, May 28, 1992:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Rating</th>
<th>Yield (%)</th>
<th>Spread (basis points)</th>
<th>Treasury Benchmark (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Electric Capital Company (GEEC)</td>
<td>Triple A</td>
<td>7.87</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>Mobil Corporation</td>
<td>Double A</td>
<td>7.77</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>Southern Bell Telephone &amp; Telegraph (SBTT)</td>
<td>Triple A</td>
<td>8.60</td>
<td>72</td>
<td>30</td>
</tr>
<tr>
<td>Bell Telephone Company of Pennsylvania (BTCP)</td>
<td>Double A</td>
<td>8.66</td>
<td>78</td>
<td>30</td>
</tr>
<tr>
<td>AMR Corporation</td>
<td>Triple B</td>
<td>9.43</td>
<td>155</td>
<td>30</td>
</tr>
</tbody>
</table>

Answer the following questions.

(a) What does rating mean?

A rating classifies the credit worthiness of the security issue. High ratings indicate very low default risk or credit risk in terms of the issuer not being able to make timely principal and/or interest payments. Most market participants rely primarily on commercial rating companies to assess the default risk of an issuer.

(b) Which of the five bonds has the greatest credit risk?

In terms of highest rating, the rating classification follows this ordering in terms of least credit risk to greatest credit risk: triple A, double A, single A, triple B, double B, single B, and below. Thus, since Mobil Co. and Bell Telephone Co. of Pennsylvania (BTCP) have double B ratings (while the other three issuers have higher ratings), these two companies have the lowest rating and thus greatest credit risk. In terms of maturity, the last three issues listed above have a higher maturity premium because each has 30 years to maturity.

However, since AMR’s bond has the highest yield it has the highest risk when all factors are considered since yield reflects both the treasury benchmark plus a risk premium caused by other factors. These factors include the type of issuer, the maturity of the issue, embedded options, the taxability of the interest received, and the liquidity of the issue.

(c) What is meant by spread?

A spread refers to the difference in yields between two securities. The difference is measured in basis points. More detail is given below on various aspects of spread.

Market participants talk of interest rates on non-Treasury securities as trading at a spread to a particular on-the-run Treasury security. The spread, called a risk premium, reflects the additional
risks the investor faces by acquiring a security that is not issued by the U.S. government. For the tax-exempt bond market, the benchmark for calculating spreads is not Treasuries. Rather, it is a generic triple A general obligation bond with a specified maturity.

In addition to measuring the spread by computing differences in yields, yield spreads can also be measured on a relative basis by taking the ratio of the yield spread to the yield level. Sometimes bonds are compared in terms of a yield ratio, the quotient of two bond yields.

The factors that affect the yield spread include (i) the type of issuer, (ii) the issuer’s perceived credit worthiness, (iii) the term or maturity of the instrument, (iv) provisions that grant either the issuer or the investor the option to do something, (v) the taxability of the interest received by investors, and (vi) the expected liquidity of the security. To illustrate one of these factors, consider the taxability factor. Municipalities are not permitted to tax the interest income from securities issued by the U.S. Treasury. Thus part of the spread between Treasury securities and taxable non-Treasury securities of the same maturity reflects the value of the exemption from state and local taxes.

The spread between the interest rate offered in two sectors of the bond market with the same maturity is referred to as an intermarket sector spread. The most common intermarket sector spread calculated is the spread between Treasury securities and some sector of the non-Treasury market with the same maturity. The spread between two issues within a market sector is called an intramarket sector spread. The spread between any two maturity sectors of the market is called a maturity spread. The spread between Treasury securities and non-Treasury securities that are identical in all respects except for quality is referred to as a quality spread or credit spread.

The presence of an embedded option has an effect on the spread of an issue relative to a Treasury security and the spread relative to otherwise comparable issues that do not have an embedded option. In general, market participants will require a larger spread to a comparable Treasury security for an issue with an embedded option that is favorable to the issuer (e.g., a call option) than for an issue without such an option. In contrast, market participants will require a smaller spread to a comparable Treasury security for an issue with an embedded option that is favorable to the investor (e.g., put option or conversion option). In fact, for a bond with an option that is favorable to an investor, the interest rate on an issue may be less than that on a comparable Treasury security.

(d) What is meant by Treasury benchmark?

The benchmark interest rate (or base interest rate) is the minimum interest rate that investors demand for investing in a non-Treasury security. The base interest rate is typically viewed as the yield to maturity (or simply the yield) offered on a comparable maturity Treasury security that was most recently issued.

(e) Why do each of the spreads reported above reflect a risk premium?

Each spread reflects a risk premium because each spread results from additional risks (most noteworthy a default risk) the investor faces by acquiring a security that is not issued by the U.S.
3. Below we repeat the corporate bond issues reported in Question 2.

<table>
<thead>
<tr>
<th>Issuer</th>
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<td>9.43</td>
<td>155</td>
<td>30</td>
</tr>
</tbody>
</table>

Given this information, answer the below questions.

(a) What is the yield spread between the Southern Bell Telephone and Telegraph (SBTT) bond issue and the Bell Telephone Company of Pennsylvania (BTCP) bond issue?

Yield spreads are typically measured in terms of the difference between the yields on two bonds. The difference is measured in basis points. Inserting in our numbers, we have:

\[
\text{yield spread} = \text{yield BTCP} - \text{yield SBTT} = 8.66\% - 8.60\% = 0.06\% \text{ which is 6 basis points.}
\]

(b) The Southern Bell Telephone and Telegraph (SBTT) bond issue is not callable, but the Bell Telephone Company of Pennsylvania (BTCP) bond issue is callable. What does the yield spread in part (a) reflect?

BTCP has a lower bond rating (double B) compared to SBTT (triple B). This can explain the 6-basis-point greater yield for BTCP. Also, the presence of an embedded option has an effect on the spread of an issue relative to otherwise comparable issues that do not have an embedded option. In general, market participants will require a larger spread to a comparable security for an issue with an embedded option that is favorable to the issuer (e.g., a call option) than for an issue without such an option. Thus, the yield spread in part (a) of 6 basis points reflects partly the fact that BTCP is callable causing investors to require a higher rate of return.

(c) AMR Corp. is the parent company of American Airlines and is therefore classified in the transportation industry. The issue is not callable. What is the yield spread between AMR Corp. and Southern Bell Telephone and Telegraph (SBTT) bond issue, and what does this spread reflect?

Yield spread = yield AMR Corp – yield SBTT = 9.43% – 8.60% = 0.83% which is 83 basis points.

This yield spread of 83 basis points represents the higher credit risk for AMR which has a triple B rated bond compared to the higher rated triple A bond for SBTT. Since both are not callable, the
The spread does not embody an extra premium required by investors for accepting a callable bond. The spread does not reflect any difference in a maturity premium since both are 30-year bonds.

(d) What is the spread between the General Electric Capital Co. (GECC) issue and the Mobil Corp. issue?

Yield spread = yield GECC – yield Mobil Corp. = 7.87% – 7.77% = 0.1% which is ten basis points.

(e) The Mobil Corp. issue is not callable. However, the General Electric Capital Co. issue is callable. How does this information help you in understanding the spread between these two issues?

GEEC has a triple A bond rating compared to Mobil’s rating of double A. Thus, everything else equal, one would think that GEEC would have a lower yield. However, everything else is not equal because GEEC’s bond is callable causing investors to demand a higher return on GEEC’s bond compared to similarly rated bonds. In this case, the callable feature appears to dominate the fact GEEC has a higher bond rating which indicates less credit risk. Thus, knowing GEEC’s bonds are callable helps us understand why its yield is higher. Since both have the same maturity, this factor cannot account for the spread of ten basis points.

4. The yield spread between two corporate bond issues reflects more than just differences in their credit risk. What other factors would the spread reflect?

In addition to the perceived riskiness of the issuer, other factors that affect the spread include the type of issuer, the term or maturity of the issue, embedded options, taxability of interest received by investors and expected liquidity of the issue.

5. In the May 29, 1992, Weekly Market Update published by Goldman, Sachs & Co., the following information was reported in an exhibit for high-grade, tax-exempt securities as of the close of business Thursday, May 28, 1992:

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Yield (%)</th>
<th>Yield (%) as a Percentage of Treasury Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.20</td>
<td>76.5</td>
</tr>
<tr>
<td>3</td>
<td>4.65</td>
<td>80.4</td>
</tr>
<tr>
<td>5</td>
<td>5.10</td>
<td>76.4</td>
</tr>
<tr>
<td>10</td>
<td>5.80</td>
<td>78.7</td>
</tr>
<tr>
<td>30</td>
<td>6.50</td>
<td>82.5</td>
</tr>
</tbody>
</table>

Answer the following questions.

(a) What is meant by a tax-exempt security?

A tax-exempt security is a security in which the investor is exempt from paying certain taxes. For example, purchasers of municipal bond issues are exempt from paying federal taxes. The
tax-exempt aspect can affect the spread. For example, part of the spread between Treasury securities and taxable non-Treasury securities of the same maturity reflects the value of the exemption from state and local taxes.

(b) What is meant by high-grade issue?

By high-grade issue, we mean a security issue that has low credit risk. Higher bond ratings such as triple A, double A, and single A are associated with high quality and would be considered having low credit risk relative to lower ratings such as triple B, double B, single B, and below.

(c) Why is the yield on a tax-exempt security less than the yield on a Treasury security of the same maturity?

The yield on a tax-exempt security is less because investors are exempt from paying certain taxes (e.g., federal, state, or local). Thus, they can require a lower rate of return.

(d) What is the equivalent taxable yield?

The equivalent taxable yield is the yield that must be offered on a taxable bond issue to give the same after-tax yield as a tax-exempt issue. In equation form, we have:

\[
\text{equivalent taxable yield} = \frac{\text{tax - exempt}}{1 - \text{marginal tax rate}}.
\]

(e) Also reported in the same issue of the Goldman, Sachs report is information on intramarket yield spreads. What are these?

An intramarket yield spread is the spread between two issues within a market sector. Examples of market sectors include the U.S. government, U.S. government agencies, municipal governments, credit (domestic and foreign corporations), and foreign governments.

6. Answer the following questions.

(a) What is an embedded option in a bond?

An option gives the buyer a right (but not the obligation) to buy or sell an asset at a price often called the exercise or strike price. The right to buy is called a call option while the right to sell is called a put option.

An embedded option is an option found in a bond that includes a provision giving either the bondholder and/or the issuer an option to take some action against the other party.

(b) Give three examples of an embedded option that might be included in a bond issue.

Below are three examples of an embedded option.
Example one is a callable bond. A bond with a call provision gives the issuer the right to call the issue by redeeming it as a designated price.

Example two is a bond convertible into common stock of the issuing company. A bond with a convertibility feature gives the lender the right to convert the bond into stock.

Example three is a sinking fund bond. A sinking fund provision gives the issuing company the power to periodically retire part of the bond issue. It is like a call option except not all of the bonds are callable at once. The redemption schedule associated with a sinking fund provision involves a sequence of principal repayments prior to the maturity date.

(c) Does an embedded option increase or decrease the risk premium relative to the base interest rate?

An embedded option that works in favor of the issuer increases the risk premium relative to the base interest rate. An embedded option that works in favor of the borrower decreases the risk premium. Thus, an embedded option can either increase or decrease the risk premium relative to the base interest rate depending on who it favors.

7. Answer the following questions.

(a) What is a yield curve?

The yield curve is the graphical depiction of the relationship between the yield on bonds of the same credit quality but different maturities. The yield curve is usually constructed from observations of prices and yields in the Treasury market.

(b) Why is the Treasury yield curve the one that is most closely watched by market participants?

The Treasury yield curve is the yield curve most closely watched by market participants for several reasons. First, Treasury securities are free from default risk and so differences in credit worthiness do not affect yields making these instruments directly comparable. Second, the Treasury market is the largest and most active bond market offering the fewest problems in terms of illiquidity and infrequent trading.

8. What is a spot rate?

The spot rate for a certain maturity is the yield on a zero-coupon Treasury of the same maturity. The graphical depiction of the relationship between the spot rate and maturity is called the spot rate curve.

9. Explain why it is inappropriate to use one yield to discount all the cash flows of a financial asset.

Because cash flows from a financial asset can occur at different points in time, it is not correct to
assume they will all have the same yield and thus should be discounted at the same interest rate. Each cash flow should be discounted at a unique interest rate appropriate for the period in which the cash flow will be received. Thus, the correct way to think about an asset is as a package of cash flows with each cash flow discounted by a rate appropriate for the period received.

10. **Explain why a financial asset can be viewed as a package of zero-coupon instruments.**

A financial asset generates cash flows over time. The value of the asset is the present value of all the cash flows. Since each cash flow can occur at a different point in time, each cash flow should be valued in today’s dollar using a discount rate that reflects the required rate of return associated with that time period. Thus, each cash flow is like a zero-coupon bond where today’s value for the zero-coupon bond is the discounted value of the zero-coupon’s promised maturity value (e.g., cash flow at a point in time). The discount rate for the zero-coupon bond is its spot rate which can differ from one time period to the next. So too, the cash flows generated by the financial asset should have discount rates that differ from one time period to the next.

11. **How are spot rates related to forward rates?**

Forward rates and spot rates are related because forward rates can be derived from spot rates. Forward rates are expected future spot rates that may differ from the actual spot rates that occur in the future. Spot rates can be used to compute the forward rate (or expected future spot rate) for any time in the future for any investment horizon.

12. You are a financial consultant. At various times you have heard comments on interest rates from one of your clients. How would you respond to each comment?

(a) “The yield curve is upward-sloping today. This suggests that the market consensus is that interest rates are expected to increase in the future.”

This is not necessarily true because investors demand a greater return as the maturity increases. The maturity premium results from the fact that more uncertainty exists for longer term maturity. Other factors causing the yield curve to be upward-sloping include liquidity considerations and supply and demand concerns. For example, if investors wanted fewer longer term bonds than were currently being supplied, then this would drive up the yield on longer term bonds.

(b) Respond to: “I can’t make any sense out of today’s term structure. For short-term yields (up to three years) the spot rates increase with maturity; for maturities greater than three years but less than eight years, the spot rates decline with maturity; and for maturities greater than eight years the spot rates are virtually the same for each maturity. There is simply no theory that explains a term structure with this shape.”

There are various theories that can account for any slope that the yield curve might take. First, there is the pure expectations theory where the forward rates exclusively represent the expected future rates. Since these rates can either increase or decrease for any time period, the yield curve can be sloped upward or downward for that time period.
Second, there is the liquidity preference theory which asserts that investors do not like uncertainty and so must be offered a higher rate of return for longer term maturities. Thus, the forward rate will not only reflect expectations about future interest rates but also a “liquidity” premium that will be higher for longer term securities. Ceteris paribus, an increasing liquidity premium implies that the yield curve will be upward sloping.

The preferred habitat theory also adopts the view that the term structure reflects the expectation of the future path of interest rates as well as a risk premium. However, the preferred habitat theory rejects the assertion that the risk premium must rise uniformly with maturity. The preferred habitat theory asserts that to the extent that the demand and supply of funds in a given maturity range do not match, some lenders and borrowers will be induced to shift to maturities showing the opposite imbalances. However, they will need to be compensated by an appropriate risk premium whose magnitude will reflect the extent of aversion to either price or reinvestment risk. Thus this theory proposes that the shape of the yield curve is determined by both expectations of future interest rates and a risk premium, positive or negative, to induce market participants to shift out of their preferred habitat. Thus, according to this theory, yield curves sloping up, down, flat, or humped are all possible.

The market segmentation theory also recognizes that investors have preferred habitats dictated by the nature of their liabilities. This theory also proposes that the major reason for the shape of the yield curve lies in asset-liability management constraints (either regulatory or self-imposed) and/or creditors (borrowers) restricting their lending (financing) to specific maturity sectors. However, the market segmentation theory differs from the preferred habitat theory in that it assumes that neither investors nor borrowers are willing to shift from one maturity sector to another to take advantage of opportunities arising from differences between expectations and forward rates. Thus for the segmentation theory, the shape of the yield curve is determined by supply of and demand for securities within each maturity sector.

(c) Respond to: “When I want to determine the market’s consensus of future interest rates, I calculate the forward rates.”

A future (expected) interest rate that can be computed from either the spot rates or the yield curve is called a forward rate.

From the yield curve we can extrapolate the theoretical spot rates. In addition, we can extrapolate what some market participants refer to as the market's consensus of future interest rates. The market prices its expectations of future interest rates into the rates offered on investments with different maturities.

A natural question about forward rates is how well they do at predicting future interest rates. Studies have demonstrated that forward rates do not do a good job in predicting future interest rates. However, from an investor’s point of view, forward rates indicate how their expectations must differ from the market's consensus in order to make the correct decision. For this reason, some market participants prefer not to talk about forward rates as being market consensus rates. Instead, they refer to forward rates as being hedgeable rates.
13. You observe the yields of the following Treasury securities (all yields are shown on a bond-equivalent basis):

<table>
<thead>
<tr>
<th>Year (Period)</th>
<th>Yield to Maturity (%)</th>
<th>Spot Rate (%)</th>
<th>Year (Period)</th>
<th>Yield to Maturity (%)</th>
<th>Spot Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 (1)</td>
<td>5.25</td>
<td>5.25</td>
<td>5.5 (11)</td>
<td>7.75</td>
<td>7.97</td>
</tr>
<tr>
<td>1.0 (2)</td>
<td>5.50</td>
<td>5.50</td>
<td>6.0 (12)</td>
<td>8.00</td>
<td>8.27</td>
</tr>
<tr>
<td>1.5 (3)</td>
<td>5.75</td>
<td>5.76</td>
<td>6.5 (13)</td>
<td>8.25</td>
<td>8.59</td>
</tr>
<tr>
<td>2.0 (4)</td>
<td>6.00</td>
<td>?</td>
<td>7.0 (14)</td>
<td>8.50</td>
<td>8.92</td>
</tr>
<tr>
<td>2.5 (5)</td>
<td>6.25</td>
<td>?</td>
<td>7.5 (15)</td>
<td>8.75</td>
<td>9.25</td>
</tr>
<tr>
<td>3.0 (6)</td>
<td>6.50</td>
<td>?</td>
<td>8.0 (16)</td>
<td>9.00</td>
<td>9.61</td>
</tr>
<tr>
<td>3.5 (7)</td>
<td>6.75</td>
<td>?</td>
<td>8.5 (17)</td>
<td>9.25</td>
<td>9.97</td>
</tr>
<tr>
<td>4.0 (8)</td>
<td>7.00</td>
<td>?</td>
<td>9.0 (18)</td>
<td>9.50</td>
<td>10.36</td>
</tr>
<tr>
<td>4.5 (9)</td>
<td>7.25</td>
<td>?</td>
<td>9.5 (19)</td>
<td>9.75</td>
<td>10.77</td>
</tr>
<tr>
<td>5.0 (10)</td>
<td>7.50</td>
<td>?</td>
<td>10.00 (20)</td>
<td>10.00</td>
<td>11.20</td>
</tr>
</tbody>
</table>

All the securities maturing from 1.5 years on are selling at par. The 0.5 and 1.0-year securities are zero-coupon instruments. Answer the following questions.

(a) Calculate the missing spot rates.

First, we compute the spot rate for year 2 by following the seven-step procedure given below.

**Step One.** Take the semiannual yield to maturity (coupon rate) for year two times $100 to get the cash flow for periods one through three. We have: (0.0600 / 2)$100 = $3.00 for t = 1, 2, and 3.

**Step Two.** Get the appropriate semiannual discount rate that will be used to get the present value of each cash flow in Step One. These rates are the spot rates given spot rate column divided by two. We get: 0.0525 / 2 = 0.02625 for t = 1, 0.0550 / 2 = 0.02750 for t = 2, and 0.05759 / 2 = 0.028798 for t = 3.

**Step Three.** Compute the sum of the present values of all of the cash flows in Step One using the appropriate discount rates from Step Two. We get cash flows of $2.92326 + $2.84156 + $2.75506 = $8.51989.

**Step Four.** Subtract the sum of the present values of the cash flows in Step Three from $100. We get: $100.00 – $8.51989 = $91.48011.

**Step Five.** Compute the cash flow in period 4 which is the $3.00 coupon payment plus the par value of $100 and divide this by the cash flow computed in Step Four. We get $103.00 / $91.48011 = 1.125928.

**Step Six.** Take the value in Step Five to the one-fourth power and subtract one. The power is determined by dividing one by the period for which we are computing the spot rate which is period four. We get: 0.030096.
**Step Seven.** Convert the value in Step Six to the bond-equivalent rate by multiplying by two and then converting to percentage form (by multiplying by 100). We get: 0.030096(2)(100) = 6.01917% or about 6.02%, which is the theoretical two-year spot rate.

We now compute the spot rate for year 2.5 in a like manner noting that we are now working with one more period and using the spot rate just computed when discounting cash flows for period four.

In Step One, we compute the cash flows for periods one through four using the semiannual coupon rate of 6.25% / 2 = 3.125% or 0.03125 for year 2.5. We get $3.125 for each cash flow. In Step Two, we gather all appropriate discount rates for each period (also incorporating the spot rate just computed for year two that will be used when discounting the cash flow for period four). For Step Three, we get: $11.6504 for the sum of the present values of all cash flows. In Step Four, we get $88.3496. In Step Five, we get 1.1672. In Step Six, we get 0.031411. In Step Seven, we get 6.2822% or about 6.28% which is the 2.5-year spot rate.

We now compute the spot rate for period 3.0.

In Step One, we get $3.25 for each cash flow. The values for Step Three through Step Seven are: $14.9008, $85.0992, 1.2133, 0.03275, and 6.5495% or about 6.55% which is the 3.0-year spot rate.

Similarly, we can compute the theoretical spot rates for the remaining four periods which are years 3.5, 4.0, 4.5, and 5.0. Rounded off, we get the respective theoretical spot rates of 6.8213% for year 3.5, 7.0985% for year 4.0, 7.3815% for year 4.5, and 7.6712% for year 5.0.

Below we fill in the missing spot rates in underlined, bold-face print (rounded-off to the nearest 0.01%). We have:

<table>
<thead>
<tr>
<th>Year (Period)</th>
<th>Yield to Maturity (%)</th>
<th>Spot Rate (%)</th>
<th>Year (Period)</th>
<th>Yield to Maturity (%)</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 (1)</td>
<td>5.25</td>
<td>5.25</td>
<td>5.5 (11)</td>
<td>7.75</td>
<td>7.97</td>
</tr>
<tr>
<td>1.0 (2)</td>
<td>5.50</td>
<td>5.50</td>
<td>6.0 (12)</td>
<td>8.00</td>
<td>8.27</td>
</tr>
<tr>
<td>1.5 (3)</td>
<td>5.75</td>
<td>5.76</td>
<td>6.5 (13)</td>
<td>8.25</td>
<td>8.59</td>
</tr>
<tr>
<td>2.0 (4)</td>
<td>6.00</td>
<td><strong>6.02</strong></td>
<td>7.0 (14)</td>
<td>8.50</td>
<td>8.92</td>
</tr>
<tr>
<td>2.5 (5)</td>
<td>6.25</td>
<td><strong>6.28</strong></td>
<td>7.5 (15)</td>
<td>8.75</td>
<td>9.25</td>
</tr>
<tr>
<td>3.0 (6)</td>
<td>6.50</td>
<td><strong>6.55</strong></td>
<td>8.0 (16)</td>
<td>9.00</td>
<td>9.61</td>
</tr>
<tr>
<td>3.5 (7)</td>
<td>6.75</td>
<td><strong>6.82</strong></td>
<td>8.5 (17)</td>
<td>9.25</td>
<td>9.97</td>
</tr>
<tr>
<td>4.0 (8)</td>
<td>7.00</td>
<td><strong>7.10</strong></td>
<td>9.0 (18)</td>
<td>9.50</td>
<td>10.36</td>
</tr>
<tr>
<td>4.5 (9)</td>
<td>7.25</td>
<td><strong>7.38</strong></td>
<td>9.5 (19)</td>
<td>9.75</td>
<td>10.77</td>
</tr>
<tr>
<td>5.0 (10)</td>
<td>7.50</td>
<td><strong>7.67</strong></td>
<td>10.00 (20)</td>
<td>10.00</td>
<td>11.20</td>
</tr>
</tbody>
</table>

**(b) What should the price of a 6% six-year Treasury security be?**

Following the process in part (a), we get respective spot rates for years 5.5 and 6.0 of 7.9684% and 8.2740%. These are more accurate than those given and can be used when computing the price of
a 6% six-year Treasury security because we have to discount the cash flows for periods eleven and twelve by the appropriate discount rate (which are the theoretical spot rates for those two periods). With the twelve spot rates known, we can proceed to compute the price of the 6% six-year Treasury security. The price of this security is the present value of its cash flows. Per $100 par value, the semiannual coupon payment is $100(0.06 / 2) = $3.00. There will be twelve payments of $3.00 plus the payment of the par value of $100 received at the end of period twelve. As noted above, the appropriate discount rate for each of the twelve cash flows are the twelve spot rates that correspond to each of the twelve periods when cash flows are received. The present value of the twelve respective cash flows is: $2.9233, $2.8412, $2.7550, $2.6645, $2.5702, $2.4726, $2.3723, $2.2696, $2.1650, $2.0590, $1.9520, and $1.844. These total $28.8894. The present value of the $100 par value discounted at the theoretical spot rate of 4.1370% is $61.4808. Thus, the price of a 6% six-year Treasury security should be $90.3702 or about $90.37.

[NOTE. The price of a zero-coupon Treasury security is the present value of its maturity value discounted using the theoretical semiannual spot rate for year six (period twelve). As seen above, the semiannual spot rate for a six-year Treasury security is 8.2740% / 2 = 4.1370%. Thus, per $100 of par value, we get: $100 / (1.041370)^{12} = $100(0.614808) = $61.48.]

(c) What is the six-month forward rate starting in the sixth year?

The six-month forward rate for period twelve can be computed by knowing the spot rates for periods twelve and thirteen. The 6-year spot rate at the beginning of year six (or period twelve) is 8.27%. The semiannual rate is 4.135%. The 6.5-year (or period-13) spot rate is 8.59% with a semiannual rate of 4.295%. The six-month forward rate at the start of the sixth year or twelfth period (f_{12}) is the rate that will satisfy the following equation:

\[
f_{12} = \frac{(1 + y)^{13}}{(1 + y)^{12}} - 1.
\]

Inserting in our values, we get:

\[
f_{12} = \frac{(1.04295)^{13}}{(1.04135)^{12}} - 1 = \frac{1.727529}{1.626152} - 1 = 1.0623428 - 1 = 0.0623428 = 6.2343\%.
\]

0.0623428 or about 6.2343%. The bond equivalent yield is 2(0.0623428) = 0.1246856 or about 12.4686%. [NOTE. We can check this out by considering two equal investment plans for a $100. The first plan is to invest $10,000 at the semiannual 6-year spot rate for 12 semiannual periods and then reinvest this amount at the six-month forward rate starting in year six. We get:

$10,000(1.04135)^{12}(1.062343%) = $17,275.29. The second plan is to invest $10,000 for 6.5 years at the 6.5-year semiannual spot rate, which is $10,000(1.04295)^{13} = $10,000(1.727529) = $17,275.29.]

14. You observe the following Treasury yields (all yields are shown on a bond equivalent basis):

<table>
<thead>
<tr>
<th>Year (Period)</th>
<th>Yield to Maturity (%)</th>
<th>Spot Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 (1)</td>
<td>10.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>

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All the securities maturing from 1.5 years on are selling at par. The 0.5 and 1.0-year securities are zero-coupon instruments. Answer the following questions.

(a) Calculate the missing spot rates.

We use the seven-step procedure described in Problem 13, part (a) to compute the theoretical 9.5-year spot rate.

Step One. Take the semiannual yield to maturity (coupon rate) for year 9.5 times $100 to get the cash flow for periods one through eighteen. We have: \( \frac{0.0550}{2} \times 100 = 2.75 \) for \( t = 1 \) through 18.

Step Two. Get the appropriate semiannual discount rate that will be used to get the present value of each cash flow in Step One. These rates are the annual spot rates given above divided by two.

Step Three. Compute the sum of the present values of all of the cash flows in Step One using the appropriate discount rates from Step Two. We get $36.1660.

Step Four. Subtract the sum of the present values of the cash flows in Step Three from $100. We get: $100.00 – $36.1660 = $63.8340.

Step Five. Compute the cash flow in period 19 which is the $2.75 coupon payment plus the par value of $100 and divide this by the cash flow computed in Step Four. We get $102.75 / $63.8340 = 1.6096.

<table>
<thead>
<tr>
<th>Year</th>
<th>Spot Rate</th>
<th>Par Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>9.75</td>
<td>9.75</td>
</tr>
<tr>
<td>1.5</td>
<td>9.50</td>
<td>9.48</td>
</tr>
<tr>
<td>2.0</td>
<td>9.25</td>
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<td>2.5</td>
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<td>8.68</td>
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**Step Six.** Take the value in Step Five to the one-nineteenth power and subtract one. The one-nineteenth power is determined by dividing one by the period for which we are computing the spot rate which is period nineteen. We get: 0.02537.

**Step Seven.** Convert the value in Step Six to the bond-equivalent rate by multiplying by two and then converting to percentage form (by multiplying by 100). We get: 0.02537(2)(100) = 5.0740% or about 5.07%, which is the theoretical 6.5-year spot rate.

Using the seven-step procedure, we get the following values when computing the theoretical 10-year spot rate. For Step One, we get the coupon semiannual coupon payment of $2.625 for each period. The values for Steps Three through Seven are: $36.1529, $63.8471, 1.6074, 0.0240, and 4.8027%. Thus, the theoretical 10-year spot rate is about 4.80%.

**(b) What should the price of a 5% four-year Treasury security be?**

The price of a 5% four-year Treasury security is the present value of its cash flows. Per $100 par value, each cash flow is the semiannual coupon payment of $100(0.05 / 2) = $2.50. There will be eight payments of $2.50 plus the payment of the par value of $100 received at the end of period eight. The appropriate discount rates for each of the eight cash flows are the eight spot rates that correspond with each of the eight periods when cash flows are received. The present values of the eight respective cash flows are: $2.3810, $2.2730, $2.1757, $2.0876, $2.0085, $1.9375, $1.9738, and $1.8169. The sum of these cash flows is $16.5540. The present value of the $100 par value discounted at the theoretical spot rate of 4.07% is $72.6768. Thus, the price of a 5% four-year Treasury security should be $16.5540 + $72.6768 = $89.2308 or about $89.23.

**15. What Treasury issues can be used to construct the theoretical spot rate curve?**

A default-free theoretical spot rate curve can be constructed from the yield on Treasury securities. The Treasury issues that are candidates for inclusion are (i) on-the-run Treasury issues, (ii) on-the-run Treasury issues and selected off-the-run Treasury issues, (iii) all Treasury coupon securities, and bills, and (iv) Treasury coupon strips.

After the securities that are to be included in the construction of the theoretical spot rate curve are selected, the methodology for constructing the curve must be determined. If Treasury coupon strips are used, the procedure is simple, because the observed yields are the spot rates. If the on-the-run Treasury issues with or without selected off-the-run Treasury issues are used, a methodology called bootstrapping is used.

**16. What are the problems with using only on-the-run Treasury issues to construct the theoretical spot rate curve?**

There are two problems with using just the on-the-run issues. First, there is a large gap between some of the maturities points, which may result in misleading yields for those maturity points when estimated using the linear interpolation method. Second, the yields for the on-the-run issues may be misleading because most offer the favorable financing opportunity in the repo market.
17. When all Treasury issues are used to construct the theoretical spot rate curve, what methodology is used to construct the curve?

The extrapolation method can be used to form the on-the-run or par coupon curve from on-the-run observed Treasury yields (for each on-the-run coupon issue, the estimated yield necessary to make the issue trade at par is used). This method extrapolates the missing yields from the surrounding maturity points on the par yield curve. The simplest extrapolation method, and the one most commonly used, is linear extrapolation. Specifically, given the yield on the par coupon curve at two maturity points, the following is calculated:

\[
\frac{\text{yield at higher maturity} - \text{yield at lower maturity}}{\text{number of semiannual period between the two maturity points}}.
\]

Then, the yield for all intermediate semiannual maturity points is found by adding to the yield at the lower maturity the amount computed here.

We run into problems if we use only on-the-run Treasury issues to construct the theoretical spot rate curve. For example, there is a large gap between some of the maturities points, and the yields for the on-the-run issues may be misleading because most offer the favorable financing opportunity in the repo market.

To overcome these problems, we convert the par yield curve into the theoretical spot rate curve using bootstrapping. To explain the process of estimating the theoretical spot rate curve from observed yields on Treasury securities, consider a six-month Treasury bill where its annualized yield is the six-month spot rate and a one-year Treasury where its annualized yield is the one year spot rate. Given these two spot rates, the price of a theoretical 1.5-year zero-coupon Treasury should equal the present value of three cash flows from an actual 1.5-year coupon Treasury, where the yield used for discounting is the spot rate corresponding to the cash flow. Doubling the computed yield, we obtain the bond-equivalent yield, which is the theoretical 1.5-year spot rate. That rate is the rate that the market would apply to a 1.5-year zero-coupon Treasury security if, in fact, such a security existed. Given the theoretical 1.5-year spot rate, we can obtain the theoretical 2-year spot rate and so forth until we derive theoretical spot rates for the remaining 15 half-yearly rates. The spot rates using this process represent the term structure of interest rates.

18. Answer the following questions.

(a) What are the limitations of using Treasury strips to construct the theoretical spot rate curve?

There are three problems with using the observed rates on strips. First, the liquidity of the strips market is not as great as that of the Treasury coupon market. Thus, the observed rates on strips reflect a premium for liquidity. Second, the tax treatment of strips is different from that of Treasury coupon securities. Specifically, the accrued interest on strips is taxed even though no cash is received by the investor. Finally, there are maturity sectors in which non-U.S. investors find it advantageous to trade off yield for tax advantages associated with a strip.
(b) When Treasury strips are used to construct the curve, why are only coupon strips used?

By using only coupon strips, the biases would be consistent in terms of liquidity, tax treatment, and international demand factors. Also, simply using observed Treasury coupon strips is easier than going through tedious computation procedures to get spot rates. This simple procedure avoids using methodologies such as bootstrapping, which use on-the-run Treasury issues with or without selected off-the-run Treasury issues to construct a theoretical spot rate. There can be significant divergence between the coupon strips and the rates generated from bootstrapping after the six-year maturity point. When the on-the-run issues are supplemented with the 20-year and 25-year off-the-run issues, the theoretical spot rate curve comes closer to the coupon strips curve. We can come even closer to the coupon strips curve by using an exponential spline methodology. For this approach, the spot rates are closer to the coupon strips, particularly after the six-year maturity point.

19. What actions force a Treasury’s bond price to be valued in the market at the present value of the cash flows discounted at the Treasury spot rates?

The price of a Treasury security should be equal to the present value of its cash flow where each cash flow is discounted at the theoretical spot rates. If this does not occur then an arbitrage situation develops where a large profit can be made with no risk involved. Thus, arbitrage forces a Treasury to be priced based on spot rates and not the yield curve. The ability of dealers to purchase securities and create value by stripping forces Treasury securities to be priced based on the theoretical spot rates.

20. Explain the role that forward rates play in making investment decisions.

Although a forward rate may never be realized in practice, it is still important for investors because it tells them about their expectations relative to what the market consensus expects. This allows them to make decisions based upon the market expects. For example, forward rates indicate how an investor’s expectations must differ from the market’s consensus in order to make the correct decision. Some investors may not speak about forward rates as being market consensus rates. Instead, they refer to forward rates as being hedgeable rates. For example, by buying the one-year security, the investor can hedge the six-month rate six months from now.

21. “Forward rates are poor predictors of the actual future rates that are realized. Consequently, they are of little value to an investor.” Explain why you agree or disagree with this statement.

To see the importance of knowing the market’s consensus for future interest rates, consider the following two investment alternatives for an investor who has a one-year investment horizon. Alternative 1 is to buy a one-year instrument. Alternative 2 is to buy a six-month instrument and then reinvest its value by purchasing another six-month instrument. With alternative 1, the investor will realize the one-year spot rate and that rate is known with certainty. In contrast, with alternative 2, the investor will realize the six-month spot rate, but the six-month rate six months from now is unknown. Therefore, for alternative 2, the rate that will be earned over one year is not known with certainty. The forward rate is the rate that will make the two alternatives equal. This rate is referred
to as the market’s expected future six-month spot rate six months from now. By knowing this rate, an investor can make a decision as to what (s)he thinks about future spot rates. For example, suppose the investor expects that six months from now, the six-month rate will be lower than the forward rate. If so, then the investor will choose alternative 1. Thus, knowing the forward rate does have practical value for an investor even if the forward rate is hindsight is not accurate.

22. Bart Simpson is considering two alternative investments. The first alternative is to invest in an instrument that matures in two years. The second alternative is to invest in an instrument that matures in one year and at the end of one year, reinvest the proceeds in a one-year instrument. He believes that one-year interest rates one year from now will be higher than they are today and therefore is leaning in favor of the second alternative. What would you recommend to Bart Simpson?

Bart has two choices. Choice 1 is to buy a two-year instrument. Choice 2 is to buy a one-year instrument and then reinvest its value by purchasing another one-year instrument. With choice 1, Bart will realize the two-year spot rate and that rate is known with certainty. In contrast, with choice 2, Bart will realize the one-year spot rate, but the one-year rate one year from now is unknown. Therefore, for choice 2, the rate that will be earned over two years is not known with certainty. The one-year forward rate is the rate that will make the two choices equal. This rate is referred to as the market’s expected future one-year spot rate one year from now. By knowing this rate, Bart can make a decision as to what he thinks about future spot rates. For example, suppose Bart expects that one year from now, the one year spot rate will be higher than the forward rate. If so, then Bart will choose choice 2. However, this rate must be higher than the forward rate and not just higher than the rate Bart may have in mind. Thus, Bart should be careful. It is just not enough to believe rates will be higher. There is a certain threshold (the forward rate) that must be met before anyone considers a rollover strategy connected with alternative 2.

23. Answer the following questions.

(a) What is the common hypothesis about the behavior of short-term forward rates shared by the various forms of the expectations theory?

There are several forms of the expectations theory: pure expectations theory, liquidity theory, and preferred habitat theory. Expectations theories share a hypothesis about the behavior of short-term forward rates are related to expected future short-term rates. These theories also assume that the forward rates in current long-term bonds are closely related to the market’s expectations about future short-term rates. These three theories differ, however, as to whether other factors also affect forward rates, and how. The pure expectations theory postulates that no systematic factors other than expected future short-term rates affect forward rates; the liquidity theory and the preferred habitat theory assert that there are other factors. Accordingly, the last two forms of the expectations theory are sometimes referred to as biased expectations theories.

(b) What is price risk and reinvestment risk and how do these two risks affect the pure expectations theory?

There are two risks that cause uncertainty about the return over some investment horizon: price
risk and reinvestment risk. Let’s begin by describe price risk.

Price risk is the uncertainty about the price of the bond at the end of the investment horizon. For example, an investor who plans to invest for four years might consider the following three investment alternatives: (i) invest in a four-year bond and hold it for four years, (ii) invest in an 8-year bond and sell it at the end of four years, and (iii) invest in a 16-year bond and sell it at the end of four years. The return that will be realized for the second and third alternatives is not known because the price of each long-term bond at the end of four years is not known. In the case of the 8-year bond, the price will depend on the yield on four-year debt securities four years from now; and the price of the 16-year bond will depend on the yield on 12-year bonds four years from now. Because forward rates implied in the current term structure for a future 8-year bond and a future 16-year bond are not perfect predictors of the actual future rates, there is uncertainty about the price for both bonds five years from now. Thus there is price risk, that is, the risk that the price of the bond will be lower than currently expected at the end of the investment horizon. An important feature of price risk is that it is greater the longer the maturity of the bond.

The second risk has to do with the uncertainty about the rate at which the proceeds from a bond can be reinvested until the expected maturity date: that is, reinvestment risk. For example, an investor who plans to invest for four years might consider the following three alternative investments: (i) invest in a four-year bond and hold it for four years, (ii) invest in a six-month instrument and when it matures, reinvest the proceeds in six-month instruments over the entire four-year investment horizon, and (iii) invest in a two-year bond and when it matures, reinvest the proceeds in a two-year bond. The risk in the second and third alternatives is that the return over the four-year investment horizon is unknown because rates at which the proceeds can be reinvested until maturity are unknown.

(c) Give three interpretations of the pure expectations theory.

There are several interpretations of the pure expectations theory that have been put forth by economists. These interpretations are not exact equivalents nor are they consistent with each other, in large part because they offer different treatments of price risk and reinvestment associated with realizing a return.

The broadest interpretation of the pure expectations theory suggests that investors expect the return for any investment horizon to be the same, regardless of the maturity strategy selected. For example, consider an investor who has a five-year investment horizon. According to this theory, it makes no difference if a five-year, 12-year, or 30-year bond is purchased and held for five years because the investor expects the return from all three bonds to be the same over five years. A major criticism of this very broad interpretation of the theory is that, because of price risk associated with investing in bonds with a maturity greater than the investment horizon, the expected returns from these three very different bond investments could differ in significant ways.

A second interpretation, referred to as the local expectations theory, a form of pure expectations theory, suggests that the returns on bonds of different maturities will be the same over a short-term investment horizon. For example, if an investor has a six-month investment horizon, buying a 5-year, 10-year, or 20-year bond will produce the same six-month return. It has been demonstrated
that the local expectations formulation, which is narrow in scope, is the only one of the interpretations of the pure expectations theory that can be sustained in equilibrium.

A third interpretation of the pure expectations theory suggests that the return that an investor will realize by rolling over short-term bonds to some investment horizon will be the same as holding a zero-coupon bond with a maturity that is the same as that investment horizon. (Because a zero-coupon bond has no reinvestment risk, future interest rates over the investment horizon do not affect the return.) This variant is called the return-to-maturity expectations interpretation. For example, let’s assume that an investor has a five-year investment horizon. By buying a five-year zero-coupon bond and holding it to maturity, the investor’s return is the difference between the maturity value and the price of the bond, all divided by the price of the bond. According to return-to-maturity expectations, the same return will be realized by buying a six-month instrument and rolling it over for five years. Most people have grave problems with the validity of this theory simply due to the unknown nature of future interest rates when assets are rolled over.

24. Answer the following questions.

(a) What are the two biased expectations theories about the term structure of interest rates?

The two biased expectations theories are the liquidity theory and the preferred habitat theory. They are considered “biased” because they argue that factors, other than the market’s expectations about future rates, affect forward rates.

(b) What are the underlying hypotheses of these two theories?

The liquidity theory states that investors will hold longer term maturities if they are offered a long-term rate higher than the average of expected future rates by a risk premium that is positively related to the term to maturity. Put differently, the forward rates should reflect both interest-rate expectations and a “liquidity” premium (really a risk premium), and the premium should be higher for longer maturities. The preferred habitat theory also adopts the view that the term structure reflects the expectation of the future path of interest rates as well as a risk premium. However, the preferred habitat theory rejects the assertion that the risk premium must rise uniformly with maturity. The market segmentation theory also recognizes that investors have preferred habitats dictated by the nature of their liabilities. However, the market segmentation theory differs from the preferred habitat theory in that it assumes that neither investors nor borrowers are willing to shift from one maturity sector to another to take advantage of opportunities arising from differences between expectations and forward rates.

25. Answer the following questions.

(a) “Empirical evidence suggests that with respect to bond risk premiums that influence the shape of the Treasury yield curve, there is a linear relationship between Treasury average returns and duration.” Explain whether you agree or disagree with this statement. If you disagree, explain the type of relationship that has been observed.
Based on empirical evidence, one would disagree with the statement because it has been shown that there is a nonlinear relation between Treasury average returns (as represented by bond risk premium) and duration (which is a measure of the price sensitivity of a bond to changes in interest rates). More details are supplied below, including the observed linear relation between duration and the theoretical expected return curve.

Ilmanen investigated the effect of the behavior of the bond risk premium using historical average returns on U.S. Treasury securities. Exhibit 5-10 shows the empirical average return curve as a function of average duration (not maturity) for the period 1972 to 2001. Exhibit 5-10 is the theoretical expected return curve based on expectations only (the first influence listed above). Notice that this curve is linear (i.e., it increases linearly with duration). In contrast, notice that the empirical evidence suggests that the bond risk premiums are not linear in duration. Instead, the empirical evidence suggests that at the front-end of the yield curve (i.e., up to a duration of 3), bond risk premiums increase steeply with duration. However, after a duration of 3 the bond risk premiums increase slowly. Ilmanen suggests that the shape shown in Exhibit 5-10 “may reflect the demand for long-term bonds from pension funds and other long-duration liability holders.”

(b) What is meant by the “convexity bias” influence on the shape of the Treasury yield curve?

Empirical evidence suggests that the three main influences on the shape of the Treasury yield curve are (i) the market’s expectations of future rate changes, (ii) bond risk premiums, and (iii) convexity bias. The convexity bias influence, according to Ilmanen, is the least well known of the three influences. Recall that convexity implies that when interest rates change by a large number of basis points, a Treasury security’s price change will not be the same for an increase and decrease in interest rates. More specifically, the price appreciation when interest rates fall will be greater than the price decline when interest rates rise by the same number of basis points. For example, if interest rates decline by 100 basis points, the price of a Treasury security might appreciate by 20%, but if interest rates increase by 100 basis points, the price of the same Treasury security might decline by only 15%. This attractive property of a bond is due to the shape of the relationship between price and yield and is referred to the bond’s convexity. The longer the maturity, the more convexity the security has. That is, longer term Treasury securities have a more attractive feature due to convexity than shorter term Treasury securities. As a result, investors are willing to pay more for longer term Treasury securities and therefore accept lower returns. This influence on the shape of the Treasury yield curve is what is referred to as the convexity bias.