International Investments

Tuesdays 6:10-9:00 p.m.
Commerce 260306
Wednesdays 9:10 a.m.-12 noon
Commerce 260508

Handout #12

Offshore Financial Markets
The Eurobond Market

Slides to highlight: 1-49, 52-66

Course web pages:
http://finance2010.pageout.net
ID: California2010  Password: bluesky
ID: Oregon2010     Password: greenland
### Reading Assignments for this Week

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<td>The Eurobond Market</td>
<td><a href="http://www.duke.edu/~mbrandt/ba450.html">http://www.duke.edu/~mbrandt/ba450.html</a></td>
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<td>Chaps 1-4</td>
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Midterm Exam: See University Calendar

Coverage: Chapters 3, 4, 5, 6, 7, 8, 9, 10 + Ben Bernanke’s semi-annual testimony

It’s a closed-book exam. However, a two-sided formula sheet (11 x 8.5) is required; calculator/dictionary is okay; notebook is NOT okay.

75 minutes, 7 questions, 100 points total; five questions require calculation and two questions require (short) essay writing.
Final Exam See University Calendar

A Three-hour Exam

Open-Book, Open Notes
Offshore Financial Markets
The Eurobond Market

International Investments
Yee-Tien Fu
Introduction

It is paradoxical that a law which was intentionally prejudicial to the interests of foreign borrowers, had the effect of creating the largest international capital market the world has known.

- Frederick G. Fisher, III
The Eurobond market is the market for long-term debt instruments issued and traded in the offshore market.

Like the Eurocurrency market, the necessary condition for the development of a Eurobond market is differences in national regulations. Increasing capital mobility and greater ease in telecommunications have provided the sufficient conditions, allowing the Eurobond market to flourish.
Introduction

From a base of zero in the late 1950s, the Eurobond market has grown to an annual volume of new issues that often nears or surpasses the annual volume of new US corporate bond issues.

Through regulatory differences as well as innovations in market processes and product offerings, the Eurobond market has carved out an important niche in the international capital market providing benefits to investors and borrowers – and on occasion profits to the parties who intermediate the transactions.
Introduction

Similar to the Eurocurrency market, the Eurobond market is in effect a parallel market, but one that has not put its chief rivals – the onshore markets for domestic and foreign bonds – out of business.

A Eurobond was once defined as a debt instrument (1) underwritten by an international syndicate, and (2) offered for sale immediately in a number of countries.
Introduction

A Eurobond is usually denominated in a currency (or unit of account) that is foreign to a large number of buyers.

A domestic bond is an obligation of a domestic issuer that is underwritten by a syndicate of domestic investment banks, denominated in domestic currency, and offered for sale in the domestic market.

A foreign bond is similar to a domestic bond except that the issuer of the foreign bond is a foreign entity, which may be beyond the legal reach of investors in the event of default.
Most tantalising of all, there is the huge mountain of Japanese personal savings, estimated at ¥1300 trillion ($12 trillion), much of it stacked up in the postal savings system, and ¥106 trillion of which is expected to mature over the next two years. As these deposits made at relatively high interest rates a decade ago mature, the post office will be able to offer only nugatory yields on reinvested or new deposits. Some of this money is expected to find its way into the stock market, so the financial industry is scrambling for a chunk of the assets.

*The Economist: Survey Online Finance May 20, 2000*
Introduction

The definition of a “domestic” or “foreign” bond that we adopt comes from the nationality of the issuer in relation to the marketplace.

The term foreign may lead to some confusion in this context.

A US$ bond issued in the United States by General Motors and a ¥ bond issued in Japan by Toyota are both domestic bonds from the standpoint of the regulations that govern their initial offering and secondary market trading.
Introduction

From the investor perspective, Americans (Japanese) would view the Toyota (General Motors) bond as “foreign” in the sense that investment is denominated in a foreign currency and traded in a foreign marketplace.

Foreign currency denominated bonds play an important role in international portfolio diversification.

Particular segments of the foreign bond market (as defined from the issuer perspective) sometimes take on colorful names.
For example, US$ obligations of non-US firms that are underwritten and issued in the US market are called Yankee bonds. Japanese yen obligations of non-Japanese firms that are underwritten and issued in the Japanese market are called Samurai bonds. And British pound sterling obligations of non-UK firms that are underwritten and issued in the UK market are called Bulldog bonds.

These names and others have proliferated along with the development of international financial markets.
## Comparative Characteristics of Bond Issues in the International Bond Market

<table>
<thead>
<tr>
<th></th>
<th><strong>U.S. Market</strong></th>
<th><strong>Non-U.S. Market</strong></th>
<th><strong>Eurobond Market</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regulatory Bodies</strong></td>
<td>Securities and Exchange Commission</td>
<td>Official agency approval</td>
<td>Minimum regulatory control</td>
</tr>
<tr>
<td><strong>Disclosure requirements</strong></td>
<td>More detailed: • High initial and ongoing expense  • Onerous to non-US firms</td>
<td>Variable</td>
<td>Determined by market practices</td>
</tr>
<tr>
<td><strong>Issuing costs</strong></td>
<td>0.75-1.00%</td>
<td>Variable to 4.0%</td>
<td>2.0-2.5%</td>
</tr>
<tr>
<td><strong>Rating requirements</strong></td>
<td>Yes</td>
<td>Usually not</td>
<td>No, but commonly done</td>
</tr>
</tbody>
</table>
## Comparative Characteristics of Bond Issues in the International Bond Market

<table>
<thead>
<tr>
<th></th>
<th>U.S. Market</th>
<th>Non-U.S. Market</th>
<th>Eurobond Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange listing</td>
<td>Usually not listed</td>
<td>Listing is usual</td>
<td>Listing is usual</td>
</tr>
<tr>
<td>Queuing</td>
<td>No queue</td>
<td>Queuing is common</td>
<td>No queue</td>
</tr>
<tr>
<td>Currency of denomination restrictions</td>
<td>United States does not restrict the use of US$</td>
<td>Part of queuing • Many countries have in the past or now restrict use of currency</td>
<td>No restrictions on use of US$ or C$</td>
</tr>
<tr>
<td>Speed of issuance</td>
<td>Relatively slow until Rule 415 on shelf registration</td>
<td>Variable</td>
<td>Usually fast - bought deal leads to fast issuance</td>
</tr>
</tbody>
</table>
## Comparative Characteristics of Bond Issues in the International Bond Market

<table>
<thead>
<tr>
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<th>U.S. Market</th>
<th>Non-U.S. Market</th>
<th>Eurobond Market</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Borrower / Issuer incentives</strong></td>
<td>+ Large market, great depth</td>
<td>+ Local visibility, diversification of funding sources</td>
<td>+ Lower annual interest expense, speed of placement</td>
</tr>
<tr>
<td></td>
<td>– Disclosure is costly to foreigners, speed</td>
<td>– Markets may be small, queuing may prevail</td>
<td>– Cannot sell issue in U.S. until seasoned</td>
</tr>
<tr>
<td><strong>Lender / Investor incentives</strong></td>
<td>+ Great depth &amp; liquidity, appeal of standardized information</td>
<td>+ Diversified currency portfolio</td>
<td>+ Diversified currency portfolio, bearer bonds, no withholding tax</td>
</tr>
<tr>
<td></td>
<td>– Reporting to tax authorities, withholding tax prior to 1984</td>
<td>– Reporting to tax authorities, withholding tax may apply</td>
<td>– Less liquidity &amp; information disclosures</td>
</tr>
</tbody>
</table>
Eurobond Underwriting

A Eurobond offering brings together the bond issuer and the bond investor.

The supply side (the issuer) and the demand side (the investor) are brought together by intermediaries that fulfill some or all of the following services: lead management, underwriting, and bond sales.
In the case of a bought deal, all of these services are provided by a single intermediary.

In a bought deal, the lead manager approaches the issuer with a proposal to raise funds under specific terms: issue size, currency, maturity, coupon payments, and other features that may enhance the volume or price of the issue.

Once the manager commits to raise funds on specific terms, the manager assumes the underwriting risks of the issue.
Underwriting risk reflects the possibility that the sales price of the bonds may not match the price promised to the issuer.

In other words, if the manager commits to raise $100 million in a seven-year bond issue with 8 percent annual coupons, she must provide this amount even if investors are willing to pay only $98 million for the bonds.

A sudden rise in interest rates, a decline in the issuer’s credit quality, or a shift away from US$-denominated investments are but three examples of underwriting risks.
Four Types of Market

direct search, brokered, dealer, auction markets

A direct search market is the least organized market -- buyers and sellers must seek each other out directly (e.g., the sale of a used refrigerator).

In markets where trading in a good is sufficiently active, brokers can find it profitable to offer search services to buyers and sellers (e.g., real estate market).

An important brokered investment market is the so-called primary market, where new issues of securities are offered to the public. In the primary market investment bankers act as brokers.
Four Types of Market

direct search, brokered, dealer, auction markets

When trading activity in a particular type of asset increases, dealer markets (e.g., over-the-counter securities market) arise.

Dealers specialize in various assets, purchasing them for their own inventory and selling them for a profit from their inventory.

Dealers, unlike brokers, trade assets for their own accounts. The dealer’s profit margin is the “bid-asked” spread.
Trading among investors of already issued securities is said to take place in secondary markets. Therefore, the over-the-counter market is one example of a secondary market. The organized stock exchanges are also secondary markets. Trading in secondary markets does not affect the outstanding amount of securities; ownership is simply transferred from one investor to another.
The most integrated market is an auction market, in which all transactors in a good converge at one place to bid on or offer a good. The New York Stock Exchange (NYSE) is an example of an auction market. An advantage of auction markets over dealer markets is that one need not search to find the best price of a good. Many assets trade in more than one type of market.
A diagram of a typical Eurobond offering is shown in Figure 10.4. The “management group” organizes most of the activities related to the initial bond offering. The group meets with the issuer to design the bond issue – issue size, currency, maturity, coupon, and so forth – and assembles other firms (labeled “underwriters”) to share in the underwriting risks of the issue. Finally, the management group organizes a “selling group” of firms that place the bonds with the ultimate investors in the issue.
Structure of a Eurobond Syndication
Structure of a Eurobond Syndication

A Eurobond offering brings together the bond issuer and investor. The process is facilitated by intermediaries.

The *lead management group* meets with the issuer to design the issue size, currency, maturity, coupon, etc...

Finally, the management group organizes a group of firms to place the bonds with the ultimate investors.

and then assembles other firms to share in the *underwriting* risks of the issue.
In practice, a single firm may play more than one role.

For example, the lead management firm typically bears some of the underwriting risk and often participates in the selling group.
The Gray Market

Excess competition => bonds decline in value in the aftermarket

In the 1970s, firms started to sell their allotment of bonds forward for delivery on a when-issued basis.

Once a bond issue was announced, a selling firm might decide to sell the bond immediately (for forward delivery) at 98 or 99% of par.
The practice of trading in Eurobonds on a when-issued basis, called the gray market or premarket, began with prices circulated over telephone lines. Then prices were published in newsletters and circulated across market participants.

This strategy would hedge the selling firm against further price declines but still allow the firm to participate in the syndicate, to appear in the tombstone announcing the deal, and to stay in good standing with the lead manager for the next deal.
Weyerhauser Capital Corp, NV (1983)

Amount: US$60 million  
Maturity: 7 years (due Nov. 15, 1990)  
Coupon: 11.5%  
Issue price: 100  
Fixed reoffer price: N.A.  
Listing: Luxembourg Stock Exchange  
Total commission: 1.875%  
  Management & underwriting fees: 0.625%  
  Selling concession: 1.25%  
Lead manager: Morgan Stanley International  
Gray market price: Minus 1.5 to 1.25  
Market commentary: “A fairly priced deal, say traders …”
Weyerhauser Capital Corp, NV (1983)

The bond was issued at par (100), but it traded in the gray market at a discount of 1.25 - 1.50 percent below par.

In the row marked “total commission,” we see that the selling concession (the amount of fees allocated to a member of the selling group) was 1.25 percent.

Thus, a European trader was willing to give up his or her entire selling concession, or a bit more, to make a sale in the gray market. This bond was apparently overpriced at par, but “fairly priced” at its gray market discount.
# Osaka Gas (1993)

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
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</thead>
<tbody>
<tr>
<td>Amount</td>
<td>US$250 million</td>
</tr>
<tr>
<td>Maturity</td>
<td>5 years (due May 26, 1998)</td>
</tr>
<tr>
<td>Coupon</td>
<td>5.75%</td>
</tr>
<tr>
<td>Issue price</td>
<td>101.489</td>
</tr>
<tr>
<td>Fixed reoffer price</td>
<td>99.889</td>
</tr>
<tr>
<td>Listing</td>
<td>London Stock Exchange</td>
</tr>
<tr>
<td>Total commission</td>
<td>1.875%</td>
</tr>
<tr>
<td>Management &amp; underwriting fees</td>
<td>0.275%</td>
</tr>
<tr>
<td>Selling concession</td>
<td>1.6%</td>
</tr>
<tr>
<td>Lead manager</td>
<td>Goldman Sachs International</td>
</tr>
<tr>
<td>Gray market price</td>
<td>100.25</td>
</tr>
<tr>
<td>Market commentary</td>
<td>“The issue blew out in 15 minutes, say traders …”</td>
</tr>
</tbody>
</table>
Osaka Gas (1993)

Although the issue price of the bond was 101.489 (percent per par), it was slated for sale initially at 99.889 (percent of par) on a fixed reoffer price basis.

Note that this 1.6 percent difference happened to be the selling concession, so again it appears that the selling group would not profit from a sale at this price.

However, the gray market price (called the premarket price in 1993) was 100.25. Thus, these bonds were apparently in heavy demand, selling for 0.361 more in the gray market than the posted initial offering price.
Another Innovation: Global Bonds

Similar to a Eurobond, a global bond issue is offered for sale in many countries simultaneously. Unlike a Eurobond however, a global bond is a registered security, usually in the US.

Global bonds are held in common depositories (such as Cedel, Euroclear, or Depository Trust Company in the US) that enhance secondary market trading in local markets and between investors in different regions.
Another Innovation: Global Bonds

The global bond strategy is designed for issuers with substantial funding needs who can benefit by reaching the widest possible investor audience.

The size of issue, combined with widespread distribution, and secondary market trading opportunities offers a liquid investment that investors find attractive.
Another Innovation: Global Bonds

The World Bank undertook the first global bond in 1989 with a $1.5 billion issue.

Since then, the global bond structure has been used by other international organizations, public enterprises, and government (sovereign) borrowers.

Global bond offerings totaled $15.4 billion in 1991 (5.0 percent of all international bonds) rising to $49.0 billion in 1994 (11.4 percent of all international bonds).
Pricing Eurobonds

As a parallel or offshore market, the Eurobond market must offer prices and terms that are advantageous to both issuers and investors to attract them from the traditional onshore markets.

In the Eurocurrency market, we saw that the wide spread between deposit and lending rates gave Eurobanks an opening to compete - offering higher rates to depositors and lower rates to borrowers, and still earning a spread for their intermediation.

The same principle could apply in the Eurobond market.
Pricing Eurobonds

Suppose that underwriting fees in the US domestic corporate bond market were 2% and our firm issues an 8% coupon bond with a seven-year maturity. After issuing the bond at par ($1,000), our firm receives only $980 after underwriting fees. The cost of funds to the firm on a current-yield basis is 8.16% (= 80/980). The cost over the seven-year period is 8.39%, acknowledging that the firm must repay $1,000 per bond at the end of year 7. The investor earns a current yield and yield-to-maturity of 8%. The effective lending and borrowing spread (for this coupon and maturity bond) is thus 8.00% – 8.39%.
Yield to Maturity

• Interest rate that makes the present value of the bond’s payments equal to its price

Solve the bond formula for $r$

$$980 = P_B = \sum_{t=1}^{7} \frac{8\% \cdot 1000}{(1+r)^t} + \frac{1000}{(1+r)^7}$$

$PB$ is the fair price (present value price) of the coupon-bearing bond.

Check: $r = 8.39\%$ ?
Pricing Eurobonds

Suppose now that underwriting fees were only 1% in an offshore market and our firm issues a seven-year bond with a higher 8.05% coupon bond in order to attract an onshore investor. If this bond is issued at par ($1,000), the firm receives $990 after underwriting fees. This makes the cost of funds to the firm on a current basis 8.13% (= 80.5/990). The all-in-cost over the seven-year period is 8.24%. Thus, the effective lending and borrowing spread is 8.05% – 8.24%, more narrow than when underwriting fee were 2%. 
Market Segmentation and the Pricing of Eurobonds

How can both issuers and investors benefit from an offshore market that typically charges higher underwriting fees than in the onshore market?

From the issuer’s side, the answer is that underwriting fees are a one-time cost and only part of the total cost. There may be certain cost savings as the Eurobond market often allows firms to issue bonds more quickly and with lower disclosure cost.
Disclosure: The submission of facts and details concerning a situation or business operation.

In general, security exchanges and the SEC require firms to disclose to the investment community the facts concerning issues that will affect the firms’ stock prices. Disclosure is also required when firms file for public offerings.
Market Segmentation and the Pricing of Eurobonds

More important is the ongoing savings that comes from a lower annual interest cost in the Eurobond market than in the onshore market.

The Eurobond market has appeared to function as a segmented capital market, where bond prices are determined primarily by Eurobond market participants who give less than full regard to how these bonds would be priced in the onshore market.
Market Segmentation and the Pricing of Eurobonds

By comparison, in an integrated capital market, a bond with specific terms and conditions would be priced identically by investors in the onshore market and in the Eurobond market. Arbitrage between the onshore and offshore bond markets leads the markets toward integration.

In the case of Eurobond market, it is often suggested that the early years of the market were dominated by smaller, retail investors who evaluated bond prices on different terms than the institutional investors who traded in the onshore markets.
Market Segmentation and the Pricing of Eurobonds

The argument is that these retail investors were less concerned about cryptic issuer ratings and more swayed by “name recognition.”

To the extent that these investors willingly paid higher prices for debt securities in the Eurobond market, issuers were offered a price incentive to issue Eurobonds instead of onshore bonds.
Eurobonds and Secrecy

Why would investors sacrifice yield by buying Eurobonds when instead they could purchase essentially identical bonds onshore?
The answer relies on the secrecy of the Eurobond market and its implication for taxes. While essentially all securities in the US are registered securities (with the name of the owner registered on the books of the issuing company), Eurobonds are bearer securities. Possession of a bearer bond is evidence of ownership because the issuer does not maintain a list. A significant fraction of Eurobonds are held in physical form.
Suppose IBM is issuing $100 million in seven-year Eurobonds priced at U.S. Treasury minus 25 basis points. There is great demand for the issue and you are willing to bid 102 for 10 percent of the issue.

A. If you actually get your bid executed, how much will you pay for the bond?

B. A year later, the IBM Eurobonds are traded on the Luxembourg Stock Exchange at 105. What is the value of your investment? What is your capital gains (loss)?

C. You decide to sell the bond at the above price to pursue other opportunities. What amount of withholding taxes are you required to pay?
A. If you actually get your bid executed, how much will you pay for the bond?

Price is 102% of par or 1,020 per bond; 
102\% \times 10\% \times 100 \text{ million} = \$10.20 \text{ million for your share of the issue.}
Suppose two similar seven-year maturity bands are issued at par, one in the U.S. domestic market and the second in the Eurodollar bond market. Underwriting fees are 2.5 percent in the U.S. market and 1 percent in the Eurobond market.

A. If the U.S. domestic bond has an initial yield of 10%, what is the effective spread between lending and borrowing rates in this market?

B. If the Eurodollar bond has an initial yield of 10.5%, what is the effective spread between lending and borrowing rates in this market?

C. Suppose that the U.S. bond is subject to a withholding tax of 20% on the interest paid. What yield would an investor accept on the Eurobond issue to make him or her indifferent between the two issues?
A. If the U.S. domestic bond has an initial yield of 10%, what is the effective spread between lending and borrowing rates in this market?

In the US bond market, after underwriting fees, the firm raises $975 for a $1,000 US domestic bond issued at par. The firm repays $10 per year for six years and $1,010 in year seven for a yield-to-maturity of 10.52%. The investor earns 10.0% yield-to-maturity for a 7-year bond. The spread is 0.52%.
B. If the Eurodollar bond has an initial yield of 10.5%, what is the effective spread between lending and borrowing rates in this market?

In the Eurodollar bond market, after underwriting fees, the firm gets $990 for a $1,000 US domestic bond issued at par. The firm repays $10.50 per year for six years and $1,050 in year seven for a yield-to-maturity of 10.71%. The investor earns 10.50% yield-to-maturity for a 7-year bond. The spread is 0.21%.
C. Suppose that the U.S. bond is subject to a withholding tax of 20% on the interest paid. What yield would an investor accept on the Eurobond issue to make him or her indifferent between the two issues?

An investor will accept a lower yield in the Eurobond market if he/she does not pay the withholding tax. An 8% yield in the Eurobond market (taken as an after-tax rate) is equivalent to a 10% yield in the US bond market, on a before tax basis and subject to 20% withholding.
Yield to Maturity

• Interest rate that makes the present value of the bond’s payments equal to its price

Solve the bond formula for $r$

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{ParValue}{(1+r)^T}$$

PB is the fair price (present value price) of the coupon-bearing bond.
The yield to maturity (or “YTM”) is the rate that makes the price of the bond just equal to the present value of its future cash flows. It is the unknown \( r \) in:

\[
932.90 = \$\text{_______} \left[ 1 - \frac{1}{(1 + r)^{10}} \right]/r + \$\text{_______}/(1 + r)^{10}
\]

The only way to find the YTM is trial and error:

a. Try 10%: $70 \quad \left[ (1 - 1/(1.10)^{10})/0.10 + \$1000/(1.10)^{10} \right] = $816

b. Try 9%: $70 \quad \left[ 1 - 1/(1.09)^{10}/.09 + \$1000/(1.09)^{10} \right] = $872

c. Try 8%: $70 \quad \left[ 1 - 1/(1.08)^{10}/.08 + \$1000/(1.08)^{10} \right] = $933

( ) The yield to maturity is 8%
Bond Pricing Theorems

• The following statements about bond pricing are *always* true.

1. Bond prices and market interest rates move in opposite directions.

2. When a bond’s coupon rate is (greater than / equal to / less than) the market’s required return, the bond’s market value will be (greater than / equal to / less than) its par value.
Bond Pricing Theorems

• The following statements about bond pricing are *always* true.

  3 Given two bonds identical but for maturity, the price of the longer-term bond will change more than that of the shorter-term bond, for a given change in market interest rates.

  4 Given two bonds identical but for coupon, the price of the lower-coupon bond will change more than that of the higher-coupon bond, for a given change in market interest rates.
If the U.S. 3-month bank deposit rate is 7%, the reserve requirement is 2.5% and FDIC fees are 0.20%, (a) what would you expect the Eurodollar rate to be? (b) What will happen if the reserve requirement increases by 0.2 percentage point?

(a) The effective cost of a domestic deposit is
\[
\left( I_{US} + \text{FDIC fees} \right) / (1 \text{- reserve requirements})
= (7\% + 0.20\%) / (1 - 2.5\%)
= 7.3846\%
\]

Thus the additional cost of the reserve requirements and FDIC fees is 38 basis points and this is the extra amount the bank can afford to pay on Eurodollar deposits to achieve the same cost of funds. Competition will generally drive the Eurodollar rate to the level that equates the cost of funds to banks in the two markets, that is, to 7.38\%.
(b) If the reserve requirement increases to 2.7%, the Eurodollar rate will be:

\[
\left( I_{US} + \text{FDIC fees} \right) / \left( 1 - \text{reserve requirements} \right) \\
= \left( 7\% + 0.20\% \right) / \left( 1 - 2.7\% \right) \\
= 7.3998\%
\]
The U.S. bank deposit rate is now 5.15%, and the Eurodollar deposit rate 5.45%. Assuming that the entire differential is attributable to the Fed's reserve requirement on bank deposits, what is likely to happen to the Eurodollar rate if the U.S. rate rises by one percentage point?

\[ I_{E\$} = 5.45\% \quad I_{US} = 5.15\% \]

Banks arbitrage their funding costs between the domestic and the Eurodollar market, so that in equilibrium:

Cost of Eurodollar deposit = Cost of domestic deposit

\[ I_{E\$} = (I_{US} + \text{FDIC fees}) / (1 - \text{reserve requirement}) \]

Assuming the differential is entirely attributable to reserve requirements, we can set FDIC fees=0 and then

\[ I_{E\$} = (I_{US} + 0) / (1 - \text{reserve requirement}) \]

Reserve requirement = 1 - (I_{US}/I_{E\$}) = 1-(5.15%/5.45%) = 5.50\%.
Innovation in the Bond Market

• Issuers constantly develop innovative bonds with unusual features - bond design can be extremely flexible.
  ▫ Issuers of *pay in kind* bonds may choose to pay interest either in cash or in additional bonds with the same face value.
  ▫ *Reverse floaters* are floating rate bonds whereby the coupon rate on the bonds *falls* when the general level of interest rates rises.
  ▫ Walt Disney has issued bonds with coupon rates tied to the financial performance of several of its films.
Innovation in the Bond Market

- Electrolux once issued a bond with a final payment that depended on whether there has been an earthquake in Japan. (*disaster bond*)

- *Indexed bonds* make payments that are tied to a general price index or the price of a particular commodity. For example, Mexico has issued bonds with payments that depend on the price of oil.

  - **More on indexed bonds:**
    - Bonds tied to the general price level have been common in countries experiencing high inflation.
Innovation in the Bond Market

- Although Great Britain is not a country experiencing extreme inflation, about 20% of its government bonds issued in the last decade have been inflation-indexed.

- The United States Treasury started issuing such inflation-indexed bonds in January 1997. They are called Treasury Inflation Protected Securities (TIPS). By tying the par value of the bond to the general level of prices, the coupon payments, as well as the final repayment of par value, will increase in direct proportion to the consumer price index. Thus, the interest rate on these bonds is a risk-free real rate.
To illustrate how TIPS work, consider one that is maturing in one year. Assume that it offers a risk-free real coupon rate of 3% per year. The *nominal* rate of return is not known with certainty in advance because it depends on the rate of inflation.

If the inflation rate turns out to be only 2%, then the realized dollar rate of return will be approximately 5%.

If the rate of inflation turns out to be 10%, then the realized dollar rate of return will be approximately 13%, consisting of the 3% coupon plus a 10% increase in the dollar value of the bond, from $1,000 to $1,100.

In early 1997, TIPS bonds were trading at a real yield to maturity a shade below 3.5%.
Treasury Inflation Protection Securities

On January 29, 1997, the U.S. Department of the Treasury issued for the first time Treasury securities that adjust for inflation. These securities are popularly referred to as Treasury inflation protection securities, or TIPS. The first issue was a 10-year note. Subsequently, the Treasury issued a five-year note in July 1997 and a 30-year bond in 1998.

TIPS work as follows. The coupon rate on an issue is set at a fixed rate. That rate is determined via the auction process described later in this section. The coupon rate is called the “real rate” since it is the rate that the investor ultimately earns above the inflation rate. The inflation index that the government has decided to use for the inflation adjustment is the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for All Urban Consumers (CPI-U).

Source: Fabozzi Bond Markets Analysis and Strategies, Seventh Ed.
The adjustment for inflation is as follows. The principal that the Treasury Department will base both the dollar amount of the coupon payment and the maturity value on is adjusted semiannually. This is called the inflation-adjusted principal. For example, suppose that the coupon rate for a TIPS is 3.5% and the annual inflation rate is 3%. Suppose further that an investor purchases on January 1 $100,000 par value (principal) of this issue. The semiannual inflation rate is 1.5% (3% divided by 2). The inflation-adjusted principal at the end of the first six-month period is found by multiplying the original par value by one plus the semiannual inflation rate. In our example, the inflation-adjusted principal at the end of the first six-month period is $101,500. It is this inflation-adjusted principal that is the basis for computing the coupon interest for the first six-month period. The coupon payment is then 1.75% (one-half the real rate of 3.5%) multiplied by the inflation-adjusted principal at the coupon payment date ($101,500). The coupon payment is therefore $1,776.25.

The principal is constantly adjusted by updated inflation figure before the coupon is calculated and paid.

Source: Fabozzi Bond Markets Analysis and Strategies, Seventh Ed.
Let’s look at the next six months. The inflation-adjusted principal at the beginning of the period is $101,500. Suppose that the semiannual inflation rate for the second six-month period is 1%. Then the inflation-adjusted principal at the end of the second six-month period is the inflation-adjusted principal at the beginning of the six-month period ($101,500) increased by the semiannual inflation rate (1%). The adjustment to the principal is $1,015 (1% times $101,500). So, the inflation-adjusted principal at the end of the second six-month period (December 31 in our example) is $102,515 ($101,500 + $1,015). The coupon interest that will be paid to the investor at the second coupon payment date is found by multiplying the inflation-adjusted principal on the coupon payment date ($102,515) by one-half the real rate (i.e., one-half of 3.5%). That is, the coupon payment will be $1,794.01.

As can be seen, part of the adjustment for inflation comes in the coupon payment since it is based on the inflation-adjusted principal. However, the U.S. government has decided to tax the adjustment each year. This feature reduces the attractiveness of TIPS as investments in accounts of tax-paying entities.

Source: Fabozzi Bond Markets Analysis and Strategies, Seventh Ed.
Bond Case--Swedish Lottery Bonds.

Profiling nonsystematic risk for a bond investor, the case describes lottery bond issues by the Swedish National Debt Office (SNDO). Swedish lottery bonds are a specific type of financial fixed income instrument for Swedish retail investors. The distinctive feature of lottery bonds is that, unlike traditional institutional bonds, the normally guaranteed interest--the coupon--here only is paid as "wins" to bondholders selected in drawings. The case takes place in March 2003, when Anders Holmlund, head of analysis, is reviewing the proposal for the next lottery bond issue. While reviewing the features of the bond issue, he also considers the larger picture: What are the benefits to the Debt Office of issuing lottery bonds, especially in view of a recently launched Internet-based sales system that allows retail investors to take part in government bond auctions?


Stockholm; Government & regulatory; 25 employees; 2003
Bond Case--Bank Leu’s Prima Cat Bond Fund.

In 2001, Bank Leu, a Swiss private bank, is considering creating the world's first public fund for catastrophe bonds. Cat bonds are securities whose payments depend on the probability of a catastrophe occurring, such as an earthquake or hurricane. Cat bonds are traditionally issued by large insurance or reinsurance companies. This case outlines the traditional reinsurance market and securitization efforts that have taken place in the past and focuses on Bank Leu's decision as a buy-side participant in the cat bond market.

To explore how insurance risks can be transferred to the capital markets and how risks in general can be brokered, securitized, and traded.


Zurich; Switzerland; Banking industry; 116 million CHF revenues; 600 employees; 2001
Bond Case--Catastrophe Bonds at Swiss Re.

In 2002, Swiss Re, the world's second largest insurance company, is considering securitizing parts of its risk portfolio in the capital markets. This would be a first for the company that, until then, had never transferred risk off its balance sheet. Peter Giessmann, head of the Retrocession Group, is considering catastrophe bonds as a way of transferring risk. "Cat bonds" are securities whose payments depend on the probability of a catastrophe occurring, such as an earthquake or hurricane. This case outlines the traditional reinsurance market and securitization efforts that have taken place in the past and then focuses on Swiss Re's decision as a sell-side participant in the cat bond market.

To explore how insurance risks can be transferred to the capital markets and how risks in general can be brokered, securitized, and traded.


Zurich; Switzerland; Insurance industry; 31 million Swiss francs revenues; 2002
**Bond Case--Mortgage Backs at Ticonderoga.**

Ticonderoga is a small hedge fund that trades in mortgage-backed securities—securities created from pooled mortgage loans. They often appear as straightforward so-called "pass-throughs," but can also be pooled again to create collateral for a mortgage security known as a collateralized mortgage obligation (CMO). CMOs allow cash flows from the underlying pass-throughs to be directed, allowing the creation of different classes of securities--tranches--with different maturities, coupons, and risk profiles. In April 2005, the general managers of Ticonderoga are looking at the market data, trying to construct a trade given their view on the prepayment speed of mortgages vs. the implied prepayment speed they derive from CMOs in the market.

To learn about the institutional details behind the mortgage-backed securities (MBS) market, covering both the actors as well as the mechanics (with special emphasis on the important prepayment feature). Also, to go through the mathematics and calculations behind MBSs--in essence, students are asked to behave as if they worked at a mortgage-back trading desk.

---

Derivatives, Finance, Hedging, Over the counter trading, Securities, Trade. London; Financial industry; 10 employees; 2005
Covers the first international nonperforming loan securitization done in Korea. The CEO of KAMCO is trying to dispose of a portfolio of nonperforming commercial loans that the organization acquired from a number of banks. A group of investment bankers have proposed securitizing the loans and selling them to institutional investors. Securitization of loans (or any other type of assets) is not common in Korea, so the CEO must think through several factors as he decides whether to accept this proposal, the most important of which is the recovery price.

To understand financial securitization--both structuring and valuation principles.

Capital markets, Debt management, Financial instruments, Financing.
Nexgen: Structuring Collateralized Debt Obligations (CDOs).

A client asks Luc Giraud, CEO of the structured finance solutions provider Nexgen Financial Solutions, to put together a solution that allows the client to add AAA-rated bonds to its portfolio. The client cannot find suitably priced top-rated bonds in the market and wonders whether Nexgen can use lower grade bonds to create AAA-equivalent instruments. The process of securitization packages together securities to create new securities with different risk and return profiles.

To examine the process of securitization--in this case, a financial intermediary creates value by putting together a package of securities and offering the client a risk tranche that the client could not otherwise obtain. In terms of credit risk, to look at the impact of correlation in credit risk in portfolios of collateralized debt securities.

Bonds, Capital markets, Credit risk, Debt management, Derivatives, Finance, Securities, Securitization.

France; Financial industry; 20 employees; 2004
Chapter 10 (C&J)
Bond Prices and Yields

• Bond Basics
• Straight Bond Prices & Yield to Maturity
• More on Yields
• Interest Rate Risk & Malkiel’s Theorems
• Duration
• Dedicated Portfolios and Reinvestment Risk
• Immunization
• Summary & Conclusions

Source: Fundamentals of Investments: Valuation and Management
By Corrado, Charles J., and Bradford D. Jordan
Bond Basics

Straight bonds and their yields
• Straight bonds
• Notes, bonds, debentures
• Other features: convertible, putable
• Yields
  ▶ Coupon rate or coupon yield
  ▶ Current yield
  ▶ Yield to maturity
Bond Calculations

Bond’s coupon rate:

\[
\text{Coupon rate} = \frac{\text{Annual coupon}}{\text{Par value}}
\]

Bond’s current yield:

\[
\text{Current yield} = \frac{\text{Annual coupon}}{\text{Bond price}}
\]

Straight bond prices:

\[
\text{Bond price} = \frac{C}{YTM} \left[ 1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}} \right] + \frac{FV}{\left(1 + \frac{YTM}{2}\right)^{2M}}
\]
Bond Prices

Straight bond prices:

\[
\text{Bond price} = \frac{C}{YTM} \left[ 1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}} \right] + \frac{FV}{\left(1 + \frac{YTM}{2}\right)^{2M}}
\]

C = annual coupon
FV = face value
M = maturity (years)
YTM = Yield to maturity

Assume a bond has 15 years to maturity, a 9% coupon, and the YTM is 8%. What is the price?

\[
\text{Bond price} = \frac{90}{0.08} \left[ 1 - \frac{1}{\left(1 + \frac{0.08}{2}\right)^{30}} \right] + \frac{1000}{\left(1 + \frac{0.08}{2}\right)^{30}} = 1,086.46
\]
More on Bond Prices

$$\text{Bond price} = \frac{C}{\text{YTM}} \left[ 1 - \frac{1}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}} \right] + \frac{\text{FV}}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}}$$

Now assume a bond has 25 years to maturity, a 9% coupon, and the YTM is 8%. What is the price? Is the bond selling at premium or discount?

$$\text{Bond price} = \frac{90}{.08} \left[ 1 - \frac{1}{\left(1 + .08/2\right)^{50}} \right] + \frac{1000}{\left(1 + .08/2\right)^{50}} = \$1,107.41$$

Now assume the same bond has a YTM of 10%. (9% coupon & 25 years to maturity) What is the price? Is the bond selling at premium or discount?

$$\text{Bond price} = \frac{90}{.10} \left[ 1 - \frac{1}{\left(1 + .10/2\right)^{50}} \right] + \frac{1000}{\left(1 + .10/2\right)^{50}} = \$908.72$$
More on Bond Prices (cont’d)

Now assume the same bond has 5 years to maturity (9% coupon & YTM of 8%) What is the price? Is the bond selling at premium or discount?

\[
\text{Bond price} = \frac{90}{.08} \left[ 1 - \frac{1}{\left(1 + \frac{.08}{2}\right)^{10}} \right] + \frac{1000}{\left(1 + \frac{.08}{2}\right)^{10}} = $1,040.55
\]

Now assume the same bond has a YTM of 10%. (9% coupon & 5 years to maturity) What is the price? Is the bond selling at premium or discount?

\[
\text{Bond price} = \frac{90}{.10} \left[ 1 - \frac{1}{\left(1 + \frac{.10}{2}\right)^{10}} \right] + \frac{1000}{\left(1 + \frac{.10}{2}\right)^{10}} = $961.39
\]
More on Bond Prices (cont’d)

Where does this leave us? We found:

<table>
<thead>
<tr>
<th>Coupon</th>
<th>Years</th>
<th>YTM</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>9%</td>
<td>25</td>
<td>8%</td>
<td>$1,107</td>
</tr>
<tr>
<td>9%</td>
<td>25</td>
<td>10%</td>
<td>$908</td>
</tr>
<tr>
<td>9%</td>
<td>5</td>
<td>8%</td>
<td>$1,040</td>
</tr>
<tr>
<td>9%</td>
<td>5</td>
<td>10%</td>
<td>$961</td>
</tr>
</tbody>
</table>

![Graph showing bond prices over different YTM and coupon years]
More on Prices

Prices and Par values
• Premium bonds
• Discount bonds
• Par bonds
• Relations among yields
• YTM > current > coupon
• YTM < current < coupon
• YTM = current = coupon
Figure 10.2: Bond prices and yields
Figure 10.1: Premium, par, and discount bond prices
Calculating Yields

The formula:

\[
\text{Bond price} = \frac{C}{\text{YTM}} \left[ 1 - \frac{1}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}} \right] + \frac{\text{FV}}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}}
\]

- Use the same formula, but solve for YTM
- How?
  - Trial and error . . .
  - Financial calculator
- Prices versus yields
Bond YTM

\[
\text{Bond price} = \frac{C}{\text{YTM}} \left[ 1 - \frac{1}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}} \right] + \frac{\text{FV}}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}}
\]

Assume a bond has 15 years to maturity, a 9% coupon, and the bond is selling for is $1,080. What is the YTM?

\[
$1,080 = \frac{90}{\text{YTM}} \left[ 1 - \frac{1}{\left(1 + \frac{\text{YTM}}{2}\right)^{30}} \right] + \frac{1000}{\left(1 + \frac{\text{YTM}}{2}\right)^{30}}
\]

\[
\text{YTM} = 4.0354\% \times 2 = 8.07\%
\]
Bond Yield to Call

Callable bond price = \[
\frac{C}{YTC} \left[ 1 - \frac{1}{\left(1 + \frac{YTC}{2}\right)^{2T}} \right] + \frac{CP}{\left(1 + \frac{YTC}{2}\right)^{2T}}
\]

Assume the previous bond has 5 years until it can be called with a $90 call premium. (9% coupon & selling for $1,080.) What is the YTM?

\[
$1,080 = \frac{90}{YTC} \left[ 1 - \frac{1}{\left(1 + \frac{YTC}{2}\right)^{10}} \right] + \frac{1090}{\left(1 + \frac{YTC}{2}\right)^{10}}
\]

YTC = 4.243% x 2 = 8.49%
Malkiel’s Theorems

Summarizes the relationship between bond prices, yields, coupons, and maturity:

Malkiel’s Theorems paraphrased (see text for exact wording); all theorems are ceteris paribus:

1) Bond prices move inversely with interest rates.
2) The longer the maturity of a bond, the more sensitive is its price to a change in interest rates.
3) The price sensitivity of any bond increases with its maturity, but the increase occurs at a decreasing rate.
4) The lower the coupon rate on a bond, the more sensitive is its price to a change in interest rates.
5) For a given bond, the volatility of a bond is not symmetrical, i.e. a decrease in interest rates raises bond prices more than a corresponding increase in interest rates lower prices.
Malkiel’s Theorems (cont’d)
Bond Prices and Yields (8% bond)

<table>
<thead>
<tr>
<th>Yields</th>
<th>5 years</th>
<th>10 years</th>
<th>20 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 percent</td>
<td>$1,041.58</td>
<td>$1,071.06</td>
<td>$1,106.78</td>
</tr>
<tr>
<td>9 percent</td>
<td>960.44</td>
<td>934.96</td>
<td>907.99</td>
</tr>
<tr>
<td>Price Difference</td>
<td>$81.14</td>
<td>$136.10</td>
<td>$198.79</td>
</tr>
</tbody>
</table>
## Malkiel’s Theorems (cont’d)
### 20-Year Bond Prices and Yields

<table>
<thead>
<tr>
<th>Yields</th>
<th>6 percent</th>
<th>8 percent</th>
<th>10 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 percent</td>
<td>$1,000.00</td>
<td>$1,231.15</td>
<td>$1,462.30</td>
</tr>
<tr>
<td>8 percent</td>
<td>802.07</td>
<td>1,000.00</td>
<td>1,197.93</td>
</tr>
<tr>
<td>10 percent</td>
<td>656.82</td>
<td>828.41</td>
<td>1,000.00</td>
</tr>
</tbody>
</table>
Malkiel’s Theorems (cont’d)
8% coupon, 20 year bond

<table>
<thead>
<tr>
<th>Yield</th>
<th>Price</th>
<th>Falls 2%</th>
<th>Rises 2%</th>
<th>Increase</th>
<th>Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>6%</td>
<td>$1,231</td>
<td>$1,547</td>
<td>$1,000</td>
<td>25.70%</td>
<td>18.80%</td>
</tr>
<tr>
<td>8%</td>
<td>$1,000</td>
<td>$1,231</td>
<td>$828</td>
<td>23.10%</td>
<td>17.20%</td>
</tr>
<tr>
<td>10%</td>
<td>$828</td>
<td>$1,000</td>
<td>$699</td>
<td>20.80%</td>
<td>15.60%</td>
</tr>
</tbody>
</table>
Duration

The key to bond portfolio management

• Macaulay duration: What is it?
  ▫ Measures the combined effect of maturity, coupon rate, and YTM on bond’s price sensitivity
  ▫ Measure of the bond’s effective maturity
  ▫ Measure of the average life of the security
  ▫ Weighted average maturity of the bond’s cash flows
Calculating Duration

Calculating Macaulay’s Duration for a par value bond is a special case, as follows:

Par value bond duration

\[ \text{Par value bond duration} = \left( \frac{1 + \frac{YTM}{2}}{YTM} \right) \left[ 1 - \frac{1}{\left( \frac{1 + \frac{YTM}{2}}{2} \right)^{2M}} \right] \]

To calculating Macaulay’s Duration for any other bond:

\[ MD = \frac{1 + \frac{YTM}{2}}{YTM} - \left( \frac{1 + \frac{YTM}{2}}{YTM} \right)^{2M} \]

\[ + \frac{M(C - YTM)}{YTM + C \left[ \left( \frac{1 + \frac{YTM}{2}}{2} \right)^{2M} - 1 \right]} \]

C = annual coupon rate
M = maturity (years)
Macaulay Duration alternative formula

Macaulay Duration = \sum_{t=1}^{n} \frac{PV(CF_t)}{Bond \ Price} \times t
### Figure 10.3: Calculating bond duration

<table>
<thead>
<tr>
<th>Years</th>
<th>Cash flow</th>
<th>Discount factor</th>
<th>Present value</th>
<th>Years x Present value / Bond price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>40</td>
<td>0.96154</td>
<td>38.4615</td>
<td>0.0192</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>0.92456</td>
<td>36.9822</td>
<td>0.0370</td>
</tr>
<tr>
<td>1.5</td>
<td>40</td>
<td>0.88900</td>
<td>35.5599</td>
<td>0.0533</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>0.85480</td>
<td>34.1922</td>
<td>0.0684</td>
</tr>
<tr>
<td>2.5</td>
<td>40</td>
<td>0.82193</td>
<td>32.8771</td>
<td>0.0822</td>
</tr>
<tr>
<td>3</td>
<td>1040</td>
<td>0.79031</td>
<td>821.9271</td>
<td>2.4658</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bond price</th>
<th>Bond duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000.00</td>
<td>2.7259</td>
</tr>
</tbody>
</table>
Duration Example

Assume you have a par value bond with 9% coupon, 9% YTM, and 15 years to maturity. Calculate Macaulay’s Duration.

\[
\text{Mac. duration} = \frac{\left(\frac{1+.09}{2}\right)}{.09} \left[ 1 - \frac{1}{\left(1+.09\right)^{30}} \right] = 8.51 \text{ years}
\]

Assume you have a bond with 9% coupon, 8% YTM, and 15 years to maturity. Calculate Macaulay’s Duration.

\[
\text{Mac. Dur.} = \frac{1+.08}{2} \left(\frac{1+.08}{2}\right) + 15(.09-.08) \left(\frac{1+.08}{2}\right)^{30} - 1 = 8.78 \text{ years}
\]
Price Change & Duration

To compute the percentage change in a bond’s price using Macaulay Duration:
\[ \% \Delta \text{ in bond price} \approx \text{MD} \times \frac{\text{Change in YTM}}{1 + \frac{\text{YTM}}{2}} \]

To compute the Modified Duration:
\[ \text{Modified duration} = \frac{\text{Macaulay duration}}{1 + \frac{\text{YTM}}{2}} \]

To compute the percentage change in a bond’s price using Modified Duration:
\[ \% \Delta \text{ in bond price} \approx \text{Modified Duration} \times \text{Change in YTM} \]
Calculating Price Change

Assume a bond with Macaulay’s duration of 8.5 years, with the YTM at 9%, but estimated the YTM will go to 11%, calculate the percentage change in bond price and the new bond price.

\[
\% \Delta \text{ in bond price} \approx 8.5 \times \left( \frac{.09 - .11}{1 + \frac{.09}{2}} \right) = -16.27\%
\]

Change in bond price, assuming bond was originally at par:

Approx. new price = $1,000 + (-16.27% x $1,000) = $837.30
Price Change & Duration

Assume you have a bond with Macaulay’s duration of 8.5 years and YTM of 9%, calculate the modified duration.

\[
\text{Modified duration} = \frac{8.5}{1 + \frac{.09}{2}} = 8.134 \text{ years}
\]

Using the bond above with modified duration of 8.134 years and a change in yields from 9% to 11%, calculate the percentage change in bond price.

\[
\% \Delta \text{ in bond price} \approx 8.134 \times (.09 - .11) = -16.27\%
\]

Note this is the same percentage change as computed previously.
Duration

The key to bond portfolio management

- **Properties:**
  - Longer maturity, longer duration
  - Duration increases at a decreasing rate
  - Higher coupon, shorter duration
  - Higher yield, shorter duration

- **Zero coupon bond:** duration = maturity
Figure 10.3: Bond duration and maturity

![Graph showing bond duration and maturity for different coupon rates, with 0%, 5%, 10%, and 15% coupon rates depicted as curves. The x-axis represents bond maturity in years, ranging from 0 to 30, and the y-axis represents bond duration in years, ranging from 0 to 12.](image)
Immunization

Target date hedging:
• Dedicated portfolios
• Reinvestment rate risk vs price risk
• Duration matching
• Rebalancing
• Dynamic immunization
Figure 10.5 Bond Price and Reinvestment Rate Risk

The graph illustrates the portfolio value over time for different yield scenarios: 6%, 8%, and 10% yield. The x-axis represents time in years, ranging from 0 to 6, while the y-axis represents the portfolio value in millions of dollars, ranging from 60 to 130.
Example of Target Date Hedging

Assume you are setting up a target portfolio. You need $1,470 in five years. You can choose a 7.9% coupon bond with 5 years to maturity or a 7.9% coupon bond with 6 years to maturity and a 5-year duration. The YTM is now 7.9%. Which do you choose?

Solution:

To compare, calculate the total wealth in five years:

If interest rates do not change the total wealth of the 5-year bond in 5 years is $1,473.14 (in five years you receive $1,000 plus 5 coupon payments of $79 each, which earn interest at 7.9%)

If interest rates change to 6%:

The 5-year bond will earn total wealth of $1,452.82 ($1,000 plus 5 coupon payments of $79, which earn interest at 6%)

The 6-year bond (MD = 5 years) will earn total wealth of $1,471.00 (5 coupon payments of $79 compounded at 6%, plus a bond with 1-year to maturity worth $1,018.18)

The duration matched bond protected your portfolio.
Problem 10-9

CIR Inc. has 7% coupon bonds on the market that have 11 years left to maturity. If the YTM on these bonds is 8.5%, what is the current bond price?

Solution:

\[
\text{Bond price} = \frac{70}{.085} \left[ 1 - \left( \frac{1}{1 + .085^2} \right)^{22} \right] + \frac{1000}{(1 + .085^2)^{22}} = \$894.16
\]
Problem 10-10

Trincor Company bonds have coupon rate of 10.25%, 14 years to maturity, and a current price of $1,225. What is the YTM? The current yield?

Solution:

\[
\frac{102.50}{YTM} \left[ 1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{28}} \right] + \frac{1000}{\left(1 + \frac{YTM}{2}\right)^{28}} = 1225
\]

\[
YTM = 3.805\% \times 2 = 7.61\%
\]

Current yield = $102.50 / $1,225 = 8.37\%
Problem 10-22

XYZ Company has a 9% callable bond outstanding on the market with 12 years to maturity, call protection for the next 5 years, and a call premium of $100. What is the YTC for this bond if the current price is 120% of par value?

Solution:

\[
\frac{1,200}{YTC} = \frac{90}{YTC} \left[ 1 - \frac{1}{\left(1 + \frac{YTC}{2}\right)^{10}} \right] + \frac{1100}{\left(1 + \frac{YTC}{2}\right)^{10}}
\]

\[
YTC = 3.024\% \times 2 = 6.05\%
\]

[see next slide for additional information]
Problem 10-22 (cont’d)

Since the bond sells at a premium to par, you know the coupon is greater than the yield. If interest rates stay at current levels, the bond issuer will likely call the bonds to refinance at the earliest possible time.

What is the YTM, with zero call premium?

Solution:

\[
\$1,200 = \frac{90}{YTM} \left[ 1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{24}} \right] + \frac{1000}{\left(1 + \frac{YTM}{2}\right)^{22}}
\]

\[
YTM = 3.283\% \times 2 = 6.57\%
\]

[see next slide for additional information]
Problem 10-22 (cont’d)

What would be the break-even call premium? (If interest rates don’t change, at what level would the call premium have to be to not call the bonds?)

Solution:

\[
$1,200 = \frac{90}{0.03283} \left[ 1 - \frac{1}{\left(1 + 0.03283 \right)^{10}} \right] + \frac{1000 + X}{\left(1 + 0.03283 \right)^{10}}
\]

\[X = $134.91\]

The bond will not be called if the call premium is greater than $134.91.
Problem 10-23

What is the Macaulay duration of an 8% coupon bond with 3 years to maturity and a current price of $937.10? What is the modified duration?

Solution:
First calculate the yield:

\[
\frac{80}{\text{YTM}} \left[ 1 - \frac{1}{(1 + \text{YTM}/2)^6} \right] + \frac{1000}{(1 + \text{YTM}/2)^6} = 937
\]

\[
\text{YTM} = 5.249 \times 2 = 10.498\%
\]
Problem 10-23 (cont’d)

Now calculate the Macaulay’s duration.

Solution:

\[
\text{Mac. Dur.} = \frac{1 + \frac{.10498}{2}}{.10498} - \frac{\left(1 + \frac{.10498}{2}\right)^3 (0.08 - 0.10498)}{.10498 + 0.08 \left[\left(1 + \frac{.10498}{2}\right)^6 - 1\right]}
\]

Mac. Duration = 2.715 years

Modified duration

\[= 2.715 / (1 + .10498/2) = 2.58 \text{ years}\]
Chapter 4 (Fabozzi) Bond Volatilities

Source: Bond Markets Analysis and Strategies, Seventh Edition
By Frank Fabozzi
Review of the Price-Yield Relationship for Option-Free Bonds

- As illustrated in Exhibit 4-1 (See Overhead 4-5):
  - An increase in the required yield decreases the present value of its expected cash flows and therefore decreases the bond’s price.
  - An decrease in the required yield increases the present value of its expected cash flows and therefore increases the bond’s price.

- As shown in Exhibit 4-2 (See Overhead 4-6):
  - The price-yield relation is not linear.
  - The shape of the price-yield relationship for any option-free bond is referred to as a convex relationship.
<table>
<thead>
<tr>
<th>Required Yield (%)</th>
<th>Price at Required Yield (coupon/maturity in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.00</td>
<td>112.7953 138.5946 100.0000 100.0000 74.4094 22.8107</td>
</tr>
<tr>
<td>7.00</td>
<td>108.3166 123.4556  95.8417  88.2722  70.8919 17.9053</td>
</tr>
<tr>
<td>8.00</td>
<td>104.0554 110.7410  91.8891  78.5178  67.5564 14.0713</td>
</tr>
<tr>
<td>8.50</td>
<td>102.0027 105.1482  89.9864  74.2587  65.9537 12.4795</td>
</tr>
<tr>
<td>8.90</td>
<td>100.3966 100.9961  88.4983  71.1105  64.7017 11.3391</td>
</tr>
<tr>
<td>8.99</td>
<td>100.0395 100.0988  88.1676  70.4318  64.4236 11.0975</td>
</tr>
<tr>
<td>9.00</td>
<td>100.0000 100.0000  88.1309  70.3570  64.3928 11.0710</td>
</tr>
<tr>
<td>9.01</td>
<td>99.9604 99.9013  88.0943  70.2824  64.3620 11.0445</td>
</tr>
<tr>
<td>9.10</td>
<td>99.6053 99.0199  87.7654  69.6164  64.0855 10.8093</td>
</tr>
<tr>
<td>9.50</td>
<td>98.0459 95.2539  86.3214  66.7773  62.8723  9.8242</td>
</tr>
<tr>
<td>10.00</td>
<td>96.1391 90.8720  84.5565  63.4881  61.3913  8.7204</td>
</tr>
<tr>
<td>11.00</td>
<td>92.4624 83.0685  81.1559  57.6712  58.5431  6.8767</td>
</tr>
<tr>
<td>12.00</td>
<td>88.9599 76.3572  77.9197  52.7144  55.8395  5.4288</td>
</tr>
</tbody>
</table>
Exhibit 4-2
Shape of Price-Yield Relationship for an Option-Free Bond

Maximum Price

Price

Yield
Price Volatility Characteristics of Option-Free Bonds

- There are *four properties* concerning the price volatility of an option-free bond:
  
  (i) Although the prices of all option-free bonds move in the opposite direction from the change in yield required, the percentage price change is not the same for all bonds.

  (ii) For very small changes in the yield required, the percentage price change for a given bond is roughly the same, whether the yield required increases or decreases.

  (iii) For large changes in the required yield, the percentage price change is not the same for an increase in the required yield as it is for a decrease in the required yield.

  (iv) For a given large change in basis points, the percentage price increase is greater than the percentage price decrease.

- An explanation for these four properties of bond price volatility lies in the *convex shape* of the price-yield relationship.
Price Volatility Characteristics of Option-Free Bonds (continued)

- Characteristics of a Bond that Affect its Price Volatility

There are two characteristics of an option-free bond that determine its price volatility: coupon and term to maturity.

1) First, for a given term to maturity and initial yield, the price volatility of a bond is greater, the lower the coupon rate.

   ✓ This characteristic can be seen by comparing the 9%, 6%, and zero-coupon bonds with the same maturity.

2) Second, for a given coupon rate and initial yield, the longer the term to maturity, the greater the price volatility.

   ✓ This can be seen in Exhibit 4-3 (See Overhead 4-9) by comparing the five-year bonds with the 25-year bonds with the same coupon.
### EXHIBIT 4-3  Instantaneous Percentage Price Change for Six Hypothetical Bonds

Six hypothetical bonds, priced initially to yield 9%:

- **9% coupon, 5 years to maturity, price = 100.0000**
- **9% coupon, 25 years to maturity, price = 100.0000**
- **6% coupon, 5 years to maturity, price = 88.1309**
- **6% coupon, 25 years to maturity, price = 70.3570**
- **0% coupon, 5 years to maturity, price = 64.3928**
- **0% coupon, 25 years to maturity, price = 11.0710**

<table>
<thead>
<tr>
<th>Yield (%) Change to:</th>
<th>Change in Basis Points</th>
<th>Percentage Price Change (coupon/maturity in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>9% / 5</td>
</tr>
<tr>
<td>6.00</td>
<td>-300</td>
<td>12.80</td>
</tr>
<tr>
<td>7.00</td>
<td>-200</td>
<td>8.32</td>
</tr>
<tr>
<td>8.00</td>
<td>-100</td>
<td>4.06</td>
</tr>
<tr>
<td>8.50</td>
<td>-50</td>
<td>2.00</td>
</tr>
<tr>
<td>8.90</td>
<td>-10</td>
<td>0.40</td>
</tr>
<tr>
<td>8.99</td>
<td>-1</td>
<td>0.04</td>
</tr>
<tr>
<td>9.01</td>
<td>1</td>
<td>-0.04</td>
</tr>
<tr>
<td>9.10</td>
<td>10</td>
<td>-0.39</td>
</tr>
<tr>
<td>9.50</td>
<td>50</td>
<td>-1.95</td>
</tr>
<tr>
<td>10.00</td>
<td>100</td>
<td>-3.86</td>
</tr>
<tr>
<td>11.00</td>
<td>200</td>
<td>-7.54</td>
</tr>
<tr>
<td>12.00</td>
<td>300</td>
<td>-11.04</td>
</tr>
</tbody>
</table>
Price Volatility Characteristics
of Option-Free Bonds (continued)

- Effects of Yield to Maturity

- We cannot ignore the fact that credit considerations cause different bonds to trade at different yields, even if they have the same coupon and maturity.

- Holding other factors constant, the higher the yield to maturity at which a bond trades, the lower the price volatility.

- To see this, compare the 9% 25-year bond trading at various yield levels in Exhibit 4-4 (See Overhead 4-11).

- The 1st column of Exhibit 4-4 shows the yield level the bond is trading at, and the 2nd column gives the initial price.

- The 3rd column of Exhibit 4-4 indicates the bond’s price if yields change by 100 basis points.

- The 4th and 5th columns of Exhibit 4-4 show the dollar price decline and the percentage price decline.

- The 4th and 5th columns of Exhibit 4-4 also show: higher the initial yield, the lower the price volatility.

- An implication of this is that for a given change in yields, price volatility is greater (lower) when yield levels in the market are low (high).
<table>
<thead>
<tr>
<th>Yield Level (%)</th>
<th>Initial Price</th>
<th>New Price a</th>
<th>Price Decline</th>
<th>Percent Decline</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$123.46</td>
<td>$110.74</td>
<td>$12.72</td>
<td>10.30</td>
</tr>
<tr>
<td>8</td>
<td>110.74</td>
<td>100.00</td>
<td>10.74</td>
<td>9.70</td>
</tr>
<tr>
<td>9</td>
<td>100.00</td>
<td>90.87</td>
<td>9.13</td>
<td>9.13</td>
</tr>
<tr>
<td>10</td>
<td>90.87</td>
<td>83.07</td>
<td>7.80</td>
<td>8.58</td>
</tr>
<tr>
<td>11</td>
<td>83.07</td>
<td>76.36</td>
<td>6.71</td>
<td>8.08</td>
</tr>
<tr>
<td>12</td>
<td>76.36</td>
<td>70.55</td>
<td>5.81</td>
<td>7.61</td>
</tr>
<tr>
<td>13</td>
<td>70.55</td>
<td>65.50</td>
<td>5.05</td>
<td>7.16</td>
</tr>
<tr>
<td>14</td>
<td>65.50</td>
<td>61.08</td>
<td>4.42</td>
<td>6.75</td>
</tr>
<tr>
<td>8</td>
<td>110.74</td>
<td>100.00</td>
<td>10.74</td>
<td>9.70</td>
</tr>
</tbody>
</table>

a As a result of a 100-basis-point increase in yield.
Measures of Bond Price Volatility

- Money managers, arbitrageurs, and traders need to have a way to measure a bond’s price volatility to implement hedging and trading strategies.

- Three measures that are commonly employed:
  1) price value of a basis point
  2) yield value of a price change
  3) duration
Measures of Bond Price Volatility (continued)

- Price Value of a Basis Point [a basis point = (1/100) of 1%]
  - The price value of a basis point, also referred to as the dollar value of an 01, is the change in the price of the bond if the required yield changes by 1 basis point.
  - Note that this measure of price volatility indicates dollar price volatility as opposed to percentage price volatility (price change as a percent of the initial price).
  - Typically, the price value of a basis point is expressed as the absolute value of the change in price.
  - Price volatility is the same for an increase or a decrease of 1 basis point in required yield.
  - Because this measure of price volatility is in terms of dollar price change, dividing the price value of a basis point by the initial price gives the percentage price change for a 1-basis-point change in yield.
Measures of Bond Price Volatility (continued)

- **Yield Value of a Price Change**
  - Another measure of the price volatility of a bond used by investors is the change in the yield for a specified price change.
  - This is estimated by first calculating the bond’s yield to maturity if the bond’s price is decreased by, say, \( X \) dollars.
  - Then the difference between the initial yield and the new yield is the yield value of an \( X \) dollar price change.
  - Price volatility is the same for an increase or a decrease of 1 basis point in required yield.
  - The smaller this value, the greater the dollar price volatility, because it would take a smaller change in yield to produce a price change of \( X \) dollars.
Measures of Bond Price Volatility (continued)

Duration

The Macaulay duration is one measure of the approximate change in price for a small change in yield:

$$\text{Macaulay duration} = \frac{\frac{1C}{(1+y)^1} + \frac{2C}{(1+y)^2} + \ldots + \frac{nC}{(1+y)^n} + \frac{nM}{(1+y)^n}}{P}$$

where $P = \text{price of the bond}$

$C = \text{semiannual coupon interest (in dollars)}$

$y = \text{one-half the yield to maturity or required yield}$

$n = \text{number of semiannual periods (number of years times 2)}$

$M = \text{maturity value (in dollars)}$
Measures of Bond Price Volatility (continued)

- **Duration**

  - Investors refer to the ratio of Macaulay duration to $1 + y$ as the modified duration. The equation is:

    \[ \text{modified duration} = \frac{\text{Macaulay duration}}{1 + y} \]

    where $y =$ *one-half the yield to maturity or required yield*.

  - The modified duration is related to the approximate percentage change in price for a given change in yield as given by:

    \[ \frac{dP}{dy} \frac{1}{P} = -\text{modified duration} \]

    where $dP =$ *change in price*, $dy =$ *change in yield*, $P =$ *price of the bond*. 

Duration

Because for all option-free bonds modified duration is positive, the modified duration equation, \((dP/dy)(1/P)\), states that there is an inverse relationship between modified duration and the approximate percentage change in price for a given yield change.

This is to be expected from the fundamental principle that bond prices move in the opposite direction of the change in interest rates.

Exhibit 4-5 (see Overhead 4-18) and Exhibit 4-6 (see Overhead 4-19) show the computation of the Macaulay duration and modified duration of two five-year coupon bonds.

- The durations computed in these exhibits are in terms of duration per period.
<table>
<thead>
<tr>
<th>Period, t</th>
<th>Cash Flow</th>
<th>PV of $1 at 4.5%</th>
<th>PV of CF</th>
<th>t × PV CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4.50</td>
<td>0.956937</td>
<td>4.306220</td>
<td>4.30622</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>0.915729</td>
<td>4.120785</td>
<td>8.24156</td>
</tr>
<tr>
<td>3</td>
<td>4.50</td>
<td>0.876296</td>
<td>3.943335</td>
<td>11.83000</td>
</tr>
<tr>
<td>4</td>
<td>4.50</td>
<td>0.838561</td>
<td>3.773526</td>
<td>15.09410</td>
</tr>
<tr>
<td>5</td>
<td>4.50</td>
<td>0.802451</td>
<td>3.611030</td>
<td>18.05514</td>
</tr>
<tr>
<td>6</td>
<td>4.50</td>
<td>0.767895</td>
<td>3.455531</td>
<td>20.73318</td>
</tr>
<tr>
<td>7</td>
<td>4.50</td>
<td>0.734828</td>
<td>3.306728</td>
<td>23.14709</td>
</tr>
<tr>
<td>8</td>
<td>4.50</td>
<td>0.703185</td>
<td>3.164333</td>
<td>25.31466</td>
</tr>
<tr>
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<td>4.50</td>
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<tr>
<td>10</td>
<td>$104.50</td>
<td>0.643927</td>
<td>67.290443</td>
<td>672.90442</td>
</tr>
</tbody>
</table>

**EXHIBIT 4-5**

**Calculation of Macaulay Duration and Modified Duration for 5-Year 9% Bond Selling to Yield 9%**

**Coupon rate:** 9.00%  **Term (years):** 5  **Initial yield:** 9.00%
### Calculation of Macaulay Duration and Modified Duration for 5-Year 6% Bond Selling to Yield 9%

**Coupon rate:** 6.00%  |  **Term (years):** 5  |  **Initial yield:** 9.00%

<table>
<thead>
<tr>
<th>Period, t</th>
<th>Cash Flow</th>
<th>PV of $1 at 4.5%</th>
<th>PV of CF</th>
<th>t × PVCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.00</td>
<td>0.956937</td>
<td>2.870813</td>
<td>2.870813</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>0.915729</td>
<td>2.747190</td>
<td>5.49437</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>0.876296</td>
<td>2.628890</td>
<td>7.88666</td>
</tr>
<tr>
<td>4</td>
<td>3.00</td>
<td>0.838561</td>
<td>2.515684</td>
<td>10.06273</td>
</tr>
<tr>
<td>5</td>
<td>3.00</td>
<td>0.802451</td>
<td>2.407353</td>
<td>12.03676</td>
</tr>
<tr>
<td>6</td>
<td>3.00</td>
<td>0.767895</td>
<td>2.303687</td>
<td>13.82212</td>
</tr>
<tr>
<td>7</td>
<td>3.00</td>
<td>0.734828</td>
<td>2.204485</td>
<td>15.43139</td>
</tr>
<tr>
<td>8</td>
<td>3.00</td>
<td>0.703185</td>
<td>2.109555</td>
<td>16.87644</td>
</tr>
<tr>
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<td>3.00</td>
<td>0.672904</td>
<td>2.018713</td>
<td>18.16841</td>
</tr>
<tr>
<td>10</td>
<td>103.00</td>
<td>0.643927</td>
<td>66.324551</td>
<td>663.24551</td>
</tr>
</tbody>
</table>

**Total**  | **88.130923** | **765.89520**   |
Measures of Bond Price Volatility (continued)

**Duration**

- In general, if the cash flows occur \( m \) times per year, the durations are adjusted by dividing by \( m \), that is,

\[
\text{duration in years} = \frac{\text{duration in } m \text{ periods per year}}{m}
\]

- We can derive an alternative formula that does not have the extensive calculation of the Macaulay duration and the modified duration.

This is done by rewriting the price of a bond in terms of its present value of an annuity and the present value of the par value, taking the first derivative, and dividing by \( P \):

\[
\text{modified duration} = \frac{C}{y^2} \left[ 1 - \frac{1}{(1+y)^n} \right] + \frac{n(100 - C/y)}{(1+y)^{n+1}}
\]

where the price is expressed as a percentage of par value.
Measures of Bond Price Volatility (continued)

❖ Properties of Duration

➢ The modified duration and Macaulay duration of a coupon bond are less than the maturity.

➢ The Macaulay duration of a zero-coupon bond equals its maturity; but a zero-coupon bond’s modified duration is less than its maturity.

➢ Lower coupon rates generally have greater Macaulay and modified bond durations.

➢ There is a consistency between the properties of bond price volatility and the properties of modified duration.

▪ For example, a property of modified duration is that, ceteris paribus, a bond with a longer maturity will have a greater modified duration.

▪ Generally, a lower coupon rate implies a greater modified duration and a greater price volatility.
Approximating the Percentage Price Change

The below equation can be used to approximate the percentage price change for a given change in required yield:

\[ \frac{dP}{P} = -(\text{modified duration})(dy) \]

where \( dP = \text{change in price}, \ P = \text{price of the bond} \) and \( dy = \text{change in yield} \).

Suppose that the yield on any bond changes by 100 basis points. Then, substituting 100 basis points (0.01) for \( dy \) into the above equation, we get:

\[ \frac{dP}{P} = -(\text{modified duration})(0.01) = -(\text{modified duration})(1\%) \]

Thus, modified duration can be interpreted as the approximate percentage change in price for a 100-basis-point change in yield.
Approximating the Dollar Price Change

Modified duration is a proxy for the percentage change in price. Investors also like to know the dollar price volatility of a bond.

For small changes in the required yield, the below equation does a good job in estimating the change in price yield:

\[ dP = -(\text{dollar duration})(dy) \]

where \( dP = \text{change in price} \) and \( dy = \text{change in yield} \).

When there are large movements in the required yield, dollar duration or modified duration is not adequate to approximate the price reaction.

Duration will overestimate the price change when the required yield rises, thereby underestimating the new price.

When the required yield falls, duration will underestimate the price change and thereby underestimate the new price.
Measures of Bond Price Volatility (continued)

- **Spread Duration**
  - Market participants compute a measure called *spread duration*.
  - This measure is used in two ways: for fixed bonds and floating-rate bonds.
  - A spread duration for a fixed-rate security is interpreted as the approximate change in the price of a fixed-rate bond for a 100-basis-point change in the spread bond.

- **Portfolio Duration**
  - Thus far we have looked at the duration of an individual bond.
  - The duration of a portfolio is simply the weighted average duration of the bonds in the portfolios.
  - Portfolio managers look at their interest rate exposure to a particular issue in terms of its contribution to portfolio duration.
  - This measure is found by multiplying the weight of the issue in the portfolio by the duration of the individual issue given as:
    \[
    \text{contribution to portfolio duration} = \text{weight of issue in portfolio} \times \text{duration of issue}.
    \]
Measures of Bond Price Volatility (continued)

- Portfolio Duration
  - Portfolio managers look at portfolio duration for sectors of the bond market.
  - The procedure is the same for computing the contribution to portfolio duration of a sector as it is for computing the contribution to portfolio duration of an individual issue.
  - A spread duration for a portfolio of fixed-rate bonds can also be computed.
    - In this case, the portfolio duration is divided into two durations.
    - The first is the duration of the portfolio with respect to changes in the level of Treasury rates.
    - The second is the spread duration.
    - The above is illustrated in Exhibit 4-7 (see Overhead 4-26) for a portfolio using six sectors.
Calculation of Duration and Contribution to Portfolio Duration for a Asset Allocation to Sectors of the Lehman Brothers U.S. Aggregate Index: October 26, 2007

<table>
<thead>
<tr>
<th>Sector</th>
<th>Portfolio Weight</th>
<th>Sector Duration</th>
<th>Contribution to Portfolio Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury</td>
<td>0.000</td>
<td>4.95</td>
<td>0.00</td>
</tr>
<tr>
<td>Agency</td>
<td>0.121</td>
<td>3.44</td>
<td>0.42</td>
</tr>
<tr>
<td>Mortgages</td>
<td>0.449</td>
<td>3.58</td>
<td>1.61</td>
</tr>
<tr>
<td>Commercial Mortgage-Backed Securities</td>
<td>0.139</td>
<td>5.04</td>
<td>0.70</td>
</tr>
<tr>
<td>Asset-Backed Securities</td>
<td>0.017</td>
<td>3.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Credit</td>
<td>0.274</td>
<td>6.35</td>
<td>1.74</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td></td>
<td>4.52</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2.000</strong></td>
<td><strong>26.52</strong></td>
<td><strong>9.04</strong></td>
</tr>
</tbody>
</table>
Measures of Bond Price Volatility (continued)

- Portfolio Duration
  - Exhibit 4-8 (see Overhead 4-28) shows the size of each sector in the Lehman Brothers U.S. Aggregate Index.
    - You can see the importance of each sector.
    - The duration for each sector is shown in the third column and uses the same values as in Exhibit 4-7.
    - The duration for the portfolio is shown in the last row of the third column.
  - The calculation of the spread duration for the recommended portfolio allocation and the Lehman Brothers U.S. Aggregate Index are shown in Exhibit 4-9 (see Overhead 4-29) and Exhibit 4-10 (see Overhead 4-30), respectively.
    - While the portfolio and the index have the same duration, the spread duration for the recommended portfolio is 4.60 vs. 3.49 for the index.
    - The larger spread duration for the recommended portfolio is expected given the greater allocation to non-Treasury sectors.
<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight in Index</th>
<th>Sector Duration</th>
<th>Contribution to Index Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury</td>
<td>0.230</td>
<td>4.95</td>
<td>1.14</td>
</tr>
<tr>
<td>Agency</td>
<td>0.105</td>
<td>3.44</td>
<td>0.36</td>
</tr>
<tr>
<td>Mortgages</td>
<td>0.381</td>
<td>3.58</td>
<td>1.36</td>
</tr>
<tr>
<td>Commercial Mortgage-Backed Securities</td>
<td>0.056</td>
<td>5.04</td>
<td>0.28</td>
</tr>
<tr>
<td>Asset-Backed Securities</td>
<td>0.010</td>
<td>3.16</td>
<td>0.03</td>
</tr>
<tr>
<td>Credit</td>
<td>0.219</td>
<td>6.35</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td></td>
<td>4.56</td>
</tr>
<tr>
<td>Total</td>
<td>2.001</td>
<td>26.52</td>
<td>9.12</td>
</tr>
<tr>
<td>Sector</td>
<td>Portfolio Weight</td>
<td>Sector Spread Duration</td>
<td>Contribution to Portfolio Spread Duration</td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>------------------</td>
<td>------------------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>Treasury</td>
<td>0.000</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Agency</td>
<td>0.121</td>
<td>3.53</td>
<td>0.43</td>
</tr>
<tr>
<td>Mortgages</td>
<td>0.449</td>
<td>3.62</td>
<td>1.63</td>
</tr>
<tr>
<td>Commercial Mortgage-Backed Securities</td>
<td>0.139</td>
<td>5.04</td>
<td>0.70</td>
</tr>
<tr>
<td>Asset-Backed Securities</td>
<td>0.017</td>
<td>3.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Credit</td>
<td>0.274</td>
<td>6.35</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td><strong>1.000</strong></td>
<td><strong>4.60</strong></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td><strong>2.000</strong></td>
<td><strong>21.88</strong></td>
<td><strong>9.20</strong></td>
</tr>
</tbody>
</table>
### EXHIBIT 4-10
Calculation of Spread Duration and Contribution to the Lehman Brothers Aggregate Index Spread Duration: October 26, 2007

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight in Index</th>
<th>Sector Spread Duration</th>
<th>Contribution to Index Spread Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury</td>
<td>0.230</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Agency</td>
<td>0.105</td>
<td>3.53</td>
<td>0.37</td>
</tr>
<tr>
<td>Mortgages</td>
<td>0.381</td>
<td>3.62</td>
<td>1.38</td>
</tr>
<tr>
<td>Commercial Mortgage-Backed Securities</td>
<td>0.056</td>
<td>5.04</td>
<td>0.28</td>
</tr>
<tr>
<td>Asset-Backed Securities</td>
<td>0.010</td>
<td>3.16</td>
<td>0.03</td>
</tr>
<tr>
<td>Credit</td>
<td>0.219</td>
<td>6.53</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.000</td>
<td>3.49</td>
</tr>
<tr>
<td>Total</td>
<td>2.001</td>
<td>21.88</td>
<td>6.98</td>
</tr>
</tbody>
</table>
Exhibit 4-11
Measures of Bond Price Volatility and Their Relationships to One Another

Notation:

\[ D = \text{Macaulay duration} \]

\[ D^* = \text{modified duration} \]

\[ PVBP = \text{price value of a basis point} \]

\[ y = \text{yield to maturity in decimal form} \]

\[ Y = \text{yield to maturity in percentage terms} \ (Y = 100 \times y) \]

\[ P = \text{price of bond} \]

\[ m = \text{number of coupons per year} \]
Exhibit 4-11

Measures of Bond Price Volatility and Their Relationships to One Another (continued)

Relationships:

\[ D^* = \frac{D}{1 + \frac{y}{m}} \quad \rightarrow \quad \text{by definition} \]

\[ \frac{\Delta P}{\Delta y} \approx D \quad \rightarrow \quad \text{to a close approximation for a small } \Delta y \]

\[ \Delta P/\Delta Y \approx \text{slope of price-yield curve} \quad \rightarrow \quad \text{to a close approximation for a small } \Delta y \]

\[ PVBP \approx \frac{D^* \times P}{10,000} \quad \rightarrow \quad \text{to a close approximation} \]

For Bonds at or near par:

\[ PVBP = \frac{D^*}{100} \quad \rightarrow \quad \text{to a close approximation} \]

\[ D^* = \frac{\Delta P}{\Delta Y} \quad \rightarrow \quad \text{to a close approximation for a small } \Delta y \]
Convexity

- Because all the duration measures are only approximations for small changes in yield, they do not capture the effect of the convexity of a bond on its price performance when yields change by more than a small amount.
- The duration measure can be supplemented with an additional measure to capture the curvature or convexity of a bond.
- In Exhibit 4-12 (see Overhead 4-34), a tangent line is drawn to the price–yield relationship at yield $y^*$. The tangent shows the rate of change of price with respect to a change in interest rates at that point (yield level).
Exhibit 4-12
Line Tangent to the Price-Yield Relationship

Actual Price

Tangent Line at $y^*$
(estimated price)

Price

$y^*$

$p^*$

Yield
Convexity (continued)

- If we draw a vertical line from any yield (on the horizontal axis), as in Exhibit 4-13 (see Overhead 4-36), the distance between the horizontal axis and the tangent line represents the price approximated by using duration starting with the initial yield \( y^* \).
- The approximation will always understate the actual price.
- This agrees with what we demonstrated earlier about the relationship between duration (and the tangent line) and the approximate price change.
- When yields decrease, the estimated price change will be less than the actual price change, thereby underestimating the actual price.
- On the other hand, when yields increase, the estimated price change will be greater than the actual price change, resulting in an underestimate of the actual price.
Exhibit 4-13. Price Approximation Using Duration

- Actual Price
- Error in Estimating Price Based only on Duration
- Tangent Line at $y^*$ (estimated price)

<table>
<thead>
<tr>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*$</td>
<td>$y_1$</td>
</tr>
<tr>
<td></td>
<td>$y_2$</td>
</tr>
<tr>
<td></td>
<td>$y^*$</td>
</tr>
<tr>
<td></td>
<td>$y_3$</td>
</tr>
<tr>
<td></td>
<td>$y_4$</td>
</tr>
</tbody>
</table>
Convexity (continued)

- Measuring Convexity
  - Duration (modified or dollar) attempts to estimate a convex relationship with a straight line (the tangent line).
  - The dollar convexity measure of the bond:
    
    \[
    \text{dollar convexity measure} = \frac{d^2P}{dy^2}
    \]
  - The approximate change in price due to convexity is:
    
    \[
    dP = (\text{dollar convexity measure})(dy)^2
    \]
  - The percentage change in the price of the bond due to convexity or the convexity measure is:
    
    \[
    \text{convexity measure} = \frac{d^2P}{dy^2} \frac{1}{P}
    \]
    - The percentage price change due to convexity is:
      
      \[
      \frac{dP}{P} = \frac{1}{2} \left(\text{convexity measure}\right)(dy)^2
      \]
Convexity (continued)

- Measuring Convexity
  - Exhibit 4-14 (see Overhead 4-39) and Exhibit 4-15 (see Overhead 4-40) demonstrate how to calculate the second derivative, annualized dollar convexity measure, and annualized convexity measure for the two five-year coupon bonds.
  - The convexity measure is in terms of periods squared.
  - In general, if the cash flows occur \( m \) times per year, convexity is adjusted to an annual figure as follows:

\[
\text{convexity measure in year} = \frac{\text{convexity measure in } m \text{ period per year}}{m^2}
\]
## Calculation of Convexity Measure and Dollar Convexity Measure for Five-Year 9% Bond Selling to Yield 9%

<table>
<thead>
<tr>
<th>Period, t</th>
<th>Cash Flow</th>
<th>$1/(1.045)^{t+2}$</th>
<th>$t(t+1)CF$</th>
<th>$\frac{t(t+1)CF}{(1.045)^{t+2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.50</td>
<td>0.876296</td>
<td>9</td>
<td>7.886</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>0.838561</td>
<td>27</td>
<td>22.641</td>
</tr>
<tr>
<td>3</td>
<td>4.50</td>
<td>0.802451</td>
<td>54</td>
<td>43.332</td>
</tr>
<tr>
<td>4</td>
<td>4.50</td>
<td>0.767895</td>
<td>90</td>
<td>69.110</td>
</tr>
<tr>
<td>5</td>
<td>4.50</td>
<td>0.734828</td>
<td>135</td>
<td>99.201</td>
</tr>
<tr>
<td>6</td>
<td>4.50</td>
<td>0.703185</td>
<td>189</td>
<td>132.901</td>
</tr>
<tr>
<td>7</td>
<td>4.50</td>
<td>0.672904</td>
<td>252</td>
<td>169.571</td>
</tr>
<tr>
<td>8</td>
<td>4.50</td>
<td>0.643927</td>
<td>324</td>
<td>208.632</td>
</tr>
<tr>
<td>9</td>
<td>4.50</td>
<td>0.616198</td>
<td>405</td>
<td>249.560</td>
</tr>
<tr>
<td>10</td>
<td>104.50</td>
<td>0.589663</td>
<td>11,495</td>
<td>6,778.186</td>
</tr>
</tbody>
</table>

**Coupon rate:** 9.00%  **Term (years):** 5  **Initial yield:** 9.00%  **Price:** 100
### Calculation of Convexity Measure and Dollar Convexity Measure for Five-Year 6% Bond Selling to Yield 9%

**Coupon rate:** 6.00%  
**Term (years):** 5  
**Initial yield:** 9.00%  
**Price:** 88.1309

<table>
<thead>
<tr>
<th>Period, t</th>
<th>Cash Flow</th>
<th>(1/(1.045)^{t+2})</th>
<th>(t(t + 1)CF)</th>
<th>(\frac{t(t + 1)CF}{(1.045)^{t+2}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00</td>
<td>0.876296</td>
<td>6</td>
<td>5.257</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>0.838561</td>
<td>18</td>
<td>15.094</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>0.802451</td>
<td>36</td>
<td>28.888</td>
</tr>
<tr>
<td>4</td>
<td>3.00</td>
<td>0.767895</td>
<td>60</td>
<td>46.073</td>
</tr>
<tr>
<td>5</td>
<td>3.00</td>
<td>0.734828</td>
<td>90</td>
<td>66.134</td>
</tr>
<tr>
<td>6</td>
<td>3.00</td>
<td>0.703185</td>
<td>126</td>
<td>88.601</td>
</tr>
<tr>
<td>7</td>
<td>3.00</td>
<td>0.672904</td>
<td>168</td>
<td>113.047</td>
</tr>
<tr>
<td>8</td>
<td>3.00</td>
<td>0.643927</td>
<td>216</td>
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<tr>
<td>9</td>
<td>3.00</td>
<td>0.616198</td>
<td>270</td>
<td>166.373</td>
</tr>
<tr>
<td>10</td>
<td>103.00</td>
<td>0.589663</td>
<td>11,330</td>
<td>6,680.891</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12,320</td>
<td>7,349.446</td>
</tr>
</tbody>
</table>
Convexity (continued)

- Approximating Percentage Price Change Using Duration and Convexity Measures
  - Using duration and convexity measures together gives a better approximation of the actual price change for a large movement in the required yield.

- Some Notes on Convexity
  - Three points to know for a bond’s convexity and convexity measure.
    i. Convexity refers to the general shape of the price-yield relationship, while the convexity measure relates to the quantification of how the price of the bond will change when interest rates change.
    ii. The approximation percentage change in price due to convexity is the product of three numbers: $\frac{1}{2}$, convexity measure, and square of the change in yield.
    iii. In practice different vendors compute the convexity measure differently by scaling the measure in dissimilar ways.
Convexity (continued)

- Value of Convexity
  - Up to this point, we have focused on how taking convexity into account can improve the approximation of a bond’s price change for a given yield change.
  - The convexity of a bond, however, has another important investment implication, which is illustrated in Exhibit 4-16 (see Overhead 4-43).
  - The exhibit shows two bonds, A and B. The two bonds have the same duration and are offering the same yield; they have different convexities, however, Bond B is more convex (bowed) than bond A.
Exhibit 4-16
Comparison of Convexity of Two Bonds

Bond B Has Greater Convexity Than Bond A
Convexity (continued)

- **Value of Convexity**
  - The market considers a bond’s convexity when pricing it.
  - If investors expect that market yields will change by very little, investors should not be willing to pay much for convexity.
  - If the market prices convexity high, investors with expectations of low interest rate volatility will probably want to “sell convexity.”

- **Properties of Convexity**
  - All option-free bonds have the following convexity properties.
    i. As portrayed in Exhibit 4-17 (*see Overhead 4-45*), the required yield increases (decreases), the convexity of a bond decreases (increases). This property is referred to as positive convexity.
    ii. For a given yield and maturity, lower coupon rates will have greater convexity.
    iii. For a given yield and modified duration, lower coupon rates will have smaller convexity.
Exhibit 4-17
Change in Duration as the Required Yield Changes

As yield ↓
Slope (duration)

As yield ↑
Slope (duration)
Additional Concerns when Using Duration

- Relying on duration as the sole measure of the price volatility of a bond may mislead investors.

- There are two other concerns about using duration that we should point out.
  i. First, in the derivation of the relationship between modified duration and bond price volatility, we assume that all cash flows for the bond are discounted at the same discount rate.
  ii. Second, there is misapplication of duration to bonds with embedded options.
Measuring a Bond Portfolio’s Responsiveness to Nonparallel Changes in Interest Rates

- **Yield Curve Reshaping Duration**
  - The yield curve reshaping duration approach focuses on the sensitivity of a portfolio to a change in the slope of the yield curve.

- **Key Rate Duration**
  - The most popular measure for estimating the sensitivity of a security or a portfolio to changes in the yield curve is *key rate duration*.
  - The basic principle of *key rate duration* is to change the yield for a particular maturity of the yield curve and determine the sensitivity of a security or portfolio to that change holding all other yields constant.
Assignment from Chapter 10
Exercises 1, 2.
Chapter 10, Exercise 1

1. Suppose IBM is issuing $100 million in 7-year Eurobonds priced at U.S. Treasury minus 25 basis points. There is great demand for the issue and you are willing to bid 102 for 10% of the issue.
   a. If you actually get your bid executed, how much will you pay for the bond?
   b. A year later, the IBM Eurobonds are traded on the Luxembourg Exchange at 105. What is the value of your investment? What is your capital gain (loss)?
   c. You decide to sell the bond at the above price to pursue other opportunities. What amount of withholding taxes are you required to pay?
SOLUTIONS:
a. Price is 102% of par or 1,020 per bond; 102% * 10% * 100 million = $10.20 million for your share of the issue.
b. Price is 105%*10%*100 = $10.50 million. Gain is $300,000.
c. No withholding taxes apply in the Eurobond market.
Chapter 10, Exercise 2

2. Suppose Credit Suisse First Boston (CSFB) is the sole lead manager in a $100 million bought deal for the World Bank. CSFB decides to price the seven-year issue at par to yield 8%.

a. What will be CSFB’s position if the Fed decides to increase short-term interest rates by 50 basis points during the offering period?

b. Instead of the Fed move described in (a) above, suppose that international trade talks break down leading to a depreciation of the dollar on currency markets. What will be CSFB’s position in this case?

c. Calculate the gain or loss for CSFB if the seven-year Eurobond rate rises to 8.25% on the offering day. (Note: Eurobonds pay interest only once each year.)

d. Suppose CSFB collects 2% in fees for lead managing the issue. Again, calculate the overall gain or loss for CSFB if the seven-year Eurobond rate rises to 8.25% on the offering day.

e. (Optional) How could CSFB hedge the risks described in (a) and (b)?
SOLUTIONS:

a. The yield required by the market on long-term bonds may change in response to the 50 basis point increase in short-term rates. If long-term interest rates rise, then by pledging to sell the Eurobonds at par, CSFB will lose the difference between par and the new lower price of the bond. Long-term interest rates may fall, however, if the market senses that the increase in short-term rates will reduce longer-run inflationary pressures. In this case, CSFB enjoys a capital gain.

b. Same as in (a). To attract investors that shy away from dollar assets, CSFB will have to lower the Eurobond price to a level attractive to lenders.

c. The Eurobond price falls to $987.09 per $1,000.00 face value. The underwriter loses 1.291% on the $100,000,000 issue or $1,291,000.

d. If CSFB collects 2.0% in fees, it transfers only $980 per bond, or $98,000,000 on the entire issue to the World Bank. CSFB's net profit is then $2,000,000 - $1,291,000 = $709,000.

e. CSFB can hedge the increase in interest rates by selling interest rate futures.
The Treasury Auction Process

The Public Debt Act of 1942 grants the Department of the Treasury considerable discretion in deciding on the terms for a marketable security. An issue may be sold on an interest-bearing or discount basis and may be sold on a competitive or other basis, at whatever prices the Secretary of the Treasury may establish.

Treasury securities are sold in the primary market through sealed-bid auctions. Each auction is announced several days in advance by means of a Treasury Department press release or press conference. The announcement provides details of the offering, including the offering amount and the term and type of security being offered, and describes some of the auction rules and procedures. Treasury auctions are open to all entities.
The Treasury auctions securities on a regular cycle Treasury bills with maturities of 4 weeks, 13 weeks (3 months), and 26 weeks (6 months). At irregular intervals the Treasury issues cash management bills with maturities ranging from a few days to about six months. The Treasury auctions 2-, 5-, and 10-year Treasury notes. The Treasury does not issue Treasury bonds on a regular basis. The Treasury had issued 30-year Treasury bonds on a regular basis but suspended doing so in October 2001. The issuance has been resumed.

**Determination of the Results of an Auction**

The auction for Treasury securities is conducted on a competitive bid basis. The competitive bid part of the auction determines the yield (or price). A noncompetitive bid is submitted by an entity that is willing to purchase the auctioned security at the yield that is determined by the auction process.
When a noncompetitive bid is submitted, the bidder only specifies the quantity sought. The quantity in a noncompetitive bid may not exceed $1 million for Treasury bills and $5 million for Treasury coupon securities. A competitive bid specifies both the quantity sought and the yield at which the bidder is willing to purchase the auctioned security.

The highest yield accepted by the Treasury is referred to as the **high yield** (or **stop-out yield**). Bidders whose bid is higher than the high yield are not distributed any of the new issue (i.e., they are unsuccessful bidders). Bidders whose bid was the high yield (i.e., the highest yield accepted by the Treasury) are awarded a proportionate amount for which they bid.
Within an hour of the auction deadline, the Treasury announces the auction results including the quantity of noncompetitive tenders, the median-yield bid, and the ratio of the total amount bid for by the public to the amount awarded to the public (called the **bid-to-cover ratio**). For notes and bonds, the announcement includes the coupon rate of the new security.
Arbitrage in the US Treasury Market?
How?
"We firmly believe that the on-the-run issues should command a high liquidity premium in the current environment. But with very high probability, the 5 1/2s of 8/15/2028 will NOT be the current bond a month from now.

The Bond/Old Bond spread is currently about 13bp. The average of this spread around auction is historically about 3bp. Hence we think that much of the premium now assigned to the current bond should be ultimately passed on to the new issue by the expected auction date. Therefore it makes sense to begin scaling into a reverse roll now, at these levels."

GLOSSARY

On-the-run: The most recently issued Treasury bond in any given sector (e.g. 10 year sector, 30 year sector) of the yield curve.

The Current Bond: the most recently issued 30 year US Treasury bond (also shortened to just "The Bond").

The Old Bond: the second most recent 30 year US Treasury issue.

Bond/Old Bond spread: the yield difference between the most recent and the previously issued 30 year Treasuries.

Basis point (bp): $1/100 \times 1\%$
The yield on the "on-the-run" 30yr bond is lower than similar bonds => it is worth more:

WHY?

Different Maturity?  NO

Better Credit?  NO

Liquidity?  YES!
Properties of the on-the-run Bond

- Tighter bid-offer spread
- Transactions costs are lower for quick and easy buying & selling
- Worth more (⇒ lower yield)

This Liquidity Premium has historically been worth about 3bp

… So why is it worth 13bp today?

Why is there an extra 10bp?

Could it be Arbitrage?

The Goldman Sachs US Treasury bond trader thinks so!
The Arbitrage Trade

**TODAY**

- **Sell** the 8/28 (today's "on-the-run" bond)
- **Buy** the 5/28 (today's "old bond")

*Net Profit:* 13bp (because the on-the-run is 13bp more costly than the old bond).

**ONE MONTH's TIME**

- **Buy** the 5/28 (today's "old bond")
- **Sell** the 5/28 (today's "old old bond")

*Net cost: zero* (now the two bonds cost the same amount; liquidity premium is now on the new "on-the-run" bond)
Chapter 10 Residential Mortgage Loans
Origination of Residential Mortgage Loans (continued)

Payment-to-Income Ratio

- The payment-to-income ratio (PTI) is the ratio of monthly payments to monthly income, which measures the ability of the applicant to make monthly payments (both mortgage and real estate tax payments).
- The lower the PTI, the greater the likelihood that the applicant will be able to meet the required monthly mortgage payments.
Loan-to-Value Ratio

- The *loan-to-value ratio* (LTV) is the ratio of the amount of the loan to the market (or appraised) value of the property.
- The lower this ratio is, the greater the protection for the lender if the applicant defaults on the payments and the lender must repossess and sell the property.
- The LTV has been found in numerous studies to be the single most important determinant of the likelihood of default.
- The rationale is straightforward: Homeowners with large amounts of equity in their properties are unlikely to default.
Types of Residential Mortgage Loans

- There are different types of residential mortgage loans.
- They can be classified according to the following attributes:
  i. lien status
  ii. credit classification
  iii. interest rate type
  iv. amortization type
  v. credit guarantees
  vi. loan balances
  vii. prepayments and prepayment penalties
Lien Status

- The *lien status* of a mortgage loan indicates the loan’s seniority in the event of the forced liquidation of the property due to default by the obligor.
- For a mortgage loan that is a *first lien*, the lender would have first call on the proceeds of the liquidation of the property if it were to be repossessed.
- A mortgage loan could also be a *second lien* or *junior lien*, and the claims of the lender on the proceeds in the case of liquidation come after the holders of the first lien are paid in full.
Types of Residential Mortgage Loans (continued)

- **Credit Classification**
  - A loan that is originated where the borrower is viewed to have a high credit quality is classified as a *prime loan*.
  - A loan that is originated where the borrower is of lower credit quality or where the loan is not a first lien on the property is classified as a *subprime loan*.
  - While the credit scores have different underlying methodologies, the scores generically are referred to as “FICO scores.”
  - FICO scores range from 350 to 850.
  - The higher the FICO score is, the lower the credit risk.
Types of Residential Mortgage Loans (continued)

- Credit Classification
  - The LTV has proven to be a good predictor of default: a higher LTV implies a greater likelihood of default.
  - When the loan amount requested exceeds the original loan amount, the transaction is referred to as a cash-out-refinancing.
  - If instead, there is financing where the loan balance remains unchanged, the transaction is said to be a rate-and-term refinancing or no-cash refinancing.
Types of Residential Mortgage Loans (continued)

- Credit Classification
  - The front ratio is computed by dividing the total monthly payments (which include interest and principal on the loan plus property taxes and homeowner insurance) by the applicant’s pre-tax monthly income.
  - The back ratio is computed in a similar manner. The modification is that it adds other debt payments such as auto loan and credit card payments to the total payments.
  - The credit score is the primary attribute used to characterize loans as either prime or subprime.
  - Prime (or A-grade) loans generally have FICO scores of 660 or higher, front and back ratios with the above-noted maximum of 28% and 36%, and LTVs less than 95%.
Types of Residential Mortgage Loans (continued)

- Interest Rate Type
  - The interest rate that the borrower agrees to pay, referred to as the note rate, can be fixed or change over the life of the loan.
  - For a fixed-rate mortgage (FRM), the interest rate is set at the closing of the loan and remains unchanged over the life of the loan.
Types of Residential Mortgage Loans
(continued)

❖ Interest Rate Type

➢ For an *adjustable-rate mortgage* (ARM), as the name implies, the note rate changes over the life of the loan.

➢ The note rate is based on both the movement of an underlying rate, called the *index* or *reference rate*, and a spread over the index called the margin.

➢ Two categories of reference rates have been used in ARMs:
  i. market-determined rates
  ii. calculated rates based on the cost of funds for thrifts
Types of Residential Mortgage Loans (continued)

- Interest Rate Type
  - The basic ARM is one that resets periodically and has no other terms that affect the monthly mortgage payment.
  - Typically, the mortgage rate is affected by other terms that include:
    i. periodic rate caps
    ii. lifetime rate cap and floor
  - A periodic rate cap limits the amount that the interest rate may increase or decrease at the reset date.
  - Most ARMs have an upper limit on the mortgage rate that can be charged over the life of the loan.
  - This lifetime rate cap is expressed in terms of the initial rate.
  - ARMs may also have a lower limit (floor) on the interest rate that can be charged over the life of the loan.
Amortization Type

The amount of the monthly loan payment that represents the repayment of the principal borrowed is called the *amortization*.

Traditionally, both FRMs and ARMs are *fully amortizing loans*.

What this means is that the monthly mortgage payments made by the borrower are such that they not only provide the lender with the contractual interest but also are sufficient to completely repay the amount borrowed when the last monthly mortgage payment is made.
Types of Residential Mortgage Loans
(continued)

- Amortization Type
  - Fully amortizing fixed-rate loans have a payment that is constant over the life of the loan.
  - For example, suppose a loan has an original balance of $200,000, a note rate of 7.5%, and a term of 30 years.
  - Then the monthly mortgage payment would be $1,398.43.
  - The formula for calculating the monthly mortgage payment is

\[
MP = MB_0 \left[ \frac{i(1 + i)^n}{(1 + i)^n - 1} \right]
\]

where \( MP = \text{monthly mortgage payment} \) ($),
\( MB_0 = \text{original mortgage balance} \) ($),
\( i = \text{note rate divided by 12} \) (in decimal), and
\( n = \text{number of months of the mortgage loan} \).
Applying annuity formula,

(i) $PV_{t=0}$ of an infinite annuity $= \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots + \frac{A}{(1+r)^n} + \cdots = \frac{A}{1+r} \{1-(1/(1+r))\} = \frac{A}{r}$

(ii) $PV_{t=n}$ of the same infinite annuity $= \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots + \frac{A}{(1+r)^n} + \cdots = \frac{A}{1+r} \{1-(1/(1+r))\} = \frac{A}{r}$

$MB_0 = PV_{t=0}$ of a finite annuity $= (i) - \frac{1}{1+(1+r)^n}(ii) = \frac{A}{r} \cdot \{1-\frac{1}{1+(1+r)^n}\}$

$\Rightarrow MP = A = MB_0 \cdot \frac{i}{1-\frac{1}{1+(1+i)^n}}$

Let $A=MP$ (monthly payment), $MB_0=$ size of mortgage loan, $r=i=$ mortgage rate, $t=n=$ say, 30 years, verify that

Derive $MP = MB_0 \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$
Sum PV time=1 to time=t (MP)
= $MP \cdot \{1/(1+i) + 1/(1+i)^2 + 1/(1+i)^3 + \cdots + 1/(1+i)^t\}$
= $MP \cdot \{1/(1+i) \cdot (1-[1/(1+i)]^t)/1-[1/(1+i)]\}$
= $MP \cdot \{1 \cdot (1-[1/(1+i)]^t)\}/i$

Note that $MP = MB_0 \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$

Sum PV time=1 to time=t (MP)
= $MP \cdot \{1/(1+i) + 1/(1+i)^2 + 1/(1+i)^3 + \cdots + 1/(1+i)^t\}$
= $MP \cdot \{1/(1+i) \cdot (1-[1/(1+i)]^t)/1-[1/(1+i)]\}$
= $MP \cdot \{1 \cdot (1-[1/(1+i)]^t)\}/i$
= $MB_0 \cdot \{1 \cdot (1-[1/(1+i)]^t)\} \cdot \{((1+i)^n) / \{(1+i)^n - 1\}\}$

$\{MB_0 – \text{Sum PV time}=1 \text{ to time}=t \ (MP)\} \cdot \{(1+i)^t\}$
= $MB_t$ (mortgage (principal) balance after $t$ months, the present value is evaluated with respect to time = $t$, not at time = 0)
\[ MB_0 - MB_0 \cdot \left\{ 1 \cdot \left( 1 - \left[ \frac{1}{1+i} \right]^t \right) \right\} \cdot \frac{(1+i)^n}{(1+i)^n - 1} \]

\[ = MB_0 \left\{ 1 \cdot \left[ (1+i)^n - 1 \right] - 1 \cdot \left( 1 - \left[ \frac{1}{1+i} \right]^t \right) \right\} \cdot \frac{(1+i)^n}{(1+i)^n - 1} \]

\[ = MB_0 \left\{ -1 + \left[ \frac{(1+i)^n}{1+i^t} \right] \right\} / \left[ (1+i)^n - 1 \right] \]

\[ = MB_0 \left\{ \left[ (1+i)^n - (1+i)^t \right] / (1+i^t) \right\} / \left[ (1+i)^n - 1 \right] \]

\[ \left\{ (1+i)^t \right\} \cdot MB_0 \left\{ \left[ (1+i)^n - (1+i)^t \right] / (1+i^t) \right\} / \left[ (1+i)^n - 1 \right] \]

\[ = MB_0 \cdot \left\{ \left[ (1+i)^n - (1+i)^t \right] / (1+i^t) \right\} / \left[ (1+i)^n - 1 \right] \]
Types of Residential Mortgage Loans
(continued)

- Amortization Type

To calculate the remaining mortgage balance at the end of any month, the following formula is used:

\[
MB_t = MB_0 \left[ \frac{(1 + i)^n - (1 + i)^t}{(1 + i)^n - 1} \right]
\]

where \( MB_t \) = mortgage (principal) balance after \( t \) months,
\( MB_0 \) = original mortgage balance ($),
\( i \) = note rate divided by 12 (in decimal), and
\( n \) = number of months of the mortgage loan.
Types of Residential Mortgage Loans
(continued)

- Amortization Type

To calculate the portion of the monthly mortgage payment that is the scheduled principal payment for a month, the following formula is used:

\[
SP_t = MB_0 \left[ \frac{i(1 + i)^{t-1}}{(1 + i)^n - 1} \right]
\]

where \(SP_t\) = scheduled principal repayment for month \(t\),

\(MB_0\) = original mortgage balance ($),

\(i = \text{note rate divided by 12 (in decimal), and}\)

\(n = \text{number of months of the mortgage loan.}\)
Amortization Type

EXAMPLE. Suppose that for month 12 \((t = 12)\), we have \(MB_0 = 200,000\); \(i = 0.00625\); \(n = 360\), then the scheduled principal repayment for month 12 is:

\[
SP_t = MB_0 \left[ \frac{i(1 + i)^{t-1}}{(1 + i)^n - 1} \right] = \\
$200,000 \left[ \frac{0.00625(1.00625)^{12-1}}{(1.00625)^{12} - 1} \right] = $158.95
\]
A Rule of Thumb:

If you take a mortgage loan of the size of $1,000,000 with a 30-year fixed mortgage rate of 6% to buy your dream house, you will be prepared to pay 1% of your borrowed amount (plus property tax, etc.) per month for 360 months.

Support or dissupport the above claim with your calculation.
Types of Residential Mortgage Loans (continued)

- Amortization Type
  - For an ARM, the monthly mortgage payment adjusts periodically.
  - Thus, the monthly mortgage payments must be recalculated at each reset date.
  - This process of resetting the mortgage loan payment is referred to as recasting the loan.
  - In recent years, several types of nontraditional amortization schemes have become popular in the mortgage market.
  - The most popular is the interest-only product (or IO product).
  - With this type of loan, only interest is paid for a predetermined period of time called the lockout period.
Types of Residential Mortgage Loans (continued)

- Credit Guarantees
  - Mortgage loans can be classified based on whether a credit guarantee associated with the loan is provided by the federal government, a government-sponsored enterprise, or a private entity.
  - Loans that are backed by agencies of the federal government are referred to under the generic term of government loans and are guaranteed by the full faith and credit of the U.S. government.
  - The Department of Housing and Urban Development (HUD) oversees two agencies that guarantee government loans.
    i. The first is the Federal Housing Administration (FHA).
    ii. The second is the Veterans Administration (VA).
Types of Residential Mortgage Loans (continued)

- Credit Guarantees
  - In contrast to government loans, there are loans that have no explicit guarantee from the federal government.
  - Such loans are said to be obtained from “conventional financing” and therefore are referred to in the market as conventional loans.
  - A conventional loan can be insured by a private mortgage insurer.
Types of Residential Mortgage Loans (continued)

- Loan Balances
  - For government loans and the loans guaranteed by Freddie Mac and Fannie Mae, there are limits on the loan balance.
  - The loan limits, referred to as *conforming limits*, for Freddie Mac and Fannie Mae are identical because they are specified by the same statute.
  - Loans larger than the conforming limit for a given property type are referred to as *jumbo loans.*
Types of Residential Mortgage Loans (continued)

- Prepayments and Prepayment Penalties
  - Homeowners often repay all or part of their mortgage balance prior to the scheduled maturity date.
  - The amount of the payment made in excess of the monthly mortgage payment is called a *prepayment*.
  - This type of prepayment in which the entire mortgage balance is not paid off is called a *partial payment* or *curtailment*.
  - When a curtailment is made, the loan is not recast.
  - Instead, the borrower continues to make the same monthly mortgage payment.
Types of Residential Mortgage Loans
(continued)

- Prepayments and Prepayment Penalties
  - The more common type of prepayment is one where the entire mortgage balance is paid off.
  - All mortgage loans have a “due on sale” clause, which means that the remaining balance of the loan must be paid when the house is sold.
  - Effectively, the borrower’s right to prepay a loan in whole or in part without a penalty is a called an option.
  - A mortgage design that mitigates the borrower’s right to prepay is the prepayment penalty mortgage.
Conforming Loans

- Freddie Mac and Fannie Mae are government-sponsored enterprises (GSEs) whose mission is to provide liquidity and support to the mortgage market.
- While Fannie Mae and Freddie Mac can buy or sell any type of residential mortgage, the mortgages that are packaged into securities are restricted to government loans and those that satisfy their underwriting guidelines.
  - The conventional loans that qualify are referred to as **conforming loans**.
  - A conforming loan is simply a conventional loan that meets the underwriting standard of Fannie Mae and Freddie Mac.
  - Thus, conventional loans in the market are referred to as **conforming conventional loans** and **nonconforming conventional loans**.
Conforming Loans (continued)

- Qualifying for a conforming loan is important for both the borrower and the mortgage originator.
- This is because the two GSEs are the largest buyers of mortgages in the United States.
  - Hence, loans that qualify as conforming loans have a greater probability of being purchased by Fannie Mae and Freddie Mac to be packaged into an MBS.
  - As a result, they have lower interest rates than nonconforming conventional loans.
Risks Associated with Investing in Mortgage Loans

- The principal investors in mortgage loans include thrifts and commercial banks.
- Pension funds and life insurance companies also invest in these loans, but their ownership is small compared to that of the banks and thrifts.
- Investors face four main risks by investing in residential mortgage loans:
  1. credit risk
  2. liquidity risk
  3. price risk
  4. prepayment risk
Risks Associated with Investing in Mortgage Loans (continued)

- **Credit Risk**
  - Credit risk is the risk that the homeowner/borrower will default.
  - For FHA- and VA-insured mortgages, this risk is minimal.
  - The LTV ratio provides a useful measure of the risk of loss of principal in case of default.
  - At one time, investors considered the LTV only at the time of origination (called the *original LTV*) in their analysis of credit risk.
  - For periods in which there are a decline in housing prices, the *current LTV* becomes the focus of attention.
Risks Associated with Investing in Mortgage Loans (continued)

- **Liquidity Risk**
  - Although there is a secondary market for mortgage loans, the fact is that bid-ask spreads are large compared to other debt instruments.
  - That is, mortgage loans tend to be rather illiquid because they are large and indivisible.
  - The degree of liquidity determines the liquidity risk.

- **Price Risk**
  - The price of a fixed-income instrument will move in an opposite direction from market interest rates.
  - Thus, a rise in interest rates will decrease the price of a mortgage loan.
Risks Associated with Investing in Mortgage Loans (continued)

- Prepayments and Cash Flow Uncertainty
  - The three components of the cash flow are:
    i. interest
    ii. principal repayment (scheduled principal repayment or amortization)
    iii. prepayment.
  - Prepayment risk is the risk associated with a mortgage’s cash flow due to prepayments.
  - More specifically, investors are concerned that borrowers will pay off a mortgage when prevailing mortgage rates fall below the loan’s note rate.
Topics in Bond-related Innovation and Challenges

The innovation of credit derivative instruments does not stop at single name credit default swaps, which shift credit exposure to a credit protection seller, and operate like standby letters of credit or insurance. Collateralized Debt Obligations (CDOs) alone, where a pool of credits such as bonds or loans are created and pieces of the pool are sold to different investors based on their risk/return appetite, is another trillion-dollar market.
Topics in Financial Innovation and Challenges

“Credit Default Swap”: Banks may sell credit default swap and pay premium in exchange for face amount of lending/loans in case of default. Investors who invest in “credit default swap” receive premium (just like selling insurance) but lose their investment when there are defaults. Therefore, “credit default swap” sells for excellent price when economy is good; and “credit default swap” loses most of its value (not attractive to investors) when economy turns sour.
Credit Default Swap: An Innovation

The investor owns the reference asset [mortgage-backed security], which was issued by XYZ. The investor buys a credit default swap and pays 40 basis points per year in exchange for the swap writer’s payment in the event of a default by XYZ. [i.e., investors bet on the potential failure of XYZ for claims, if insurer survives to pay.] In theory, no limit on the amount of default swaps that can be created.

In 2008, the CDS market ballooned to 54.6 trillions, when world GDP was at 54.3 trillions.

Figure 5: Depiction of the cash flows in a credit default swap. Source: McDonald, Derivatives, 2008.
Topics in Financial Innovation and Challenges

Banks and investors sell credit-default swaps to protect against losses on bonds or loans they hold; many traders and hedge funds also use the swaps to bet on the fortunes of companies or sectors. The swaps moves don't always correlate to the actual securities.
Topics in Financial Innovation and Challenges

In the past week of June 28, 2008, the ABX index of swaps, which tracks the performance of subprime-mortgage bonds, dropped to new lows as the cost of default insurance on these assets soared. The index that tracks triple-A subprimemortgage-backed securities fell to 45.9 cents on the dollar, down 17% from a month ago and 38% in the year to date, according to data from Markit.
Topics in Financial Innovation and Challenges

The ABX has been criticized as an inaccurate indicator of subprime mortgage losses -- even the Bank for International Settlements said in a recent report that loss estimates implied by the triple-A slice of the index may be overstated. Financial institutions continue to use the index to hedge their holdings of mortgage assets, and their buying of protection has the effect of pushing the index lower still. Credit-default swaps tied to GM imply it has a 31% chance of defaulting in the next year, even though GM has billions in cash to tide it over the near term.
The CDO market started with cash CDOs where the underlying pool contains actual bonds or loans. The life insurer is the largest investor's group of the cash CDO securities. CDO’s future cash flow comprises interests receivable and principal receivable.
Then synthetic CDOs, which have a collateral pool consisting of a portfolio of single name default swaps, quickly followed suit. This innovation overcame the constraint that only limited actual assets could be taken as collateral in the cash CDO markets. Be alert!!!
Protection Money

The market for “synthetic collateralized debt obligations” — an investment instrument that divvies up bond risk — as measured in bond equivalent.*

In billions.

1997 '98 '99 2000 '01 '02 '03 '04

*Synthetic CDOs don’t contain actual bonds but instead represent insurance on a hypothetical face amount of bonds.

Source: J.P. Morgan Chase
Topics in Financial Innovation and Challenges

How Does a Synthetic CDO Work?
Synthetic CDOs provide investors with income by selling insurance against defaults, typically on a pool of 100 or more companies.

Via the Process Known As Tranches
CDS

Source: SMCP, Alan Alanson: How Octave Notes proved a costly mistake for some

Yee-Tien Fu

Disappearing Act
Las Vegas home prices peaked in 2006, so do home prices in general in the States.

Credit Default Swap: An Innovation
The investor owns the reference asset [mortgage-backed security], which was issued by XYZ. The investor buys a credit default swap and pays 40 basis points per year in exchange for the swap writer’s payment in the event of a default by XYZ. [i.e., investors bet on the potential failure of XYZ for claims, if insurer survives to pay.] In theory, no limit on the amount of default swaps that can be created.

In 2008, the CDS market ballooned to 54.6 trillions, when world GDP was at 54.3 trillions.

Only to See their Own “Tickers”
The time when investment firms selling CDS of other companies see their own CDS market value (insurance premiums) surge, i.e., approach default or bankruptcy. Of course, if the insurer bankrupts, all the CDS issued by them loses all the value.

Get Money’s Worth from Distressed Assets
The time when investment firms selling CDS of other companies see their own CDS market value (insurance premiums) surge, i.e., approach default or bankruptcy. Of course, if the insurer bankrupts, all the CDS issued by them loses all the value.

International Recovery Time With Liquidity
International coordination and orchestrating, known as mushrooming or bambooing, are observed for the first time in the global bailout effort.

Ironing Out the Details
Congress is going Treasury’s plan to issue guidelines on pricing assets in the US and the US treasury is about to buy them. Here’s a report on what’s likely to work.
Risk Reshapes the Bond Markets

Safety first
Treasury yields plummeted as prices rose when investors rushed to buy the safest assets during the credit crunch, and the Federal Reserve lowered its target federal-funds rate by one percentage point.

Investors demanded to be paid more for riskier investments...
The difference between yields on corporate bonds and Treasury bonds widened sharply as the subprime mortgage crisis deepened.

...As junk debt issuance slowed to a trickle
Bankers found fewer buyers for lower-rated corporate debt during the second half of 2007.

Sources: Merrill Lynch (spreads); Ryan ALM, Reuters via WSJ Market Data Group (Treasury, target rate); Dealogic (debt issuance)
Topics in Financial Innovation and Challenges

The modelling of default events using survival time distribution is very similar to the modelling of the death of a human life. A credit curve, which describes the term structures of default probabilities for an obligor, is very much like a mortality table. Pricing a default swap is not much different from pricing a life insurance contract. David Li on Synthetic CDO
What's in your nest egg?

Convertible A convertible bond is generally one which starts out its life as a bond but at some point can be converted into equity in the bond issuer, at a predetermined price at the bondholder's discretion.

Callable A bond that can be redeemed by the issuer. If Nocash Corp issues a callable bond with a 5% coupon and interest rates drop to 4%, Nocash Corp would be able to buy back the callable bonds, probably to reissue them at a lower interest rate.

Step-up A bond that pays two or more rates of interest. It might pay 3% interest for the first three years, and then 5% for the next three.

Junk A junk bond is relatively riskier and usually has a credit rating of less than BB. These are often called high-yield bonds as they tend to pay more interest because of their low credit quality.

Floating-rate While most bonds have fixed interest rates or coupons, some can have an interest rate that moves over time. A floating-rate bond's interest rate will be defined by reference to an index, such as LIBOR + 0.20%.

Bearer If you are a villain in a movie, you will demand the ransom in bearer bonds. These things really exist and were once very popular. Whoever has possession of a bearer bond at the time of its maturity is entitled to repayment of the principal.

Subordinated Not all bonds are created equal. It is possible for Nocash Corp to issue two sets of bonds that have different rights to repayment. If Nocash Corp issues a standard bond and a subordinated bond, the standard bond (called senior) gets paid first and the subordinated bond gets paid only if there is enough money left over. The subordinated bond is riskier and so has a higher yield.

Mortgage-backed Bonds backed by a pledge of mortgages and payable from the issuer's general funds.

Inflation-linked Usually issued by governments, they generally pay a lower interest rate than normal bonds, but the investors' return is linked to inflation. The higher the inflation rate, the higher the yield.

Source: SCMP, Alan Alanson