Chapter 7. Utility Maximization

7.1 The preferences exhibit local nonsatiation, except at (0,0). The consumer will choose this consumption point when faced with positive prices.

7.2 The demand function is

\[
x_1 = \begin{cases} 
  \frac{m}{p_1} & \text{if } p_1 < p_2 \\
  \text{any } x_1 \text{ and } x_2 \text{ such that } p_1 x_1 + p_2 x_2 = m & \text{if } p_1 = p_2 \\
  0 & \text{if } p_1 > p_2 
\end{cases}
\]

The indirect utility function is \( v(p_1, p_2, m) = \max\{m/p_1, m/p_2\} \), and the expenditure function is \( e(p_1, p_2, u) = u \min\{p_1, p_2\} \).

7.3 The expenditure function is \( e(p_1, p_2, u) = u \min\{p_1, p_2\} \). The utility function is \( u(x_1, x_2) = x_1 + x_2 \) (or any monotonic transformation), and the demand function is

\[
x_1 = \begin{cases} 
  \frac{m}{p_1} & \text{if } p_1 < p_2 \\
  \text{any } x_1 \text{ and } x_2 \text{ such that } p_1 x_1 + p_2 x_2 = m & \text{if } p_1 = p_2 \\
  0 & \text{if } p_1 > p_2 
\end{cases}
\]

7.4.a Demand functions are \( x_1 = m/(p_1 + p_2) \), \( x_2 = m/(p_1 + p_2) \).

7.4.b \( e(p_1, p_2, u) = (p_1 + p + 2)u \)

7.4.c \( u(x_1, x_2) = \min\{x_1, x_2\} \)

7.5.a Quasilinear preferences.

7.5.b Less than \( u(1) \).

7.5.c \( v(p_1, p_2, m) = \max\{u(1) - p_1 + m, m\} \)

7.6.a Homothetic.

7.6.b \( e(p, u) = u/A(p) \)

7.6.c \( \mu(p; q, m) = mA(q)/A(p) \)

7.6.d It will be the same, since this is just a monotonic transformation.

\[
\text{max exactly } \quad u(\hat{p}; \hat{q}, m) = \frac{A(\hat{q})}{A(\hat{p})} m b
\]

Chapter 8. Choice

8.1 We know that

\[
x_j(p, m) \equiv h_j(p, v(p, m)) \equiv \partial v(p, v(p, m))/\partial p_j. \quad (0.1)
\]
(Note that the partial derivative is taken with respect to the first occurrence of \( p_j \).) Differentiating equation (0.1) with respect to \( m \) gives us

\[
\frac{\partial x_i}{\partial m} = \frac{\partial^2 e(p, v(p, m))}{\partial p_j \partial u} \frac{\partial v(p, m)}{\partial m}.
\]

Since the marginal utility of income, \( \partial v/\partial m \), must be positive, the result follows.

8.2 The Cobb-Douglas demand system with two goods has the form

\[
x_1 = \frac{a_1m}{p_1}, \quad x_2 = \frac{a_2m}{p_2},
\]

where \( a_1 + a_2 = 1 \). The substitution matrix is

\[
\begin{pmatrix}
-a_1 m p_1^{-2} & a_1^2 m p_1^{-2} & -a_1 a_2 m p_1^{-1} p_2^{-1} \\
-a_1 a_2 m p_1^{-1} p_2^{-1} & -a_2 m p_2^{-2} & -a_2^2 m p_2^{-2}
\end{pmatrix}.
\]

This is clearly symmetric and negative definite.

8.3 The equation is \( d\mu/dt = at + bu + c \). The indirect money metric utility function is

\[
\mu(q, p, m) = e^{b(q-p)} \left[ m + \frac{c}{b} + \frac{a}{b^2} + \frac{c}{b} p \right] - \frac{c}{b} - \frac{a}{b^2} - \frac{aq}{b}.
\]

8.4 The demand function can be written as \( z = e^{c+ap+bm} \). The integrability equation is

\[
\frac{d\mu}{dt} = e^{at+bu+c}.
\]

Write this as

\[
e^{-bu} \frac{d\mu}{dt} = e^c e^{at}.
\]

Integrating both sides of this equation between \( p \) and \( q \), we have

\[
-\frac{e^{-bu}}{b} \left. pq \right|_p^q = \frac{e^c e^{at}}{a} \left. \right|_p^q.
\]

Evaluating the integrals, we have

\[
e^{b\mu(q;p,m)} = e^{-bm} - \frac{be^c}{a} \left[ e^{ap} - e^{aq} \right].
\]
8.5 Write the Lagrangian

\[ \mathcal{L}(x, \lambda) = \frac{3}{2} \ln x_1 + \ln x_2 - \lambda(3x_1 + 4x_2 - 100). \]

(Be sure you understand why we can transform \( u \) this way.) Now, equating the derivatives with respect to \( x_1, x_2, \) and \( \lambda \) to zero, we get three equations in three unknowns

\[
\begin{align*}
\frac{3}{2x_1} &= 3\lambda, \\
\frac{1}{x_2} &= 4\lambda, \\
3x_1 + 4x_2 &= 100.
\end{align*}
\]

Solving, we get

\[ x_1(3, 4, 100) = 20, \text{ and } x_2(3, 4, 100) = 10. \]

Note that if you are going to interpret the Lagrange multiplier as the marginal utility of income, you must be explicit as to which utility function you are referring to. Thus, the marginal utility of income can be measured in original "utils" or in "ln utils". Let \( u^* = \ln u \) and, correspondingly, \( v^* = \ln v \); then

\[
\lambda = \frac{\partial u^*(p, m)}{\partial m} = \frac{\partial v(p, m)}{\partial m} = \frac{\mu}{v(p, m)},
\]

where \( \mu \) denotes the Lagrange multiplier in the Lagrangian

\[ L(x, \mu) = x_1^\frac{1}{2} x_2 - \mu(3x_1 + 4x_2 - 100). \]

Check that in this problem we'd get \( \mu = \frac{20}{4} \), \( \lambda = \frac{1}{40} \), and \( v(3, 4, 100) = 20\frac{1}{10}. \)

8.6 The Lagrangian for the utility maximization problem is

\[ \mathcal{L}(x, \lambda) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} - \lambda(p_1 x_1 + p_2 x_2 - m), \]

taking derivatives,

\[
\begin{align*}
\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} &= \lambda p_1, \\
\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} &= \lambda p_2, \\
p_1 x_1 + p_2 x_2 &= m.
\end{align*}
\]

Solving, we get

\[
\begin{align*}
x_1(p, m) &= \frac{3}{5} m, \\
x_2(p, m) &= \frac{2}{5} m.
\end{align*}
\]
20 ANSWERS

Plugging these demands into the utility function, we get the indirect utility function

\[ u(p, m) = U(x(p, m)) = \left( \frac{3}{5} \frac{m}{p_1} \right)^{\frac{1}{2}} \left( \frac{2}{5} \frac{m}{p_2} \right)^{\frac{1}{3}} = \left( \frac{m}{5} \right)^{\frac{1}{2}} \left( \frac{3}{p_1} \right)^{\frac{1}{3}} \left( \frac{2}{p_2} \right)^{\frac{1}{3}}. \]

Rewrite the above expression replacing \( u(p, m) \) by \( u \) and \( m \) by \( e(p, u) \). Then solve it for \( e(\cdot) \) to get

\[ e(p, u) = 5 \left( \frac{p_1}{3} \right)^{\frac{1}{2}} \left( \frac{p_2}{2} \right)^{\frac{1}{3}} u. \]

Finally, since \( h_i = \frac{\partial e}{\partial p_i} \), the Hicksian demands are

\[ h_1(p, u) = \left( \frac{p_1}{3} \right)^{-\frac{1}{2}} \left( \frac{p_2}{2} \right)^{\frac{1}{3}} u, \]

and

\[ h_2(p, u) = \left( \frac{p_1}{3} \right)^{\frac{1}{2}} \left( \frac{p_2}{2} \right)^{-\frac{1}{3}} u. \]

8.7 Instead of starting from the utility maximization problem, let's now start from the expenditure minimization problem. The Lagrangian is

\[ L(x, \mu) = p_1 x_1 + p_2 x_2 - \mu((x_1 - \alpha_1)^{\beta_1}(x_2 - \alpha_2)^{\beta_2} - u); \]

the first-order conditions are

\[ p_1 = \mu^\beta_1 (x_1 - \alpha_1)^{\beta_1-1}(x_2 - \alpha_2)^{\beta_2}, \]
\[ p_2 = \mu^\beta_2 (x_1 - \alpha_1)^{\beta_1}(x_2 - \alpha_2)^{\beta_2-1}, \]
\[ (x_1 - \alpha_1)^{\beta_1}(x_2 - \alpha_2)^{\beta_2} = u. \]

Divide the first equation by the second

\[ \frac{p_1^\beta_2}{p_2^\beta_1} = \frac{x_2 - \alpha_2}{x_1 - \alpha_1}, \]

using the last equation

\[ x_2 - \alpha_2 = ((x_1 - \alpha_1)^{-\beta_1} u)^{\frac{1}{\beta_1}} ; \]

substituting and solving,

\[ h_1(p, u) = \alpha_1 + \left( \frac{p_2^\beta_1}{p_1^\beta_2} u^{\frac{1}{\beta_1}} \right)^{\frac{\beta_2}{\beta_1+\beta_2}}. \]
and
\[ h_2(p, u) = \alpha_2 + \left( \frac{p_1\beta_2}{p_2\beta_1 u^{\beta_1}} \right)^{\frac{\beta_2}{\beta_1 + \beta_2}}. \]

Verify that
\[ \frac{\partial h_1(p, m)}{\partial p_2} = \left( \frac{u}{\beta_1 + \beta_2} \right) \left( \frac{p_2\beta_1}{p_1\beta_2} \right)^{\beta_1} \left( \frac{p_1\beta_2}{p_2\beta_1} \right)^{\beta_2} \frac{\beta_2}{\beta_1 + \beta_2} = \frac{\partial h_2(p, m)}{\partial p_1}. \]

The expenditure function is
\[ e(p, u) = p_1 \left( \alpha_1 + \left( \frac{p_2\beta_1}{p_1\beta_2} u^{\beta_1} \right)^{\frac{\beta_2}{\beta_1 + \beta_2}} \right) + p_2 \left( \alpha_2 + \left( \frac{p_1\beta_2}{p_2\beta_1} u^{\beta_2} \right)^{\frac{\beta_1}{\beta_1 + \beta_2}} \right). \]

Solving for \( u \), we get the indirect utility function
\[ v(p, m) = \left( \frac{\beta_1}{\beta_1 + \beta_2} \left( \frac{m - \alpha_2 p_2}{p_1} - \alpha_1 \right) \right)^{\beta_1} \left( \frac{\beta_2}{\beta_1 + \beta_2} \left( \frac{m - \alpha_1 p_1}{p_2} - \alpha_2 \right) \right)^{\beta_2}. \]

By Roy's law we get the Marshallian demands
\[ x_1(p, m) = \frac{1}{\beta_1 + \beta_2} \left( \beta_1 \alpha_2 + \beta_2 \frac{m - \alpha_1 p_1}{p_2} \right), \]
and
\[ x_2(p, m) = \frac{1}{\beta_1 + \beta_2} \left( \beta_2 \alpha_1 + \beta_1 \frac{m - \alpha_2 p_2}{p_1} \right). \]

8.8 Easy—a monotonic transformation of utility doesn't change anything about observed behavior.

8.9 By definition, the Marshallian demands \( x(p, m) \) maximize \( \phi(x) \) subject to \( px = m \). We claim that they also maximize \( \psi(\phi(x)) \) subject to the same budget constraint. Suppose not. Then, there would exist some other choice \( x' \) such that \( \psi(\phi(x')) > \psi(\phi(x(p, m))) \) and \( px' = m \). But since applying the transformation \( \psi^{-1}() \) to both sides of the inequality will preserve it, we would have \( \phi(x') > \phi(x(p, m)) \) and \( px' = m \), which contradicts our initial assumption that \( x(p, m) \) maximized \( \phi(x) \) subject to \( px = m \). Therefore \( x(p, m) = x^*(p, m) \). (Check that the reverse proposition also holds—i.e., the choice that maximizes \( u^* \) also maximizes \( u \) when the same budget constraint has to be verified in both cases.)

\[ u^*(p, m) = \psi(\phi(x^*(p, m))) = \psi(\phi(x(p, m))) = \psi(u(p, m)). \]
the first and last equalities hold by definition and the middle one by our previous result; now

$$e^*(p, u^*) = \min \{px : \psi(\phi(x)) = u^*\}$$
$$= \min \{px : \phi(x) = \psi^{-1}(u^*)\}$$
$$= e(p, \psi^{-1}(u^*))$$;

again, we’re using definitions at both ends and the properties of $\psi(\cdot)$ — namely that the inverse is well defined since $\psi(\cdot)$ is monotonic— to get the middle equality; finally using definitions and substitutions as often as needed we get

$$h^*(p, u^*) = x^*(p, e^*(p, u^*)) = x(p, e^*(p, u^*)) = x(p, e(p, \psi^{-1}(u^*))) = h(p, \psi^{-1}(u^*))$$.

8.10.a Differentiate the identity $h_j(p, u) \equiv x_j(p, e(p, u))$ with respect to $p_i$ to get

$$\frac{\partial h_j(p, u)}{\partial p_i} = \frac{\partial x_j(p, m)}{\partial p_i} + \frac{\partial x_j(p, e(p, u))}{\partial m} \frac{\partial e(p, u)}{\partial p_i}$$.

We must be careful with this last term. Look at the expenditure minimization problem

$$e(p, u) = \min \{p(x - \overline{x}) : u(x) = u\}$$.

By the envelope theorem, we have

$$\frac{\partial e(p, u)}{\partial p_i} = h_i(p, u) - \overline{x}_i = x_i(p, e(p, u)) - \overline{x}_i$$.

Therefore, we have

$$\frac{\partial h_j(p, u)}{\partial p_i} = \frac{\partial x_j(p, m)}{\partial p_i} + \frac{\partial x_j(p, e(p, u))}{\partial m} (x_i(p, m) - \overline{x}_i)$$,

and reorganizing we get the Slutsky equation

$$\frac{\partial x_j(p, m)}{\partial p_i} = \frac{\partial h_j(p, u)}{\partial p_i} + \frac{\partial x_j(p, e(p, u))}{\partial m} (\overline{x}_i - x_i(p, m))$$.

8.10.b Draw a diagram, play with it and verify that Dave is better off when $p_2$ goes down and worse off when $p_1$ goes down. Just look at the sets of allocations that are strictly better or worse than the original choice—i.e., the sets $SB(x) = \{z : z > x\}$ and $SW(x) = \{z : z < x\}$. When $p_1$ goes down the new budget set is contained in $SW(x)$, while when $p_2$ goes down there’s a region of the new budget set that lies in $SB(x)$. 
8.10. The rate of return—also known as "own rate of interest"—on good \( x \) is \( (p_1/p_2) - 1 \).

8.11 No, because his demand behavior violates GARP. When prices are \( (2, 4) \) he spends 10. At these prices he could afford the bundle \( (2, 1) \), but rejects it; therefore, \( (1, 2) \succ (2, 1) \). When prices are \( (6, 3) \) he spends 15. At these prices he could afford the bundle \( (1, 2) \) but rejects it; therefore, \( (2, 1) \succ (1, 2) \).

8.12 Inverting, we have \( c(p, u) = u/f(p) \). Substituting, we have

\[
\mu(p, q, y) = v(q, y)/f(p) = f(q)y/f(p).
\]

8.13a Draw the lines \( x_2 + 2x_1 = 20 \) and \( x_1 + 2x_2 = 20 \). The indifference curve is the northeast boundary of this \( X \).

8.13b The slope of a budget line is \(-p_1/p_2\). If the budget line is steeper than 2, \( x_1 = 0 \). Hence the condition is \( p_1/p_2 > 2 \).

8.13c Similarly, if the budget line is flatter than \( 1/2 \), \( x_2 \) will equal 0, so the condition is \( p_1/p_2 < 1/2 \).

8.13d If the optimum is unique, it must occur where \( x_2 - 2x_1 = x_1 - 2x_2 \). This implies that \( x_1 = x_2 \), so that \( x_1/x_2 = 1 \).

8.14a This is an ordinary Cobb-Douglas demand: \( S_1 = \frac{a}{\alpha + \beta + \gamma} Y \) and \( S_2 = \frac{b}{\alpha + \beta + \gamma} Y \).

8.14b In this case the utility function becomes \( U(C, S_1, L) = S_1^\alpha L^\beta C^\gamma \). The \( L \) term is just a constant, so applying the standard Cobb-Douglas formula \( S_L = \frac{\alpha}{\alpha + \beta + \gamma} Y \).

8.15 Use Slutsky's equation to write \( \frac{\partial L}{\partial w} = \frac{\partial L}{\partial w} + (L - L) \frac{\partial L}{\partial m} \). Note that the substitution effect is always negative, \( (L - L) \) is always positive, and hence if leisure is inferior, \( \frac{\partial L}{\partial w} \) is necessarily negative. Thus the slope of the labor supply curve is positive.

8.16a True. With the grant, the consumer will maximize \( u(x_1, x_2) \) subject to \( x_1 + x_2 \leq m + g_1 \) and \( x_1 \geq g_1 \). We know that when he maximizes his utility subject to \( x_1 + x_2 \leq m \), he chooses \( x_1^* \geq g_1 \). Since \( x_1 \) is a normal good, the amount of \( x_1^* \) that he will choose if given an unconstrained grant of \( g_1 \) is some number \( x_1^1 \geq x_1^* \geq g_1 \). Since this choice satisfies the constraint \( x_1^1 \geq g_1 \), it is also the choice he would make when forced to spend \( g_1 \) on good 1.
8.16.b False. Suppose for example that \( g_1 = x_1^* \). Then if he gets an unconstrained grant of \( g_1 \), since good 1 is inferior, he will choose to reduce his consumption to less than \( x_1^* = g_1 \). But with the constrained grant, he must consume at least \( g_1 \) units of good 1. Incidentally, he will accept the grant, since with the grant he can always consume at least as much of both goods as without the grant.

8.16.c If he got an unconstrained grant of \( g_1 \), he would spend \( (48 + g_1)/4 \) on good 1. This is exactly what he will spend if \( g_1 \leq (48 + g_1)/4 \). But if \( g_1 > (48 + g_1)/4 \), he will spend \( g_1 \) on good 1. The curve therefore has slope 1/4 if \( g_1 < 16 \) and slope 1 if \( g_1 > 16 \). Kink is at \( g_1 = 16 \).
Chapter 11. Uncertainty

11.1 The proof of Pratt's theorem established that

\[ \pi(t) \approx \frac{1}{2} \sigma^2 t^2. \]

But the \( \sigma^2 t^2 \) is simply the variance of the gamble \( t \).

11.2 If risk aversion is constant, we must solve the differential equation \( u''(x)/u'(x) = -r \). The answer is \( u(x) = -e^{-rx} \), or any affine transformation of this. If relative risk aversion is constant, the differential equation is \( u''(x)x/u'(x) = -r \). The solution to this is \( u(x) = x^{1-r}/(1-r) \) for \( r \neq 1 \) and \( u(x) = \ln x \) for \( r = 1 \).

11.3 We have seen that investment in a risky asset will be independent of wealth if risk aversion is constant. In an earlier problem, we've seen that
constant absolute risk aversion implies that the utility function takes the form \( u(w) = -e^{-cw} \).

11.4 Marginal utility is \( u'(w) = 1 - 2bw \); when \( w \) is large enough this is a negative number. Absolute risk aversion is \( 2b/(1 - 2bw) \). This is an increasing function of wealth.

11.5.a The probability of heads occurring for the first time on the \( j \)th toss is \((1 - p)^{j-1}p\). Hence the expected value of the bet is \( \sum_{j=1}^{\infty} (1 - p)^{j-1}p2^j = \sum_{j=1}^{\infty} 2^{-j}2^j = \sum_{j=1}^{\infty} 1 = \infty \).

11.5.b The expected utility is

\[
\sum_{j=1}^{\infty} (1 - p)^{j-1}p \ln(2^j) = p \ln(2) \sum_{j=1}^{\infty} j(1 - p)^{j-1}.
\]

11.5.c By standard summation formulas:

\[
\sum_{j=0}^{\infty} (1 - p)^j = \frac{1}{p}.
\]

Differentiate both sides of this expression with respect to \( p \) to obtain

\[
\sum_{j=1}^{\infty} j(1 - p)^{j-1} = \frac{1}{p^2}.
\]

Therefore,

\[
p \ln(2) \sum_{j=1}^{\infty} j(1 - p)^{j-1} = \frac{\ln(2)}{p}.
\]

11.5.d In order to solve for the amount of money required, we equate the utility of participating in the gamble with the utility of not participating. This gives us:

\[
\ln(w_0) = \frac{\ln(2)}{p}.
\]

Now simply solve this equation for \( w_0 \) to find

\[
w_0 = e^{\ln(2)/p}.
\]

11.6.a Note that

\[
E[u(R)] = \int_{-\infty}^{\infty} u(s) \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left( \frac{s - \mu}{\sigma} \right)^2 \right\} ds = \phi(\mu, \sigma^2).
\]
11.6.b Normalize \( u(\cdot) \) such that \( u(\mu) = 0 \). Differentiating, we have

\[
\frac{\partial E[u(R)]}{\partial \mu} = \frac{1}{\sigma^2} \int_{-\infty}^{\infty} u(s)(s - \mu)f(s)ds > 0,
\]

since the terms \([u(s)(s - \mu)]\) and \(f(s)\) are positive for all \(s\).

11.6.c Now we have

\[
\frac{\partial E[u(R)]}{\partial \sigma^2} = \frac{1}{\sigma^3} \int_{-\infty}^{\infty} u(s)((s - \mu)^2 - \sigma^2)f(s)ds
\]

\[
< \frac{1}{\sigma^3} \int_{-\infty}^{\infty} u'(\mu)(s - \mu)((s - \mu)^2 - \sigma^2)f(s)ds
\]

\[
= \frac{u'(\mu)}{\sigma^3} \left\{ \int_{-\infty}^{\infty} (s - \mu)^3f(s)ds - \sigma^2 \int_{-\infty}^{\infty} (s - \mu)f(s)ds \right\}
\]

\[
= 0.
\]

The first inequality follows from the concavity of \(u(\cdot)\) and the normalization imposed; the last equality follows from the fact that \(R\) is normally distributed and, hence, \(E[(R - E[R])^k] = 0\) for \(k\) odd.

11.7 Risk aversion implies a concave utility function. Denote by \(\alpha \in [0,1]\) the proportion of the initial wealth invested in asset 1. We have

\[
E[u(\alpha w_0(1 + R_1) + (1 - \alpha)w_0(1 + R_2))]
\]

\[
= \int \int u(\alpha w_0(1 + r_1) + (1 - \alpha)w_0(1 + r_2))f(r_1)f(r_2)dr_1dr_2
\]

\[
> \int \int [\alpha u(w_0(1 + r_1)) + (1 - \alpha)u(w_0(1 + r_2))]f(r_1)f(r_2)dr_1dr_2
\]

\[
= \int u(w_0(1 + r_1))f(r_1)dr_1 = \int u(w_0(1 + r_2))f(r_2)dr_2 = E[u(w_0(1 + R_1))] = E[u(w_0(1 + R_2))].
\]

The inequality follows from the concavity of \(u(\cdot)\).

For part \(b\), proceed as before reversing the inequality since now \(u(\cdot)\) is convex.

11.8.a Start by expanding both sides of \(E[u(\tilde{w} - \pi_u)] = E[u(\tilde{w} + \epsilon)]:\)

\[
E[u(\tilde{w} - \pi_u)] = pu_1(w_1 - \pi_u) + (1 - p)u(w_2 - \pi_u)
\]

\[
\approx p(u(w_1) - u'(w_1)\pi_u) + (1 - p)(u(w_2) - u'(w_2)\pi_u);
\]

\[
E[u(\tilde{w} + \epsilon)] = \frac{p}{2}(u(w_1 - \epsilon) + u(w_1 + \epsilon)) + (1 - p)u(w_2)
\]

\[
\approx p\left(u(w_1) + \frac{u''(w_1)\epsilon^2}{2}\right) + (1 - p)u(w_2).
\]
Combining, we obtain

\[-(pu'(w_1) + (1 - p)u'(w_2))\pi_u \approx \frac{1}{2} pu''(w_1) \epsilon^2,\]

or

\[\pi_u \approx \frac{-\frac{1}{2} pu''(w_1) \epsilon^2}{pu'(w_1) + (1 - p)u'(w_2)}.\]

11.8.b For these utility functions, the Arrow-Pratt measures are \(-u''/u' = a\), and \(-v''/v' = b\).

11.8.c We are given \(a > b\) and we want to show that a value of \((w_1 - w_2)\) large enough will eventually imply \(\pi_v > \pi_u\), thus we want to get

\[\frac{ae^{-aw_1}}{pe^{-aw_1} + (1 - p)e^{-aw_2}} < \frac{be^{-bw_1}}{pe^{-bw_1} + (1 - p)e^{-bw_2}};\]

cross-multiplying we get

\[ape^{-w_1(a+b)} + a(1 - p)e^{-(aw_1+bw_2)} < be^{-w_1(a+b)} + b(1 - p)e^{-(aw_1+bw_2)},\]

which implies

\[(a - b) \frac{p}{1 - p} < be^{a(w_1 - w_2)} - ae^{b(w_1 - w_2)}.\]

The derivative of the RHS of this last inequality with respect to \(w_1 - w_2\) is

\[ab \left( e^{a(w_1 - w_2)} - e^{b(w_1 - w_2)} \right) > 0\]

whenever \(w_1 > w_2\); the LHS does not depend on \(w_1\) or \(w_2\). Therefore, this inequality will eventually hold for \((w_1 - w_2)\) large enough. According to the Arrow-Pratt measure, \(u\) exhibits a higher degree of risk aversion than \(v\). We've shown that \(v\) could imply a higher risk premium than \(u\) to avoid a fair lottery provided there's an additional risk "big" enough. In this case, higher risk premium would no longer be synonymous with higher absolute risk aversion.

11.9 Initially the person has expected utility of

\[\frac{1}{2} \sqrt{4} + 12 + \frac{1}{2} \sqrt{4} + 0 = 3.\]

If he sells his ticket for price \(p\), he needs to get at least this utility. To find the break-even price we write the equation

\[\sqrt{4} + p = 3.\]
Solving, we have $p = 5$.

11.10 The utility maximization problem is $\max \pi \ln(w+x) + (1-\pi) \ln(w-x)$. The first-order condition is

$$\frac{\pi}{w+x} = \frac{1-\pi}{w-x},$$

which gives us $x = w(2\pi - 1)$. If $\pi = 1/2$, $x = 0$.

11.11 We want to solve the equation

$$\frac{p}{w_1} + \frac{1-p}{w_2} = \frac{1}{w}.$$

After some manipulation we have

$$w = \frac{w_1w_2}{pw_2 + (1-p)w_1}.$$

11.12.a

$$\max_{x} \alpha px$$

such that $x$ is in $X$.

11.12.b $\psi(p, \alpha)$ has the same form as the profit function.

11.12.c Just mimic the proof used for the profit function.

11.12.d It must be monotonic and convex, just as in the case of the profit function.