Sec. 5.1: The Goal, Logic and Art of Structural Equation Modeling

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Psych 253
Sources

• For accounts of SEM at different levels of technical difficulty, see HO’s, and HW’s.

• For applications of SEM to single- and multi-group designs, see articles on Coursework.

• Data sets are given in HO’s & HW’s.

• Demonstrate SEM by applying lavaan to
  – Noise pollution data
  – Gene*Environment on ‘schizo’, etc.
  – Scripts in Mx available upon request
Methods of Analysis

• **ANOVA**: For designs with random assignment, manipulation of factors, ‘small’ \( n \). Also, for quasi-expts w/o random assignment, with 1-3 IV’s.

• **Regression**: Can subsume ANOVA; however, usually for correlational designs, ‘large’ \( n \)

• **SEM**: For correlational designs, ‘very large’ \( n \); interest in structure of relations among all observable variables; often introducing **latent** variables
Regression Issues
(all of which recur in SEM)

• Sample size (10 obs. per IV?), capitalizing on chance, need for cross-validation and replication.

• $t$-tests for the effects of individual IV’s; $B$ (raw) versus Beta (standardised) coeffs.

• Global goodness-of-fit of the model ($MSE$, $MS_{reg}$, $R^2$ and $F$ test, CV-PRESS); testing nested models using $\Delta R^2$; Mallows’ criterion, $C_p \approx p$, for ‘best’ model; $AIC$
Regression Issues

• Transforming DV’s or IV’s to linearise a relationship and to stabilize variance, defining interaction or non-linear terms, logistic regression. Centering quantitative variables.

• Multicollinearity; violations of the LINE model.

• Suppressor, mediating and moderating variables. (Handled naturally within SEM.)
Regression Limitations

- No interest in relations among IV’s.
- With 2 or more simultaneous regressions, no overall test for the set of equations.
- No provision for correlated errors.
- No easy way to compare regressions across groups. (Multi-group option in SEM)
- SEM addresses these limitations and can be a very convenient summary of the set of equations. But use regression before SEM to narrow search for acceptable solutions.
SEM

Matrices
Correlations
(KMatrix)
Or
Covariances
(CMatrix)

Regression

SEM
SEM

Matrices
Correlations (KMatrix)
Or
Covariances (CMatrix)

Regression

SEM
A simple example
(all variables standardised)

From Mult Reg: \( \beta_1 = \frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2} \), etc.

Both models have 3 path coeffs; both are consistent with the multiple regression approach in which we care little about the reln between X1 & X2. 0.45 & 0.12 are beta coeffs. 0.84 in Model 2 is the error variance in the regression of X2 on X1, i.e., \( 1 - (r_{12})^2 = 1 - (0.4)^2 = 0.84 \).

These models have as many parameters (3) as there are data points (covariances); they fit the data perfectly and are called saturated models.
# String literals, i.e., characters enclosed within
# single quotes, '/*', are used often in lavaan
# ‘slavex1.r’

library(lavaan)
# Enter correls

lower = '1 .4 1 .5 .3 1

labels1 = c("X1", "X2", "Y")
ex.cov = getCov(lower, names = labels1)
print(ex.cov)

<table>
<thead>
<tr>
<th></th>
<th>X1</th>
<th>X2</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>1.0</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>X2</td>
<td>0.4</td>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Y</td>
<td>0.5</td>
<td>0.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>
# Specify the model with different operators for different relations: =~; ~; ~~. (See Ex. on next slide.)

ex.model = ' latent variable definitions
X =~ U + V

regressions
X + Y ~ U + V + W

variances and covariances
X ~~ X; X ~~ Y

# Note that it isn't necessary to specify covars among exogenous variables; lavaan automatically estimates them, as long as we use sem(..., fixed.x = F, ...)

  intercepts
  X ~ 1            # If one is fitting mean structure as well as covar structure
,'
**Example:** Observed variables in rectangles; latent variables in circles. Black 2-headed arrows are variance or covariance paths (“~~”); blue 1-headed arrows are regression paths (“~”); red 1-headed arrows connect to the indicators of the latent variable (“=~”).
ex.model1 = ' Y ~ X1 + X2,'

# Fit the model
ex.fit1 = sem(ex.model1, fixed.x = F, sample.cov = ex.cov, sample.nobs = 50)

cat('\\n Fit of Model 1, in brief \n')
print(ex.fit1)
cat('\\n Fit of Model 1, in detail \n')
print(summary(ex.fit1))

semPaths(ex.fit1, what = 'std')
Output from `semPaths(ex.fit1, what = 'std')`
ex.model2 = ' 
    Y ~ X1 + X2 
    X2 ~ X1 
',

# Fit the model
ex.fit2 = sem(ex.model2, fixed.x = F, 
sample.cov = ex.cov, sample.nobs = 50)

cat('
    Fit of Model 2, in brief 
')
print(ex.fit2)
cat('
    Fit of Model 2, in detail 
')
print(summary(ex.fit2))

semPaths(ex.fit1, what = 'std')
Both models are saturated, with no df for testing model fit.

Fit of Model 1, in brief
lavaan (0.5-10) converged normally after 16 iterations

Number of observations                  50
Estimator                               ML
Minimum Function Chi-square            0.000
Degrees of freedom                     0
P-value                                1.000
Model 1:

|                | Estimate | Std.err | Z-value | P(>|z|) |
|----------------|----------|---------|---------|---------|
| **Regressions:**|          |         |         |         |
| \( Y \sim \)   |          |         |         |         |
| X1             | 0.452    | 0.133   | 3.413   | 0.001   |
| X2             | 0.119    | 0.133   | 0.898   | 0.369   |
| **Covariances:**|          |         |         |         |
| X1 \( \sim \) X2| 0.392    | 0.149   | 2.626   | 0.009   |
| **Variances:**  |          |         |         |         |
| Y              | 0.723    | 0.145   |         |         |
| X1             | 0.980    | 0.196   |         |         |
| X2             | 0.980    | 0.196   |         |         |
Model 2: Note change in var($X_2$)

| Regressions: | Estimate | Std.err | Z-value | P($>|z|)$ |
|-------------|----------|---------|---------|----------|
| $Y \sim$    |          |         |         |          |
| $X_1$       | 0.452    | 0.133   | 3.413   | 0.001    |
| $X_2$       | 0.119    | 0.133   | 0.898   | 0.369    |
| $X_2 \sim$  |          |         |         |          |
| $X_1$       | 0.400    | 0.130   | 3.086   | 0.002    |

<table>
<thead>
<tr>
<th>Variances:</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>0.723</td>
<td>0.145</td>
<td></td>
<td></td>
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<tr>
<td>$X_2$</td>
<td>0.823</td>
<td>0.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_1$</td>
<td>0.980</td>
<td>0.196</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Variable definitions and correlation matrices used in constructing plausible causal models

1. Noise pollution data (‘mlm.anlat.txt’)

The 2 participants in each house are labeled ‘1’ and ‘2’; hence ‘age1’, ‘age2’, etc. ‘Latency’ is the delay in falling asleep; ‘annoy’ is the participant’s level of annoyance; and ‘noise’ is the average noise level across the nights of the study in that house. Mixed models analyses of Level-1 data suggest (i) ‘age’ affects ‘latency’, and ‘noise’ does not affect ‘latency’, (ii) ‘noise’ and ‘latency’ affect ‘annoy’; ‘age’ does not affect ‘annoy’. What relationships hold for Level-2 data (i.e., data averaged across ‘nights’)? Use SEM, which, like Factor Analysis, is a method based on covariances or correlations.

However, the sample size needed for reliable results in SEM is greater than that needed in regression (which, in turn, is greater than that needed in t-tests). We made the choice to treat the ‘house’ as our unit of analysis (Why?); our main interest is in examining the social interaction between the 2 adults in a house. The sample size is then \( n = 24 \) houses in the original data set. To generate a fictional sample that is (a) ‘large’, and (b) faithful to the original sample, we sampled, with replacement, 100 houses from the sample of 24 houses, and used the resulting data. Are the correlations in the fictional sample approximately equal to those in the original sample? ‘Yes’, according to the plot below. We use the covariance matrix as our input to Mx whenever we have one, because convergence tends to be more likely than if the correlation matrix is used. Note that \( \text{cov}(X, Y) = r_{XY} \cdot \text{sd}(X) \cdot \text{sd}(Y) \).

![Bootstrap corr vs actual corr](image-url)

<table>
<thead>
<tr>
<th>n=100 corr matrix</th>
<th>noise</th>
<th>age1</th>
<th>age2</th>
<th>latency1</th>
<th>latency2</th>
<th>annoy1</th>
<th>annoy2</th>
</tr>
</thead>
<tbody>
<tr>
<td>noise</td>
<td>1.0000</td>
<td>0.1336</td>
<td>0.1049</td>
<td>-0.0305</td>
<td>0.1329</td>
<td>0.0917</td>
<td>-0.0551</td>
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<tr>
<td>age1</td>
<td>0.1336</td>
<td>1.0000</td>
<td>0.9080</td>
<td>-0.2993</td>
<td>0.1662</td>
<td>0.4616</td>
<td>0.5082</td>
</tr>
<tr>
<td>age2</td>
<td>0.1049</td>
<td>0.9080</td>
<td>1.0000</td>
<td>-0.1972</td>
<td>0.0733</td>
<td>0.4547</td>
<td>0.5787</td>
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<tr>
<td>latency1</td>
<td>-0.0305</td>
<td>-0.2993</td>
<td>-0.1972</td>
<td>1.0000</td>
<td>0.2349</td>
<td>-0.0059</td>
<td>0.0293</td>
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<tr>
<td>latency2</td>
<td>0.1329</td>
<td>0.1662</td>
<td>0.0733</td>
<td>0.2349</td>
<td>1.0000</td>
<td>0.3698</td>
<td>0.0792</td>
</tr>
<tr>
<td>annoy1</td>
<td>0.0917</td>
<td>0.4616</td>
<td>0.4547</td>
<td>-0.0059</td>
<td>0.3698</td>
<td>1.0000</td>
<td>0.6240</td>
</tr>
<tr>
<td>annoy2</td>
<td>-0.0551</td>
<td>0.5082</td>
<td>0.5787</td>
<td>0.0293</td>
<td>0.0792</td>
<td>0.6240</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Lecture 2: Noise pollution data

• Average across ‘nights’ (level 1) to examine level 2 (‘house’) data. At this level, ‘noise’ does not predict (ave) ‘latency’ or ‘annoy’.

• Use lavaan to examine relations among ‘age’, ‘latency’ and ‘annoy’ for the 2 adults in a home. Focus on social interaction.

• Possible Strategy: Try models of increasing complexity to find good fit (while deleting ‘weak’ links to save degrees of freedom).

• Next steps: Treat this good-fitting model as the unconstrained model. Constrain certain parameters to examine social interaction.
Data Ninput=6 Nobs=100

CMatrix

1.0788
0.9258 0.9637
-0.2018 -0.1257 0.4217
0.1377 0.0574 0.1216 0.6362
0.4189 0.3899 -0.0034 0.2577 0.7631
0.4748 0.5111 0.0171 0.0568 0.4904 0.8094

Labels age1 age2 latency1 latency2 annoy1 annoy2

See ‘smlm4.r’
Modeling options

• Whence the structural model? Usually from one’s curiosity or theory. E.g.,
• Suppose one finds age $\rightarrow$ annoy. Curiosity might lead one to seek a mediator for this effect, e.g., latency, and test, e.g., full vs partial mediation:
  
  $\text{age} \rightarrow \text{latency} \rightarrow \text{annoy}$

• Add to the mix a 2$^{\text{nd}}$ adult with same data. How to model social-interactive effects? Ans. Try causal modeling and SEM – with 1- and 2-headed arrows.
Use lavaan() to fit **student-generated** models in class. ‘smlm4.r’ can be used for this analysis.
This is model1 in ‘smlm4.r’. Comment on covar paths. Which paths have been omitted, and why? This model has 13 paths, i.e., 13 parameters (excl variances). The # of data points = # of covs. With $k$ variables, # of covs = \textbf{# of ways to choose 2 from} \quad k = k(k-1)/2 = 15, \quad \text{when} \quad k = 6. \quad \text{So 2 df left for model tests.}^{24}
mlm.model1 = '  
    # regressions  
    latency1 ~ age1  
    latency2 ~ age2  
    annoy1 + annoy2 ~ latency1 + latency2 + age1 + age2

    # variances and covariances  
    latency1 ~~ latency2
,  

mlm.fit1 = sem(mlm.model1, fixed.x = T, sample.cov = mlm.cov, sample.nobs = 100)

df('mlm.semplot.pdf')
semPaths(mlm.fit1, what = 'std')
graphics.off()

**Result:** chisq = 13.2 with 2 df - poor fit, even though model is almost saturated! To fix this, replace unhelpful paths with helpful ones, using the **corr matrix** or **residuals(mlm.fit1)** for hints.
### Regressions:

| Equation       | Estimate | Std.err | Z-value | P(>|z|) |
|----------------|----------|---------|---------|---------|
| latency1 ~ age1 | -0.211   | 0.059   | -3.557  | 0.000   |
| latency2 ~ age2 | 0.032    | 0.080   | 0.396   | 0.692   |
| annoy1 ~ latency1 | 0.022    | 0.122   | 0.181   | 0.857   |
|                 | latency2 | 0.354   | 0.095   | 3.741   | 0.000   |
|                 | age1     | 0.089   | 0.167   | 0.532   | 0.595   |
|                 | age2     | 0.301   | 0.175   | 1.721   | 0.085   |
| annoy2 ~ latency1 | 0.208    | 0.122   | 1.703   | 0.089   |
|                 | latency2 | -0.001  | 0.095   | -0.013  | 0.989   |
|                 | age1     | 0.004   | 0.168   | 0.024   | 0.981   |
|                 | age2     | 0.554   | 0.176   | 3.155   | 0.002   |

### Covariances:

| Covariance       | Estimate | Std.err | Z-value | P(>|z|) |
|------------------|----------|---------|---------|---------|
| latency1 ~ latency2 | 0.147    | 0.051   | 2.879   | 0.004   |
| annoy1 ~ annoy2   | 0.271    | 0.058   | 4.666   | 0.000   |

### Variances:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std.err</th>
</tr>
</thead>
<tbody>
<tr>
<td>latency1</td>
<td>0.381</td>
<td>0.054</td>
</tr>
<tr>
<td>latency2</td>
<td>0.627</td>
<td>0.089</td>
</tr>
<tr>
<td>annoy1</td>
<td>0.512</td>
<td>0.072</td>
</tr>
<tr>
<td>annoy2</td>
<td>0.516</td>
<td>0.073</td>
</tr>
</tbody>
</table>
> residuals(mlm.fit1)
$ cov

<table>
<thead>
<tr>
<th></th>
<th>ltncy1</th>
<th>ltncy2</th>
<th>annoy1</th>
<th>annoy2</th>
<th>age1</th>
<th>age2</th>
</tr>
</thead>
<tbody>
<tr>
<td>latency1</td>
<td>-0.011</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>latency2</td>
<td>-0.020</td>
<td>0.002</td>
<td></td>
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<tr>
<td>annoy1</td>
<td>0.015</td>
<td>0.018</td>
<td>0.013</td>
<td></td>
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<td></td>
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<tr>
<td>annoy2</td>
<td>0.036</td>
<td>0.011</td>
<td>0.009</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>age1</td>
<td>0.025</td>
<td>0.107</td>
<td>0.039</td>
<td>0.005</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>age2</td>
<td>0.069</td>
<td>0.026</td>
<td>0.011</td>
<td>0.014</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

cov(age1, latency2) & cov(age2, latency1) poorly predicted. One reason might be the reciprocal influences between the 2 people.

Try adding age1 --> lat2 and age2 --> lat1 paths.
We show the significant or theoretically interesting links. Focus on the 'interactive' effects, (Gu, Hu) and (Ju, Ku). Is fit worsened by equating Gu & Hu, and Ju & Ku?
To equate path coeffs, simply assign the same label to paths. Replace Hu by Gu, and Ku by Ju. $X^2$ increases from 3.722 to 5.935, df increases from 4 to 6; so test $\Delta = 2.213$ with 2 df; n.s. Conclude that simpler, symmetric interaction model is OK.
This is model3 in ‘smlm4.r’; it is a little different from that on preceding slide. Comment on symmetry.
Symbolic version of the model drawn in previous slide

mlm.model3 = '
    # regressions
    # X + Y ~ U + V + W
    # s for self, o for other, a for age, l for lat
    latency1 ~ sa1*age1 + oa1*age2
    latency2 ~ oa1*age1 + sa1*age2
    annoy1 ~ ol1*latency2 + sa2*age1
    annoy2 ~ ol1*latency1 + sa2*age2
    # variances and covariances
    # X ~~ Y
    latency1 ~~ latency2
    annoy1 ~~ annoy2
    '
Bushman (Psychol Sci, 2005): TV content & ads

- Adults (N = 336) watched a TV program containing (i) violence or sex, or (ii) neither violence nor sex; all programs contained the same 12 ads.
- Result: ‘Violence or sex’ reduced
- (a) P(remember brand) – recall & recog,
- (b) Interest in buying that brand, and
- (c) P(selecting a coupon for brand) = ‘Buying behav’.
- These effects occurred regardless of whether Ps liked programs containing violence and sex.
- What are the roles of memory & intent in ‘buying behavior’? Use SEM to find an answer.
Bushman covariances

Data Ninput=5 Nobs=336
CMatrix

0.18
-0.085  1.174
-0.349  1.737  7.618
-0.241  0.856  3.388  5.833
-0.207  0.655  1.988  2.959  11.229

Labels TVViolSx Recall Recog Intent Behavior
The Role of Memory and Buying Intentions in Coupon Choices

Structural equation models were computed with AMOS using maximum likelihood estimation (Arbuckle, 1999). The hypothesized model was that brand memory and buying intentions mediate the effect of TV violence and sex on coupon choices. In specifying the model, a dummy variable was used to represent TV violence and sex (violent or sexual program = 1, neutral program = 0). Brand memory was treated as a latent variable, measured using brand recall and recognition. The variance-covariance matrix used for the analyses is given in Table 2.
The hypothesized mediation model fit the data extremely well, $\chi^2(5, N = 336) = 4.76$, $p < .45$ (sic!), GFI (goodness-of-fit index) = .995, CFI (comparative fit index) = 1, RMSEA (root mean square error of approximation) = .005.

As can be seen in Figure 1, TV violence and sex impaired brand memory. Brand memory and buying intentions were important mediators. People intended to buy brands they could remember, and they chose coupons for brands they intended to buy.
TV Content Effects on Sales
(Try to reproduce this model in class!)

• Bushman (2005)
We don’t need a latent variable in Model 1
Model 1

tvv.model1 ='
  # regressions
  # X + Y ~ U + V + W
  recall ~ tvviol
  recog ~ tvviol
  intent ~ tvviol + recog
  behavior ~ intent

  # variances and covariances
  # X ~~ Y

  recall ~~ recog
,'
But latent variable models are certainly elegant – if you can get them to fit the data! **Model 2:**
Nested Models

- Starting with a baseline model, construct a more constrained model by fixing the value of a parameter (e.g., to 0), or by equating two parameters.
- $\chi^2$ will be greater for the more constrained model; whether it is significantly greater depends on the difference in df of the two models.
- If fit for constrained model = fit for baseline model, we choose the former because it is more parsimonious.
2. Gene-Environment interactions (‘comt0.csv’)

Data on environmental, behavioral, genetic and control variables in columns, ‘earlyon’–‘chpsycq’.

1. Environmental risk = adolescent-onset use of cannabis (‘earlyon’ = 1/0, i.e., yes/no).
2. Genotypes of the COMT gene with two alleles, valine (V) and methionine (M), that affect the level of dopamine activity. ‘genotype’ could be a quantitative variable (‘COMT’) with values 0 (for MM individuals), 1 (for VM) and 2 (for VV).
3. ‘schizo’ (= 1/0), indexes whether or not a person is diagnosed with schizophreniform disorder at age 26, and
4. ‘psysym’ is a self-report of the severity of psychotic symptoms at that age. (This variable is synthetic.)

Many control variables were measured, such as,

1. ‘chpsycq’, a quantitative measure of the severity of childhood psychotic symptoms,
2. ‘chpsych’, a 0/1 variable based on ‘chpsycq’;
3. ‘conduct’ (= 1/0), adolescent conduct disorder;
4. ‘adultuse’ (= 1/0), use of other drugs.

Correl matrix for 7 vars

<table>
<thead>
<tr>
<th></th>
<th>earlyon</th>
<th>comt</th>
<th>schizo</th>
<th>psysym</th>
<th>conduct</th>
<th>chpsycq</th>
<th>genxenv</th>
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</thead>
<tbody>
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<td>0.110</td>
<td>0.360</td>
<td>0.353</td>
<td>0.014</td>
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<td>1.000</td>
<td>0.008</td>
<td>0.081</td>
<td>0.036</td>
<td>-0.077</td>
<td>0.334</td>
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<td>0.008</td>
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<td>-0.099</td>
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<td>0.334</td>
<td>0.139</td>
<td>0.340</td>
<td>0.260</td>
<td>-0.057</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Use this correlation matrix as a guide to selecting models in the covariance-based SEM.
Other Examples. COMT-Environment
(Maybe omit COMT and Chpsyq!)

Appears OK  ML ChiSq
Group Fit: 11.7178
Fits 11.718 (5.005, 26.030)
Probability 0.020
AIC 3.718 (-2.995, 18.030)
RMSEA 0.049 (0.018, 0.083)
Degrees of freedom 4
Free parameters 24
Observed Statistics 28
Constraints 0
SEM Lecture 3: Latent variables

• Reprise of Bushman et al: Ads embedded in TV shows with violence or sex are less persuasive than ads in TV shows without such content. What are some of the mediators of this effect? ‘Memory’, ‘intent’?

• Interpersonal Theory and complementarity. See Sadler & Wells

• Time permitting, introduce Multi-group option
The Role of **Memory** and Buying **Intentions** in Coupon **Choices**

The hypothesized model was that brand memory and buying intentions mediate the effect of TV violence and sex on coupon choices. In specifying the model, a **dummy variable** was used to represent TV violence and sex (violent or sexual program = 1, neutral program = 0). Brand memory was treated as a **latent variable**, measured using brand recall and recognition. The **variance-covariance matrix** used for the analyses is given in Table 2.
The coeffs shown above are standardized; those in a previous model are unstandardized. What is the `lavaan()` syntax for the model?
Model 2

tvv.model2 = '  
  # Latent variable definition for brand.mem
  
  brand.mem =~ recog + recall  
  # recog, 1st variable on RHS, is perfect index of brand.mem
  
  # regressions
  brand.mem ~ tvviol
  intent ~ brand.mem
  behavior ~ intent
  
  # variances and covariances
  recall ~~ 0*recog
  '
Summary of Model 2 with latent brand.mem

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>336</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Function Test Statistic</td>
<td>4.920</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>5</td>
</tr>
<tr>
<td>P-value (Chi-square)</td>
<td><strong>0.426</strong></td>
</tr>
</tbody>
</table>

Parameter estimates:

| Latent variables: | Estimate | Std.err | Z-value | P(|z|) |
|-------------------|----------|---------|---------|-------|
| brand.mem =~ | | | | |
| recog | 1.000 | | | |
| recall | 0.262 | 0.031 | 8.499 | 0.000 |

| Regressions: | Estimate | Std.err | Z-value | P(|z|) |
|--------------|----------|---------|---------|-------|
| brand.mem ~ tvviol | -1.965 | 0.336 | -5.846 | 0.000 |
| intent ~ brand.mem | 0.514 | 0.065 | 7.921 | 0.000 |
| behavior ~ intent | 0.507 | 0.070 | 7.200 | 0.000 |
Parameters of Model 2 with latent brand.mem

Covariances:
recog ~~ recall  0.000

Variances:
recog           1.012    0.637
recall          0.719    0.071
intent          4.075    0.359
behavior        9.699    0.748
brand.mem       5.890    0.809

Comment: Earlier we fit Model 1 without latent variables. However, Model 2 is more economical!
In Model 1, $tvviol \rightarrow \{\text{recall, recog, intent}\}$; $\text{recog} \leftrightarrow \rightarrow \text{recall}$; $\text{recog} \rightarrow \text{intent}$; and $\text{intent} \rightarrow \text{behavior}$.

In Model 2, $tvviol \rightarrow \text{brand.mem}$ [latent]; $\text{brand.mem} \rightarrow \text{intent}$; and $\text{intent} \rightarrow \text{behavior}$. $\text{Brand.mem}$ is indicated mainly by $\text{recog}$, but also by $\text{recall}$. [In your research, try to use 3 or more indicators of each latent variable.]

Can we agree that Model 2 is the more elegant?!
Interpersonal Theory and Complementarity

• Sadler & Woody (2003)’s elegant SEM for testing an interpersonal theory of dyadic interaction: ‘Interactive behavior’ = ‘Dominance’ or, in a separate analysis, ‘Affiliation’

• Person M’s ‘behavior’ when interacting with person F (i.e., M’s situational behavior) is influenced by
  – M’s long-term tendency to elicit the behavior (i.e., M’s behavioral trait), and
  – F’s situational behavior.

• The same is assumed true for Person F, mutatis mutandis.
Key aspects of this analysis

• Choices of latent and manifest variables
• Reciprocal effects
• Covariance arrows between manifest variables to indicate unexplained correlations, e.g., due to the multiple measures from the same observer. For aesthetic reasons, these paths are omitted in article!
• > 1 behavior, ‘dominance’ and ‘affiliation’, to test the theory
• Identification of certain structural parameters with the main features of the theory, so as to predict how these parameters should vary with behavior (i.e., dominance vs affiliation)
Figure 1. Model for situational dominance as a function of trait dominance and reciprocity. The letters $d_1$ through $d_4$ and $e_1$ through $e_6$ designate error variables reflecting imperfect measurement by the respective indicators of the underlying traits, and $Z_1$ and $Z_2$ represent unexplained variance in the two situational latent variables. Paths $A$ and $B$ represent the effect of individuals' traits on their situational behavior (actor effects), and Paths $C$ and $D$ represent the effects of individuals' situational behavior on each other (mutual-influence effects).
• M and F were strangers, so M’s trait (unknown to F) would not affect F’s situational behavior.
• However, if M and F were friends, such links should be included in the model.
• Trait is measured by self-report and a friend’s report; and situational behavior is rated by self, partner and an observer. Rater biases are introduced through the use of covariance arrows. For example,
• M’s self-rating of trait, M’s self-rating of situational behavior, and M’s rating of F’s situational behavior are all obtained from M and, therefore, are likely to be correlated. Hence, covariance arrows are added to connect the 3 pairs of manifest variables in this set of 3 variables.
HW exercises
(‘sdyad1.r’)

• **Test** the null hypothesis that the reciprocal effects, $C$ and $D$, are equal in magnitude.

• Are the values of $C$ and $D$ **consistent** with interpersonal theory.

• Which **rater biases** are significant (i.e., which biases need to be considered when testing the theory)?

• Repeat for ‘affiliation’
dyad.model1 = '
  # Latent variable definitions for trait, 
  # FTZ & MTZ, and situation, FSZ & MSZ
  # F =~ X1 + X2 + X3

  MTZ =~ MTSR + MTFR
  MSZ =~ MSSR + MSIR + MSOR
  FTZ =~ FTSR + FTFR
  FSZ =~ FSSR + FSIR + FSOR
dyad.model1 = ‘ ... ...'

# regressions
# X + Y ~ U + V + W

FSZ ~ B*FTZ + C*MSZ
MSZ ~ A*MTZ + D*FSZ
Covariance paths not shown in graph

# \text{corr}(MTZ, FTZ) = 0, \text{ because } M \& F \text{ are strangers}

MTZ \sim 0*FTZ

# No path is shown, none is intended!
Covariance paths not shown in graph

# Residual correls among M ratings, MTSR, MSSR, FSIR
MTSR + MSSR \sim FSIR
MSSR \sim MTSR
Covariance paths not shown in graph

# Residual correls among F ratings, FTSR, FSSR, MSIR
FTSR + FSSR ~~ MSIR
FSSR ~~ FTSR
Covariance paths not shown in graph

# Residual correls among Obser situ ratings, MSOR, FSOR

MSOR ~ FSOR
This causal graph includes **some** of the covariance arrows included in the symbolic 'dyad.model1'. It is already more 'messy' than the graph in the Sadler & Woody's article!
# Fit the model

dyad.fit1 = sem(dyad.model1, fixed.x = F, sample.cov = dom.cov, sample.nobs = 112)

cat('\\n\\nSummary of Model 1 for Dominance \\
\\n')
print(summary(dyad.fit1))

## Model fit is good.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Function Test Statistic</td>
<td>18.614</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>24</td>
</tr>
<tr>
<td>P-value (Chi-square)</td>
<td>0.772</td>
</tr>
</tbody>
</table>
# Fit the model

dyad.fit1 = sem(dyad.model1, fixed.x = F, sample.cov = dom.cov, sample.nobs = 112)

cat('

Summary of Model 1 for Dominance 
"
"
print(summary(dyad.fit1))

## Model fit is good. Prune the model of unhelpful paths
## The only resid cov paths that approach sig are the trait & situational ratings of a person

## Go to R Console to discuss output.
### Latent variables:

|       | Estimate | Std.err | Z-value | P(>|z|) |
|-------|----------|---------|---------|---------|
| **FTZ =~** |          |         |         |         |
| FTSR  | 1.000    |         |         |         |
| FTFR  | 1.139    | 0.238   | 4.781   | 0.000   |
| **MTZ =~** |          |         |         |         |
| MTSR  | 1.000    |         |         |         |
| MTFR  | 0.962    | 0.332   | 2.895   | 0.004   |
| **FSZ =~** |          |         |         |         |
| FSSR  | 1.000    |         |         |         |
| FSIR  | 0.830    | 0.169   | 4.924   | 0.000   |
| FSOR  | 0.935    | 0.178   | 5.263   | 0.000   |
| **MSZ =~** |          |         |         |         |
| MSSR  | 1.000    |         |         |         |
| MSIR  | 1.118    | 0.207   | 5.410   | 0.000   |
| MSOR  | 1.666    | 0.303   | 5.495   | 0.000   |

1. The 2 indicators of latent trait, self & friend, are equally reliable.
2. Ditto for the 3 indicators of latent state, except for MSOR?
Regressions:

FSZ ~
FTZ  (B)  0.705  0.135  5.217  0.000
MSZ  (C)  -0.275  0.160  -1.726  0.084

MSZ ~
MTZ  (A)  0.395  0.145  2.722  0.006
FSZ  (D)  -0.266  0.114  -2.335  0.020

$A > 0, B > 0$, as expected – why? Ditto for $C < 0, D < 0$.
Test for asymmetry?
1. Introducing model constraints on labeled parameters, C & D

```r
model2 = '...
    FSZ ~ B*FTZ + C*MSZ
    MSZ ~ A*MTZ + D*FSZ
  ...
',
fit2 = sem(model2, ...)

model2a = '...
    FSZ ~ B*FTZ + C*MSZ
    MSZ ~ A*MTZ + C*FSZ
  ...
',
fit2a = sem(model2a, ...)

anova(fit2a, fit2)
```
model2 = ‘...
    FSZ ~ B*FTZ + C*MSZ
    MSZ ~ A*MTZ + D*FSZ
    ...
’

model2a is model2 with ‘D’ replaced by ‘C’.
fit2 = sem(model2, ..); fit2a = sem(model2a, ..)
anova(fit2a, fit2)

OR, equivalently,

fit2b = sem(model2, ..., constraints = 'C - D == 0', start = fit2)
anova(fit2b, fit2)

## ‘C - D == 0’ same as ‘C == D’
A difficulty with constraints = ‘…’

On Monday, May 27, 2013 10:50:13 AM UTC-7, Ewart Thomas wrote:
- hide quoted text -
one can assign 2 paths the same label, e.g., A, so as to force the 2 path coeffs to be equal.

but, instead of retyping an entire model with the new constraint, e.g., replacing B by A in the 2nd path, is there a shortcut using, e.g., lavaanify(model1, constraints = 'A == B')? this particular shortcut does not work. does anyone know of a workable shortcut? thanks.
ewart
On 05/28/2013 06:43 PM, Alex Schoemann wrote:
This seems to be a start value issue. When you use the
following to fit dyad.fit3a things work (it looks the same as
dyad.fit3):

dyad.fit3a = sem(dyad.model2, fixed.x = F, sample.cov =
dom.cov, sample.nobs = 112, constraints = 'C - D == 0',
start = dyad.fit2)

The start argument is using values from the model dyad.fit2
as starting values. If I had to guess, the optimizer is doing
different things with explicit equality constraints (e.g. using the
same label) and external constraints (but that's just a guess)
and things were going haywire somewhere.

Alex
This is indeed the case (at least in 0.5-13 and older). Explicit constraints (using the constraints argument) are handled by the 'constrained' optimizer (nlminb.constr), which is much more sensitive to starting values than the unconstrained optimizer (nlminb).

If you can, it is always better to use the 'same label, same value’ trick in the model syntax. Here, we simply reduce the dimension of the parameter space and apply unconstrained optimization. Much faster, much more more stable.

One day, lavaan will 'detect' simple equality constraints provided by the 'constraints' argument, and handle them accordingly.

Yves.
Last Lecture

- Multiple Groups option: Testing constraints on parameters
- Required sample size in SEM
- Use Clark & Smith example to illustrate the matrix approach to SEM
- Review the range of fit functions or ‘objective’, $D$, which is a function of the model parameters (path coeffs and variances). The computer finds the parameter values that minimize $D$ using a ‘gradient’ method.
- More SEM examples
Lecture 4: Multiple Groups option for analyzing group differences

Psych 253
(using slides made by Louise Chim & Daniella Furman
5/13/2010)
An Example of possible Group differences

- \( X \) = perceived responsibility of a defendant for a harm
- \( Y \) = severity of punishment
- Compare 2 groups of potential jurors
  - Group difference in \( E(X) \)?
  - Group difference in \( E(Y) \)?
  - Group difference in \( r_{XY} \)? This group difference in relationship or regression or covariance is described as a structural difference, and is perhaps most interesting.
- Test \( Y = b_0 + b_1X + b_2G + b_3X*G(0/1) \); is the interaction term, \( b_3 \), significant?
• When there are more than 2 variables, group differences in **structure** are hard to investigate using multiple regression. Factor Analysis, Cluster Analysis, and other multivariate methods are useful here. So is SEM.

• Groups might be defined by experimental manipulation, gender, culture, etc.

• Use J. Tsai’s “Affect Valuation Theory” to study cultural differences in structure.
Affect Valuation Theory

Cultural Factors → “Ideal” Affect → “Actual” Affect

Tsai, Knutson, & Fung (2006); Tsai, Louie, Chen, & Uchida (2007); Tsai, Miao, & Seppala (2007); Tsai, Miao, Seppala, Fung, & Yeung (in press); Tsai (in press).
• Is fit between data and a general model good for all groups? Benefits of the discipline of testing *nested* models.

• Start with a fairly general structure that can subsume the separate group diagrams. Any group diagram can then be obtained from the general model by
  – Setting some parameters equal to 0 (e.g., a relationship is present in EA but not CH), or
  – Equating parameters across groups (e.g., a relationship is the same in EA and CA, but not CH)

• For multiple group fits, we need to explicitly label the relevant paths so as to keep track of which parameters we want to equate, or not equate, across groups (we equate 2 parameters simply by giving them the same name).
Ideal versus Actual Affect: Measuring Ideal Affect

• Please use the following rating scale to indicate how much you would IDEALLY like to feel the following states on average…….

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Never</td>
<td>A small amount of the time</td>
<td>Half the time</td>
<td>Most of the time</td>
<td>All the time</td>
</tr>
</tbody>
</table>
High Arousal (HA)

Aroused
Astonished
Surprised

HAP
Enthusiastic
Elated
Excited
Euphoric

Unpleasant (N)

Unhappy
Sad
Lonely

HAN
Fearful
Hostile
Nervous

Pleasant (P)

Happy
Content
Satisfied

LAP
Relaxed
Calm
Peaceful
Serene

LAN
Dull
Sleepy
Sluggish

Low Arousal (LA)

Idle
Passive
Inactive

Two-Dimensional Map of Affective States
Ideal versus Actual Affect: Measuring Actual Affect

- Please use the following rating scale to indicate how much you ACTUALLY feel the following states on average......

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>A small amount of the time</td>
<td>Half the time</td>
<td>Most of the time</td>
<td>All the time</td>
</tr>
</tbody>
</table>
European Americans Value HAP More and LAP Less Than Hong Kong Chinese

**Graph:**
- X-axis: HAP (excited) to LAP (calm)
- Y-axis: Value
- Three conditions: EA, CA, CH
- Legend:
  - EA: Red square
  - CA: Green circle
  - CH: Blue triangle

+Controlling for actual HAP and LAP

Affect Valuation Theory

Cultural Factors

Temperament

“Ideal” Affect → Mood Producing Behavior

“Actual” Affect → Physiological Activation

Facial and Vocal Behavior

Conceptions of Mental Health

Tsai, Knutson, & Fung (2006); Tsai, Louie, Chen, & Uchida (2007); Tsai, Miao, & Seppala (2007); Tsai, Miao, Seppala, Fung, & Yeung (in press); Tsai (in press).
Statistical Issues That Can Be Addressed By SEM

• Are the measures equivalent across cultures? This is the issue of ‘factorial invariance’.

• Are there cultural differences in response styles? E.g., differences in mean response to a given stimulus?

• Does AVT hold across cultures? Age? SES? E.g., Are the structural parameters from an SEM the same across cultures (groups)?
1-factor model with 2 independent groups: Are the loadings, \( \{u_{1i}\} \) and \( \{u_{2i}\} \), the same for the 2 groups – factorial invariance?

For Group 1 (ditto for Group 2),

\[
X_{1i} = u_{1i} F_1 + e_{1i} ; \quad X_{1j} = u_{1j} F_1 + e_{1j}
\]
Factorial Invariance: Do we have enough data points (or enough indicator variables) to estimate all parameters?

- With $k$ variables in a Group, we have $\frac{k(k-1)}{2}$ covariances and $k$ variances, for a total of $\frac{k(k+1)}{2}$ data points. In the above model, we have, for each Group, 1 factor variance, $k$ path coefficients and $k$ error variances, for a total of $2k+1$ parameters.

- We would not be able to estimate the parameters unless we have more data points than parameters:

$$\frac{1}{2}k(k+1) \geq 2k + 1; \quad i.e., \, k^2 - 3k - 2 \geq 0; \quad i.e., \, k \geq 4$$

- A worked example of factorial invariance is in HO-6-SEM2.pdf
Testing AVT across cultures

ActualHAP: A composite of ‘I actually feel “enthusiastic”, “excited”, “elated” and “euphoric”.

IdealHAP: A composite of ‘I prefer to feel “enthusiastic”, “excited”, “elated” and “euphoric”.

Depression: On a 1-50 scale

RigorAct: Level of rigorous physical activities

CultAtt: Average of Independence and Schwartz Influence

TemperAtt: The composite, Extraversion – Neuroticism
Model 1
d = read.csv("jt-data1.csv")

avt.model1 = '
    # regressions
    ideahap + actuhap ~ cultatt + temperatt
    rigoract + depress ~ ideahap + actuhap + cultatt + temperatt

    # variances and covariances
    rigoract ~~ 0*depress
',

avt.fit1 = sem(avt.model1, fixed.x = F, data = d)

<table>
<thead>
<tr>
<th>Estimator</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Function Test Statistic</td>
<td>9.547</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>2</td>
</tr>
<tr>
<td>P-value (Chi-square)</td>
<td>0.008</td>
</tr>
</tbody>
</table>
avt.model2 = ' 
  # regressions
  # X + Y ~ U + V + W

  ideahap + actuhap ~ cultatt + temperatt
  rigoract ~ ideahap + temperatt
  depress ~ actuhap + temperatt

  # variances and covariances
  # X ~~ Y
  rigoract ~~ 0*depress
  ideahap ~~ actuhap

, '

Estimator                                         ML
  Minimum Function Test Statistic                3.556
  Degrees of freedom                                 5
  P-value (Chi-square)                           0.615
## Multiple Group analyses: Does Model 2 apply to any one of the 3 groups, EA, AA and CH?

```r
grplab1 = c("EA", "AA", "CH")
```

# Fit Model 2 to all 3 groups, params differ across groups

```r
avt.group1 = sem(avt.model2, fixed.x = F, data = d, group = "group", meanstructure = F)
```

print(summary(avt.group1))
Summary of unconstrained Model 2 fit to all groups

<table>
<thead>
<tr>
<th>Number of observations per group</th>
<th>Used</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>67</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>83</td>
<td>84</td>
</tr>
</tbody>
</table>

Estimator: ML
Minimum Function Test Statistic: 15.149
Degrees of freedom: 15
P-value (Chi-square): 0.441

Chi-square for each group:

1: 9.798
2: 2.776
3: 2.574

## The fit to each group is acceptable, except that the fit to Group 1 is almost sig: chisq = 9.8 with 5 df, p = .08.
## The parameters were free to vary across groups. What if we were to equate params across groups? See ‘savt1.r’: avt.group2 & avt.model3.
Last Lecture

• Multiple Groups option
• Required sample size in SEM
• Use Clark & Smith example to illustrate the matrix approach to SEM
• Review the range of fit functions or ‘objective’, $D$, which is a function of the model parameters (path coeffs and variances). The computer finds the parameter values that minimize $D$ using a ‘gradient’ method.
2. Required sample size: 2x2 designs

• For a 2x2 design, it can be verified that, with a Type I error rate of .05, the power of the study to detect a ‘moderate’ effect size (e.g., $d = 0.4$) is about 80% if there are about $n = 25$ participants in each of the 4 cells of the design. (See the applet at: http://euclid.psych.yorku.ca/cgi/power.pl.) This would suggest a minimum sample size of 100.
Required sample size: SEM analyses

- For a longitudinal study in which many variables are measured repeatedly, the recommended sample sizes are in the 100-400 range. A summary of these recommendations can be found at [http://faculty.chass.ncsu.edu/garson/PA765/structur.htm](http://faculty.chass.ncsu.edu/garson/PA765/structur.htm).

- Dave Kenny has updated his recommendations at [http://davidakenny.net/cm/fit.htm](http://davidakenny.net/cm/fit.htm).
For example, an R script I got from the applet at http://timo.gnambs.at/en/scripts/powerforsem generates the recommended sample size, \( n \), when certain parameters are specified. These parameters are:

- Type I error rate and power (set at .05 and .80)
- Model ‘complexity’ (e.g., the degrees of freedom, \( df \), for assessing the \( \chi^2 \) index of lack-of-fit; we set \( df = 10 \) to represent a moderately complex model)
- The expected lack-of-fit of the measurement or null model, RMSEA0, and of the full model, RMSEA1. RMSEA is defined as \( \sqrt{[\chi^2 - df/(n*df)]} \), and we set RMSEA0 = .05 and RMSEA1 = .10, values that are typically used.
• With these settings, the recommended sample size is \( n = 313 \).
• Reducing the \( df \) (i.e., model complexity) from 10 to 5 changes \( n \) to 560.
• Increasing the \( df \) (i.e., model complexity) from 10 to 15 changes \( n \) to 229.
Concluding comments

• Thanks to Eric and Kevin!
• Allow your theory to motivate your data analysis
• Learning by doing; general approaches?
• Do try techniques in R, SPSS that you don’t fully ‘understand’
• Recall the different regression packages: \texttt{lm()}, \texttt{glm()}, \texttt{glmpath()}, \texttt{clm()}, \texttt{mlogit()}; \texttt{lmer()}, \texttt{clmm()}, \texttt{lavaan()}
• Good luck with your researches!
• Any time for more SEM examples?
Appendix for Last Lecture

• Imposing, then testing, constraints on parameters
• Required sample size in SEM
• Use Clark & Smith example to illustrate the matrix approach to SEM
• Review the range of fit functions or ‘objective’, $D$, which is a function of the model parameters (path coeffs and variances). The computer finds the parameter values that minimize $D$ using a ‘gradient’ method.
Clark & Smith example on Metacognition

What is the role of the "objective difficulty" (D) of the questions, and the person's "feeling of knowing" the right answer (C, for "confidence") in predicting recognition (G) and latency (L) performance? C&S regress G on (D,C,L).

\[
Z = u_1 D + \varepsilon_1 ,
\]
\[
C = u_2 Z + \varepsilon_2 ,
\]
\[
G = u_3 Z + \varepsilon_3 ,
\]
\[
L = u_4 Z + \varepsilon_4 .
\]
Equations in convenient *matrix* form:

\[ X \text{ is a 5x1 vector of all variables,} \]
\[ U \text{ is a 5x5 matrix of parameters,} \]
\[ \varepsilon \text{ is a 5x1 vector of error terms.} \]

\[
\begin{pmatrix}
Z \\
D \\
C \\
G \\
L
\end{pmatrix}
= 
\begin{pmatrix}
0 & u_1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
u_2 & 0 & 0 & 0 & 0 \\
u_3 & 0 & 0 & 0 & 0 \\
u_4 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
Z \\
D \\
C \\
G \\
L
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_1 \\
0 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4
\end{pmatrix}, \quad \text{or} \quad X = UX + \varepsilon,
Test Procedure

• To test models like this:
• Calculate the covariance (or correlation) matrix predicted by the model for the observed variables,
• Compare the observed and predicted covariances using a goodness-of-fit test. Hence the term, Covariance Structure Analysis (CSA).
• The above matrix equation can be used to derive the expected covariance matrix in terms of elements of $U$ and of the variance-covariance matrix for $\varepsilon$. 
Expected Correlation Matrix:
Parameters, $u_i$, are chosen so as to minimise the deviation between the expected and observed corr matrices

- **D**    C    G    L
- **D**    1
- **C**    $u_1u_2$    1
- **G**    $u_1u_3$    $u_3u_2$    1
- **L**    $u_1u_4$    $u_4u_2$    $u_3u_4$    1
• The results are encouraging.
• The parameter estimates are:
  – \( u_1 = 0.679, \ u_2 = 0.814, \ u_3 = 0.633, \ \text{and} \ u_4 = 0.856. \)
  – Predicted (& Observed) simple correlations are: \( r_{DL} = .581 (.62); \ r_{DG} = .430 (.42); \ r_{DC} = .552 (.53), \ r_{GL} = .542 (.52); \ r_{CL} = .696 (.68); \ r_{GC} = .515 (.55). \)
  – Predicted (& Observed) partial correlations are: \( r_{CG.D} = .369 (.43); \ r_{DG.C} = .204 (.19). \)
  One can do a formal ‘goodness-of-fit’ test involving \( \chi^2 \) with 2 degrees of freedom (6 data points, i.e., the 6 simple correlations, minus 4 parameters, the \( u_i \)).
Last Lecture

• Imposing, then testing, constraints on parameters
• Required sample size in SEM
• Use Clark & Smith example to illustrate the matrix approach to SEM
• Review the range of fit functions or ‘objective’, $D$, which is a function of the model parameters (path coeffs and variances). The computer finds the parameter values that minimize $D$ using a ‘gradient’ method.
Range of fit functions

• Let $D$ be the ‘distance’ or discrepancy between the expected covariance matrix (derived, using Matrix Algebra, from the specifications in an SEM) and the observed covariance matrix (calculated from the raw data).

• How to reasonably define $D$? What is the distribution of $D$ under $H_0$: That the SEM is the true model of the raw data? (We can use this distrn to decide if $D$ is ‘large’.)
• **D** is the ‘size’ or *magnitude* of the matrix that represents the *difference* between 2 matrices
  – How to define the difference between 2 matrices? This will be a matrix.
  – How to define the magnitude of a matrix?
• With **scalars**, *a* and *b*, 2 indexes of *difference* are \((a - b)^2\) and \((a*b^{-1} - 1)\). With matrices, the analogous expressions are: \((\Sigma - S)^2\) and \((S \Sigma^{-1} - I)\). *I* is the identity matrix with trace = \(p\), its *dim*.
• Two indexes of the ‘size’ or *magnitude* of a matrix, *A*, are the **trace**, \(\text{Tr}(A) = \sum a_{ii}\), and the **determinant**, \(\text{Det}(A) = |A|\). The dominant term in the \(|A|\) is the **product** of the diagonal elements
• The dominant term in $|A|$ is the **product** of the diagonal elements, $\prod a_{ii}$, which is conceptually similar to the **trace**, $\text{Tr}(A) = \sum a_{ii}$.

• Combining these indexes of *difference* and *magnitude* yields a range of *fit* indexes, $D$, e.g., $\text{Tr}((\Sigma - S)^2)$, $(\text{Tr}(S\Sigma^{-1}) - p)$, or $(\log(|\Sigma|) - \log(|S|))$.

• How does the computer minimize $D$?
\( \theta \) is the parameter, \( D(\theta) \) is the model deviance at \( \theta \). How to ensure that our iterative search for the minimum converges on \( \theta^* \), and not on \( \theta' \)?

(a) If the process is in region \( V_1 \) and the step-size, \( s \), is too ‘large’, process will exit from \( V_1 \).

(b) If the process is in region \( V_2 \) and \( s \) is too ‘small’, process will never leave \( V_2 \).
• Choose different **starting points** for iteration.

• At each iteration, choose the **direction** of the move to the next value carefully.

• Choose the **size** of the step or move carefully - too small a step can cause one to be trapped in a local minimum; too large a step can lead to an infinite sequence that misses the global minimum.
More examples of SEM
(most without latent variables)
Coupling between distinct metacognitive systems for memory and visual perception
Li Yan McCurdy\textsuperscript{1}, Brian Maniscalco\textsuperscript{1}, Janet Metcalfe\textsuperscript{1}, Ka Yuet Liu\textsuperscript{2}, Floris de Lange\textsuperscript{3}, Hakwan Lau\textsuperscript{1, 3}

What is the neural basis for metacognition, i.e. the introspective ability to monitor one’s own mental processes? A recent study found that, across individuals, gray matter volume in the frontal polar region was correlated with capacity for visual metacognition (i.e. how well one’s confidence ratings distinguish between correct and incorrect judgments). A question arises as to whether the putative metacognitive mechanisms in this region may be common for other processes, e.g. long term memory. ... and a formal model comparison analysis suggests this structural covariation is sufficient to account for the behavioral correlation of metacognition in the two tasks.
Figure 5. Model comparison. A) Schematic description of the models. These models characterize the possible ways to explain the positive behavioral correlation between memory and visual metacognitive efficiencies (type II performance).

In Model I, the dark arrows indicate that the prefrontal cortex (specifically, aPFC) is mainly functionally responsible for visual type I performance, and the precuneus is mainly functionally responsible for memory type II performance. The dashed lines indicate that there may be some degree of functional crosstalks between the two systems, such that the precuneus may also be partially responsible for visual type II performance and the prefrontal cortex may also be partially responsible for memory type II performance.
Model II and III are variants in which the crosstalk is one-sided.

In Model IV, there is no functional crosstalk. i.e. the precuneus is functionally responsible for only memory but not visual type II performance, and the prefrontal cortex is functionally responsible for only visual but not memory type II performance. In this model, the behavioral correlation between visual and memory type II performance is accounted for entirely by the covariation in volume between the precuneus and the prefrontal cortex across individuals (as shown in Figure 4).
Akaike Information Criterion (AIC) model selection was used to estimate which of the four models (in Figure 5) was most likely. Lower AIC values indicate that the model incurs less information loss. AIC values also take into account how parsimonious the model is, thus the number of parameters involved in the model is proportionate to the AIC value. Akaike weights calculated from the AIC values can be interpreted as the probability that that particular model is best. Thus Model IV fits the data best.
Appears OK  ML ChiSq
Group Fit: 6.97043
Fits 6.970 [3.006, 18.783]
Probability 0.073
AIC 0.970 [-2.994, 12.783]
RMSEA 0.200 [0.008, 0.399]
Degrees of freedom 3
Free parameters 7
Observed Statistics 10
Constraints 0
Options in Model Construction

• Possible links between A and B
  – Uni- or bi-directional (or reciprocal) effects
  – Uncorr vs corr errors (due, e.g., to omitted variables)
• Fixing a path coefficient, e.g., to 1
• Fixing a factor variance, e.g., to 1
• Fixing an error variance, e.g., to 0, to equate a latent variable with an observed variable
• Equating 2 path coefficients (symmetry, equivalence between groups):
  – (i) A \rightarrow B = B \rightarrow A?
  – (ii) A_t \rightarrow B_{t+1} = B_t \rightarrow A_{t+1}?
  – (iii) Is A \rightarrow B same in Groups 1 and 2?
Possible relations between A and B

1 parameter

2 params
u = v?

1 parameter
SEM Issues

• Use chi-square and other indices to assess a model. Because chi-square is often significant (indicating a poor model fit), one should look at other indices to see if our model is useful, or more useful than an alternative model.

• We draw a covariance (2-headed) path between two variables, X and Y, when we are not sure of the direction of causality, or when a third, unmeasured variable affects X and Y. Sometimes, it is possible to estimate $X \rightarrow Y$, $Y \rightarrow X$, and $X \leftrightarrow Y$! Each of these paths has its own interpretation.

• Use the multi-group option to study interactions, e.g., culture by ‘structure’, and factorial invariance across groups.
Examples: In c. the covariance arrow between $X$ and $Z$ indicates correlated errors, in addition to $e_1$ and $e_3$. In all models: $Y = aX + e_2$; $Z = b_1X + b_2Y + e_1$. In addition, (b) $X = cZ + e_3$, $\text{corr}(e_1, e_3) = 0$; and (c) $X = cZ + e_3$, $\text{corr}(e_1, e_3) = r$.

a. Uncorrel errors
b. Bi-directional
c. Correl errors effects
Example
Correlated Errors
In analyzing time series, SEM allows for different structural parameters, \( w_i \), and a different measurement model, \( u_i \), at each time period. SEM is thus more general than linear mixed models, or longitudinal growth models, typically applied to such data.
More examples

• We have already examined the **range of technical assumptions** the modeler can use in SEM, and the **applications** in areas of, e.g., ‘TV content and buying behavior’, ‘Dual process model of metacognition’, etc. A brief review of more examples shd be helpful.
  – Models of Justice
  – Interpersonal Theory
  – Antecedents to Reactions to Penalties
  – TV Violence Viewing
  – Gender Discrimination
SEM applications to a Theory of Justice and to Interpersonal Theory

1. In Tyler’s article, note the acceptance of models with significant chi-square values, provided these models are relatively good.
Tyler’s Justice Models

Model A: Saturated justice model

Model B: Resource-dominated justice model

Model C: Relational-dominated justice model

Model D: Resource-only justice model

Model E: Relational-only justice model

Model F: Dual-process justice model

Figure 1. Models of the psychology of justice.
Table 1

*Psychological Models of Justice: Legal Authority* (Note the use of the *relative*, not absolute, value of Chi-Sq, to pick Model C!)

<table>
<thead>
<tr>
<th>Model</th>
<th>Chi-Sq</th>
<th>df</th>
<th>NNFI</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.00</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>201.01</td>
<td>3</td>
<td>.183</td>
<td>.955</td>
</tr>
<tr>
<td>C</td>
<td>33.72</td>
<td>3</td>
<td>.873</td>
<td>.993</td>
</tr>
<tr>
<td>D</td>
<td>345.53</td>
<td>6</td>
<td>.300</td>
<td>.924</td>
</tr>
<tr>
<td>E</td>
<td>81.86</td>
<td>6</td>
<td>.884</td>
<td>.983</td>
</tr>
<tr>
<td>F</td>
<td>264.81</td>
<td>6</td>
<td>.466</td>
<td>.942</td>
</tr>
</tbody>
</table>
Table 2

*Psychological Models of Justice: Managerial Authority*

<table>
<thead>
<tr>
<th>Model</th>
<th>Chi-Sq</th>
<th>df</th>
<th>NNFI</th>
<th>CFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.00</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>89.15</td>
<td>4</td>
<td>.491</td>
<td>.969</td>
</tr>
<tr>
<td>C</td>
<td>65.99</td>
<td>5</td>
<td>.709</td>
<td>.978</td>
</tr>
<tr>
<td>D</td>
<td>316.01</td>
<td>7</td>
<td>-.055</td>
<td>.888</td>
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<tr>
<td>E</td>
<td>141.15</td>
<td>9</td>
<td>.649</td>
<td>.952</td>
</tr>
<tr>
<td>F</td>
<td>154.61</td>
<td>8</td>
<td>.652</td>
<td>.947</td>
</tr>
</tbody>
</table>
• Figure 1. Basic social-cognitive process model for reactions to penalties. R.W. Authorit. = right-wing authoritarianism.
Structural Conceptual Model (SCM)
We can ‘analyze’ the reln between *Chldagg* and *ChldTVviol* if we add the background variables, *ParEduc* and *ChldIQ*. 
Assessing Possible Gender Discrimination

![Diagram showing relationships between variables: GENDER, EXPER, RANK, EDUC, SALARY.](image)

- **GENDER**
  - 1.00
  - 0.57 (0.33, 0.80)
  - 0.24 (0.12, 0.37)

- **EXPER**
  - 0.68
  - 0.78 (0.55, 0.90)
  - 0.12

- **RANK**
  - 0.57 (0.33, 0.80)
  - 0.24 (0.12, 0.37)

- **EDUC**
  - 1.00
  - 0.44 (0.38, 0.50)

- **SALARY**
  - 0.04

**Statistics**
- **ML Chi-Sq**
  - Group Fit: 1.37703
  - Fits 1.377 (4.000, 6.807)
  - Probability 0.848
  - AIC -6.623 (-4.000, -1.193)
  - RMSEA 0.000 (0.000, 0.120)
  - Degrees of freedom 4
  - Free parameters 11
  - Observed Statistics 15
  - Constraints 0