Self-Enhancing Transmission Bias and Active Investing

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Individual investors often invest actively and lose thereby. Social interaction seems to exacerbate this tendency. In our model, senders’ propensity to discuss their strategies’ returns, and receivers’ propensity to be converted, are increasing in sender return. The rate of conversion of investors to active investing is convex in sender return. Unconditionally, active strategies (high variance, skewness, and personal involvement) dominate the population unless the mean return penalty to active investing is too large. Thus, the model can explain overvaluation of ‘active’ asset characteristics even when investors have no inherent preference over them.

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1 Introduction

A neglected topic in financial economics is how investment ideas are transmitted from person to person. In most investments models, the influence of individual choices on others is mediated by price or by quantities traded in impersonal markets. However, more direct forms of social interaction are also important for investment decisions. As Shiller (1989) put it, “...Investing in speculative assets is a social activity. Investors spend a substantial part of their leisure time discussing investments, reading about investments, or gossiping about others’ successes or failures in investing.” In one survey, individual investors were asked what first drew their attention to the firm whose stock they had most recently bought. Almost all referred to direct personal contact; personal interaction was also important for institutional investors (Shiller and Pound 1989). Furthermore, an empirical literature finds that social interactions affect investment decisions by individuals and money managers, including selection of individual stocks.\(^2\)

Our purpose here is to model how the process by which ideas are transmitted affects social outcomes, with an application to active versus passive investment behavior. We view the transmission process here as including both in-person and electronic means of conversation, as well as one-to-many forms of communication such as blogging and news media. Our approach is based on the idea that conversational biases can favor superficially-appealing but mistaken ideas about personal investing (Shiller (2000a, 2000b)).

It is a remarkable fact is that individual investors trade actively and have invested in active investment funds for decades, and thereby have on average underperformed net of costs relative to a passive strategy such as holding a market index (on individual trading: Barber and Odean (2000b), Barber et al. (2009)).\(^3\) This phenomenon can be called the active investing puzzle. In addition to net underperformance relative to benchmarks, trading in individual stocks and investing in active funds adds idiosyncratic portfolio volatility. Calvet, Campbell, and Sodini (2007) report that idiosyncratic risk exposure of Swedish households accounts for half of the return variance for the median household. Active investing often reflects a belief of individual investors that they can identify stocks or managers who will


\(^3\)Carhart (1997) and Daniel et al. (1997) find that active funds do not tend to outperform against passive benchmarks. French (2008) documents very large fees paid in the aggregate by investors to active funds.
outperform the market. Financial scams such as the Madoff scheme also rely on investors’ belief that they can identify superior investment managers.

A plausible explanation for excessive investor trading is overconfidence (DeBondt and Thaler (1995), Barber and Odean (2000b)), a basic feature of individual psychology. However, trading aggressiveness seems to be greatly exacerbated by social interactions. For example, more than other investors, participants in investment clubs seem to select individual stocks based on reasons that are easily exchanged with others (Barber, Heath, and Odean (2003)); select small, high-beta, growth stocks; turn over their portfolios very frequently; and underperform the market (Barber and Odean (2000a)). Contagion in stock picking by individuals and institutions spreads a type of speculative behavior. There is also evidence that self-reported stock market participation increases with measures of social connectedness (Hong, Kubik, and Stein (2004)).

These considerations suggest looking beyond direct individual-level psychological biases alone, to an explanation based on social interaction. However, the sheer fact of behavioral contagion, as analyzed and documented in recent work, does not in itself explain a tilt toward active trading, since either active or passive strategies can spread from person to person. So bias in the transmission process is an essential ingredient for addressing this issue.

The explanation we propose here is that investors like to recount to others their investment victories more than their defeats, and that listeners do not fully discount for this. We call this behavior self-enhancing transmission bias, or SET.

Both a rational concern for reputation and psychological bias can contribute to SET. A literature from psychology and sociology on self-presentation and impression management finds that people seek to report positively about themselves, as constrained by the need to be plausible and to satisfy presentational norms (Goffman (1961), Schlenker (1980)). In a review of the impression management field, Leary and Kowalski (1990) discuss how people tend to avoid lying, but selectively omit information, so that “Impression management often involves an attempt to put the best parts of oneself into public view” (pp. 40-1). Consistent with SET, for a wide set of consumer products, positive word-of-mouth discussion of user experiences tends to predominate over negative discussion (see the review of East, Hammond, and Wright (2007)), perhaps because users want to persuade others.

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4During the millennial high-tech boom, investors who switched early to online trading subsequently began to trade more actively and speculatively, and earned reduced trading profits (Barber and Odean (2002)). Early internet investors probably had greater access to and interest in online forms of social interaction, such as e-mail and investment chat rooms. Internet discussions rooms were, according to media reports, important in stimulating day trading.
that they are expert at product choice (Wojnicki and Godes (2010)).

There is also evidence of internal self-enhancing thought processes, such as the tendency of people to attribute successes to their own virtues, and failures to external circumstances or luck (Bem (1972), Langer and Roth (1975)).\(^5\) Self-enhancing psychological processes encourage people to think more about their successes than their failures (as in the model Benabou and Tirole (2002)). It is a small step from thinking in a self-enhancing way to talking that way.

In the model, investors adopt either an Active (A) or Passive (P) investment strategy. We interpret A as the riskier option, or alternatively, the more engaging one (meaning that adopters are, *ceteris paribus*, more likely to talk about it, perhaps because it is more novel, affect-laden, or arousing). SET creates an upward selection bias in the reports received by other investors about the profitability of the chosen strategy: they hear about good outcomes more than bad outcomes. The size of the selection bias increases with return variance; for example, if variance is zero the selection bias vanishes. So if A has higher variance than P and if listeners do not fully discount for the biased sample they observe, they will overestimate the value of adopting A relative to P, so A spreads through the investor population.

Furthermore, if receivers attend more to extreme outcomes, high skewness strategies will tend to spread, because such strategies more often generate the extreme high returns which are most often reported, attended to, and influential. As a result, A spreads through the population unless it has a sufficiently strong offsetting disadvantage (lower expected return).

Finally, if A is more engaging than P as a conversation topic (more *conversable*, in the terminology of our model), then A is recommended and its return reported to current adopters of P more often than reports about P are made to adopters of A. This favors the spread of A.

For the reasons discussed above (and also in Subsection 2.7.2), SET is a key factor in generating the results of the model. However, even without SET and naivete on the part of receivers about selection bias, naive extrapolation by receivers of reported returns combined with salience of extremes can also cause high variance strategies to spread. This

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\(^5\) The ‘totalitarian ego’ describes the tendency in many contexts for people to filter and interpret information to their own credit (Greenwald (1980)). People differentially recall information in ways favorable to their self-esteem (see, e.g., Section 5.3 of von Hippel and Trivers (2011)). There is evidence of self-enhancing behaviors in investing; Karlsson, Loewenstein, and Seppi (2009) and Sicherman et al. (2012) find that Scandinavian and U.S. investors reexamine their portfolios more frequently when the market has risen than when it has declined.
direct attention-to-extremes effect is complementary to the SET effect. SET promotes A owing to its higher variance and (if there is greater attention to extreme outcomes), its higher skewness. The direct attention-to-extremes effect (discussed in Subsection 2.7.2) promotes A only when there is high attention-to-extremes, and only via a variance effect, not a skewness effect.

SET helps explain a range of patterns in trading and returns, including the convexity of new participation in investment strategies as a function of past performance, the participation of individuals in lotteries with negative expected return; the preference of some investors for high variance or high skewness ('lottery') stocks; overvaluation of growth firms, distressed firms, firms that have recently undertaken Initial Public Offerings (IPOs), and high idiosyncratic volatility firms; heavy trading and overvaluation of firms that are attractive as topics of conversation (such as sports, entertainment, and media firms, firms with hot consumer products, and local firms); and the association of these effects, and the extent of stock market trading in general, with proxies for social interactiveness. The approach also offers new empirical implications.

A general theoretical literature on social interaction in economics focuses on its effects on the efficiency of information flows, and on behavioral convergence (herding). The effects of social interactions on the spread of cultural traits have been analyzed in fields such as anthropology (Henrich and Boyd (1998)), zoology (Lachlan, Crooks, and Laland (1998), Dodds and Watts (2005)), and social psychology (Cialdini and Goldstein (2004)). Finance models have examined how social interactions affect information aggregation, and potentially can generate financial crises. This paper differs from this literature in examining how SET affects the evolutionary outcome.

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Economists have also modelled how cultural evolutionary processes affect ethnic and religious traits, and altruistic preferences (Bisin and Verdier (2000, 2001)). The focus here is on understanding investment and risk-taking behavior.

2 The Model

2.1 Social Interactions, Timing of Events, and Population Shifts

The Population

We consider a population of \( n \) individuals who adopt one of two types of investment strategies, A (Active) and P (Passive), which have different return probability distributions (specified in Section 2.6). In each period (generation), a pair of individuals is randomly selected to meet. In each pair, one investor randomly becomes the sender and the other the receiver.\(^8\) The returns of the sender and receiver from their current strategies over the period are realized. The sender reports his return to the receiver with probability \( s(R_i) \) for sender of type \( i \), which is increasing in the sender’s return. Finally, a receiver who receives a message is transformed into the type of the sender with probability \( r(R_i) \), also increasing in the sender’s return.

Let \( n_i \) be the number of type \( i \) in the current period or generation and \( n'_i \) be the number in the next generation, and let

\[
\begin{align*}
  f_i & \equiv \frac{n_i}{n} \\
  f'_i & \equiv \frac{n'_i}{n}, \quad i = A, P,
\end{align*}
\]

where \( f_A + f_P = f'_A + f'_P = 1 \). In the rest of the paper, we use \( f \equiv f_A \) to denote the population frequency of type A individuals. The total number of individuals in population, \( n \), is constant over time.

A standard model for allele (gene type) frequency change in evolutionary biology is the Moran process (Moran (1962)), in which in each generation exactly one individual is born and one dies, leaving population size constant. Here we apply a Moran process to the spread of a cultural trait or, in the terminology of Dawkins (1976), a ‘meme’.

When an AA or PP pair is drawn, there is no change in the population frequency of each strategy. When A and P meet, with probability \( s \) the sender communicates his

\(^8\)In actual conversations, often both parties recount their experiences. Our sharp distinction between being a sender and a receiver in a given conversation is stylized, but since we allow for the possibility that either type be the sender, is unlikely to be misleading.
performance and upon receiving the message, the receiver converts to the type of the sender with probability \( r \). We assume that the value of the sending and receiving functions \((s, r)\) depend only on the sender’s return, i.e., for given return, they are independent of whether the sender or receiver are \( A \) or \( P \). Nevertheless, transformations do depend on which type becomes the sender, as this affects the distribution of the sender’s return.

Let \( T_{ij}(R_i) \) be the probability that the sender, who is of type \( i = A, P \), transforms the receiver, who is of type \( j \), into type \( i \), as a function of the sender’s return \( R_i \). The change in \( f \), the frequency of type \( A \) individuals, after two individuals meet is

\[
f' - f = \begin{cases} 
\frac{1}{n} & \text{with probability } \left( \frac{3}{2} \right) T_{AP}(R_A) \\
- \frac{1}{n} & \text{with probability } \left( \frac{3}{2} \right) T_{PA}(R_P) \\
0 & \text{with probability } 1 - \left( \frac{3}{2} \right) [T_{AP}(R_A) + T_{PA}(R_P)],
\end{cases}
\]

(2)

where \( \chi \) is the probability that a mixed pair is drawn,

\[
\chi = \frac{(n_A/n) (n - n_A/n - 1) + (n - n_A/n) (n_A/n - 1)}{2nf(1-f)}.
\]

(3)

We model the transformation probability as the result of conversational initiations and sendings of performance information by senders, and of the receptiveness of receivers. The next subsection considers senders, and the one that follows considers receivers.

### 2.2 Self-Enhancement and the Sending Function

Owing to self-enhancing transmission bias, we assume that the probability that the sender of type \( i \) sends is increasing in the performance of the sender’s strategy, \( R_i \), so \( s'(R_i) > 0 \). Potentially consistent with SET, Shiller (1990) provides survey evidence that people talked more about real estate in U.S. cities that have experienced rising real estate prices than those that have not. A sender may, of course, exaggerate or simply fabricate a story of high return. But if senders do not always fabricate, the probability of sending will still depend upon the actual return.

We apply a linear version of SET,

\[
s(R_i) = \beta R_i + \gamma, \quad \beta, \gamma > 0,
\]

(4)

where \( i \) is the type of the sender. Since the sending function is type-independent, \( \beta \) and \( \gamma \) have no subscripts. To ensure that \( 0 \leq s(R_i) \leq 1 \), we require that \(-\gamma/\beta \leq R_i \leq (1-\gamma)/\beta \)
with high probability. Under reasonable parameter values for \( \beta \) and \( \gamma \), we can ensure that the probability of return falling outside this range is negligible.

The assumption that sending is stochastic reflects the fact that raising a topic in a conversation depends on both social context and on what topics the conversation partner happens to raise. A high return encourages a sender to discuss his investments, but senders also prefer to obey conversational norms for responsiveness and against bragging. Reporting favorable information about one’s achievements and competence often lead to negative reactions in onlookers when the information is not provided in response to a specific question (Holtgraves and Srull (1989)). Owing to conversational norms, in some contexts even a reluctant sender with poor return will feel pressure to discuss his performance, and in others an eager sender may not talk about a high return for fear of seeming immodest.

The positive slope \( \beta \) of the sending schedule reflects SET. Consistent with \( \beta > 0 \), in a database from a Facebook-style social network for individual investors, Simon and Heimer (2012) report that the frequency with which an investor contacts other traders is increasing in the investor’s short-term return.

SET creates a selection bias in the set of returns observed by receivers. The more tightly bound is the sender’s self-esteem or reputation to return performance, the stronger is SET, and therefore the higher is \( \beta \). The constant \( \gamma \) reflects the conversability of the investment choice. When the investment is an attractive topic for conversation the sender raises the topic more often. The sender also raises the topic more often when conversations are more extensive, as occurs when individuals are more sociable (how much they talk and share information with each other). So \( \gamma \) also reflects investor sociability.

### 2.3 The Receiving Function

For a mixed pair of individuals, consider now the likelihood of a receiver of type \( j \) being converted to the sender’s type \( i \). Given a sender return \( R_i \) and that this return is indeed sent, the conditional probability that the receiver is converted is denoted \( r(R_i) \).

Messages from a sender with strong performance are more persuasive than messages from a sender with weak performance, so we assume that \( r'(R_i) > 0 \). This accommodates the possibility that receivers have a degree of skepticism about the selection bias in the messages they receive, and about sender lying and exaggeration.

There is extensive evidence in various contexts that observers do not fully discount for selection biases in the data they observe, a phenomenon called selection neglect.\(^9\) Selection

\(^9\)See, e.g., Nisbett and Ross (1980) and Brenner, Koehler, and Tversky (1996). People tend to place
neglect is to be expected if individuals with limited processing power automatically process data in intuitive ways, and do not always take the extra cognitive step of adjusting for selection bias.

There is also evidence that investors overweight past performance as an indicator of future performance. Despite the existence of some well-known return predictability anomalies, one or a few recent observations of the performance of a trading strategy generally convey little information about its future prospects. But investors think otherwise.\(^\text{10}\)

If a receiver believes that past performance is indicative of strategy value, and does not adequately adjust for the fact that sending increases with strategy returns as implied by the sending function (4), then the receiver will overvalue the sender’s strategy. This tends to raise the probability of type switching, which promotes a frothy churning in beliefs from generation to generation.

We further assume that \(r''(R_t) \geq 0\), to capture general evidence that extreme news is more salient than moderate news, and therefore is more often noticed and encoded for later retrieval (Fiske (1980), Moskowitz (2004)). This auxiliary assumption is mainly needed for the model’s skewness predictions. When cognitive processing power is limited, a focus on extremes is a useful heuristic, as extreme news tends to be highly informative. Intuitively, this convexity assumption overlays higher attention to extreme values of \(R_t\) on an otherwise-linear relationship between receptiveness and \(R_t\). Salience as a motivation for convexity of the receiving function is broadly consistent with the attentional explanation given by Barber and Odean (2008) for their finding that individual investors are net buyers of stocks that experience extreme one-day returns of either sign.

\(^{10}\)There is evidence from experimental markets that investors have extrapolative expectations (Smith, Suchanek, and Williams (1988) and Choi, Laibson, and Madrian (2010)). Survey evidence indicates that return expectations are higher after high housing price growth (Case and Shiller (1988)) and high portfolio returns (Vissing-Jorgensen (2003)). Using survey and experimental data, DeBondt (1993) reports that the expectations of individual investors follow past trends. Benartzi (2001) finds that employees invest more in their firm’s stock after it experiences high returns. There are strong inflows from investors into top-performing mutual funds (Chevalier and Ellison (1997), Sirri and Tufano (1998)), and mutual fund investors reallocate toward stock funds after stock price increases (Ederington and Golubeva (2010)). Barber and Odean (2002) find that early adopters switched to online trading after unusually good performance, and subsequently traded more actively. Dichev and Yu (2011) report that hedge fund investors chase past high returns, resulting in lower performance.
We apply a quadratic version of these assumptions,
\[ r(R_i) = a(R_i)^2 + bR_i + c, \quad a, b, c > 0, \]
under implicit parameter constraints ensuring that for all except arbitrarily small probability of return realizations, \( r \) is monotonic and takes value between 0 and 1.

The parameter \( c \) measures the susceptibility of receivers to influence, deliberate or otherwise, by the sender. The parameter \( b \) reflects the degree to which the receiver tends to naively extrapolate past strategy returns, or at least to be persuaded by high returns. The quadratic parameter \( a \) reflects the tendency, after allowing for the effect of \( b \), for extreme returns to be more persuasive.

In this specification, the probability that the receiver is converted is smoothly increasing in the sender return, and is positive even when the sender has a negative return. One motivation for this is a rational endorsement effect: the very fact that another individual has adopted the trading strategy suggests that he possessed favorable information about it. Furthermore, according to the mere exposure effect (Zajonc (1968), Bornstein and D’Agostino (1992), Moreland and Beach (1992)), a receiver who had little prior awareness of the strategy will start to like it more simply by being exposed to it. Another way to have low sender returns occasionally convert receivers is to have the switch decision depend on the difference in return between sender and receiver, as in Subsection 2.7.3.

In 5, a trader’s decision whether to switch strategy is based on the single return message received. Over time, if rational individuals remember the entire history of returns, we would expect the system to converge eventually to the best action. One limit on the speed and efficacy of learning is that strategy return realizations are often noisy indicators about which strategy is better. If investors learn slowly, and new generations of naive investors continually arrive, it is no longer obvious that the system will converge toward the best action. Our model captures this in an extreme way by allowing investors to retain return messages for only single period.

### 2.4 Transformation Probabilities

We first examine \( T_{AP} \), the transformation probability for a sender of type \( A \) and receiver of type \( P \). By definition,

\[
T_{AP}(R_A) = r(R_A)s(R_A) \\
= (aR_A^2 + bR_A + c)(\beta R_A + \gamma) \\
= a\beta R_A^3 + BR_A^2 + CR_A + c\gamma, \quad (6)
\]
where

\[ B = a\gamma + b\beta \]
\[ C = b\gamma + c\beta. \]  

(7)

Similarly,

\[ T_{PA}(R_P) = a\beta R^3_P + BR^2_P + CR_P + c\gamma. \]  

(8)

By assumption, \( r', s' > 0 \), so \( T'_{AP}(R_A), T'_{PA}(R_P) > 0 \).

Since the \( r \) and \( s \) functions are type-independent and the only random variable they depend upon is the sender return, the expected transformation between types derives from the effect of \( A \) versus \( P \) on the distribution of sender returns \( R \), as reflected in mean, variance, and skewness.

### 2.5 Evolution of Types Conditional on Realized Return

We first show that, owing to SET, high return favors active investing. Given returns \( R_P \) and \( R_A \), we can calculate the expected change in the fraction of type \( A \) in the population after one social interaction between two randomly selected individuals. In the four possible pairing \( AA, PP, AP, \) or \( PA \) (the first letter denotes the sender, the second the receiver), the change in the frequency of type \( A \) given \( AA \) or \( PP \) is zero. The expected changes in the frequency of type \( A \) given a meeting \( AP \) or \( PA \) and realized returns are

\[ E[\Delta f|AP,R_A] = \left( T_{AP}(R_A) \times \frac{1}{n} \right) + [(1 - T_{AP}(R_A)) \times 0] = \frac{T_{AP}(R_A)}{n} \]
\[ E[\Delta f|PA,R_P] = \left[ T_{PA}(R_P) \times \left( -\frac{1}{n} \right) \right] + [(1 - T_{AP}(R_A)) \times 0] = -\frac{T_{PA}(R_P)}{n}. \]  

(9)

So taking the expectation across the different possible combinations of sender and receiver types \( (AA, PP, AP, PA) \), by (2) and (9),

\[ \left( \frac{2n}{\chi} \right) E[\Delta f|R_A, R_P] = T_{AP}(R_A) - T_{PA}(R_P). \]  

(10)

So for given returns, the fraction of type \( A \) increases on average if and only if \( T_{AP}(R_A) > T_{PA}(R_P) \).

Recalling that \( T_{AP}(R_A) = s(R_A)r(R_A) \), we can derive some basic predictions of the model from the features of the sending and receiving functions. If \( R_A \) and \( R_P \) are not perfectly correlated, we can calculate the effect of increasing \( R_A \) with \( R_P \) held constant.
Partially differentiating \((10)\) with respect to \(R_A\) twice and using the earlier conditions that \(r'(R_A), s'(R_A), s''(R_A) > 0\), that \(s''(R_A) = 0\) by \((4)\), and that \(r''(R_A) > 0\) by \((5)\) gives

\[
\frac{1}{2n} \frac{\partial^2 E[\Delta f|R_A,R_P]}{\partial (R_A)^2} = \frac{\partial^2 T_{AP}(R_A)}{\partial (R_A)^2} = r''(R_A)s(R_A) + 2r'(R_A)s'(R_A) > 0. \tag{12}
\]

Since \(R_A\) affects \(T_{AP}\) but not \(T_{PA}\), these formulas describe how active return affects both the expected net shift in the fraction of actives, which reflects both inflows and outflows, and the expected unidirectional rate of conversion of passives to actives, such as the rate at which investors who have never participated in the stock market start to participate.

**Proposition 1** If the returns to \(A\) and \(P\) are not perfectly correlated, both the one-way expected rate of transformation from \(P\) to \(A\) and the expected change in frequency of \(A\) are increasing and convex in return \(R_A\).

Several studies provide evidence that is broadly consistent with these predictions. Chevalier and Ellison (1997) and Sirri and Tufano (1998) find that investor funds flow into mutual funds with better performance. This is a non-obvious effect since evidence of persistence in fund performance is very limited. Furthermore, the flow-performance relationship is convex; flows are disproportionately into the best-performing funds.

Lu (2011) finds that 401(k) plan participants place a greater share of their retirement portfolios in risky investments (equity rather than fixed income) when their coworkers earned higher equity returns in the preceding period. Kaustia and Knüpfer (2012) report a strong relation between returns and new participation in the stock market in Finland in the range of positive returns. Specifically, in this range, a higher monthly return on the aggregate portfolio of stocks held by individuals in a zip code neighborhood is associated with increased stock market participation by potential new investors living in that neighborhood during the next month.

The greater strength of the effect in the positive than in the negative range is consistent with the convexity prediction. Furthermore, within the positive range the effect is stronger for higher returns. The model does not imply a literally zero effect in the negative range, but a weaker effect within this range (as predicted by Proposition 1) would be statistically harder to detect.

Kaustia and Knupfer explain their findings based on what we call SET. Proposition 1 captures this insight, and further reinforcing effects. SET is captured by \(s'(R_A) > 0\). The
willingness of receivers to convert is increasing with return, as reflected in \( r'(R_A) > 0 \). By (12), these together contribute to convexity of expected transformation as a function of \( R_A \). A further contributor is the convexity of the receiver function, \( r''(R_A) \), reflecting high salience of extreme outcomes.

If we interpret \( A \) as active trading in the market for individual stocks with a preponderance of long positions, then a high market return implies high returns to \( A \) investors. Proposition 1 therefore implies that when the stock market rises, volume of trade in individual stocks increases. This implication is consistent with episodes such as the rise of day trading, investment clubs, and stock market chat rooms during the millennial internet boom, and with extensive evidence from 46 countries including the U.S. that investors trade more when the stock market has performed well (Statman, Thorley, and Vorkink (2006), Griffin, Nardari, and Stulz (2007)).

2.6 Strategy Return Components and the Meaning of Active Investing

Let \( r \) be the common component of returns (e.g., the market portfolio) shared by \( A \) and \( P \), where \( E[r] = 0 \), and let \( \epsilon_i \) be the strategy-specific component, \( E[\epsilon_i] = 0 \), \( i = A, P \). We assume that \( r, \epsilon_A \) and \( \epsilon_P \) are independent, and write the returns to the two strategies as

\[
R_A = \beta_A r + \epsilon_A - D \\
R_P = \beta_P r + \epsilon_P,
\]

where \( \beta_A \) is the sensitivity of strategy return to the common return component. We assume that the active strategy has higher systematic risk, \( \beta_A > \beta_P \geq 0 \). (The condition \( \beta_P \geq 0 \) is not needed for most of the results.) We further assume that \( \sigma_A^2 > \sigma_P^2, \gamma_{1A} > 0, \gamma_{1P} \approx 0, \) and \( \gamma_{1r} \geq 0 \), where \( \sigma_A^2, \sigma_P^2 \) are the variances of \( \epsilon_A \) and \( \epsilon_P \), \( \gamma_{1r} \) is the skewness of \( r \), and \( \gamma_{1A}, \gamma_{1P} \) are the skewnesses of \( \epsilon_A \) and \( \epsilon_P \). We also let \( \sigma_r \) denote standard deviation of the common factor \( r \).

In summary, in the choice between possible return distributions, active investing means a strategy with higher volatility and possibly also higher skewness. This corresponds fairly well with ordinary usage, but it is not hard to think of exceptions. For example, a long-short strategy that achieved low risk, or a dynamic hedging strategy using a stock and its option that is used to generate a riskfree payoff would not be active in the sense we are using.

Since \( E[r] = E[\epsilon_i] = 0 \), (13) implies that \( P \) has expected return of zero, and \( D \) is the return penalty (or if negative, premium) to active trading. We call \( D \) the return penalty.
rather than the ‘cost’ of active trading, because a major part of the welfare loss may come from lack of diversification and excessive idiosyncratic risk-bearing. So even if $D < 0$, the As may be worse off than Ps.\textsuperscript{11}

In an explicit model of trading decisions and equilibrium price-setting (see Section 3), risk premia and mispricing affect $E[R_A]$ and $E[R_P]$. In a multiperiod setting, owing to shifting fractions of the investor types, equilibrium prices and therefore the probability distributions of $R_P$ and $R_A$ fluctuate from period to period. The difference in expected returns between the two strategies $E[R_A] - E[R_P]$ in general varies over time and depends on the fraction of type A investors. However, it is possible to think of scenarios in which the expected returns do not shift stochastically (e.g., if different A investors merely place side-bets against each other, and therefore do not need to offer a risk premium to persuade P investors to participate in these bets).

Furthermore, in a market equilibrium setting a rise in A in the population tends to be self-limiting. As the frequency of A rises, prices tend to move against the strategies they employ (so long as the As trade with Ps rather than just taking side-bets against each other). The reduction in the expected value to A relative to P would be reflected by a higher expected return differential $D$. So in an equilibrium setting there will be, apart from random fluctuations, a balanced frequency of A and P between zero and one.

### 2.7 Unconditional Expected Evolution of Types

To see how the frequency of active investing evolves without conditioning on returns, we take the expectation of the change in the population fraction of A over $R_A$ and $R_P$ in (10), which gives

$$
\left(\frac{2n}{\chi}\right) E[\Delta f] = E[T_{AP}(R_A)] - E[T_{PA}(R_P)].
$$

(14)

so the fraction of type A increases on average if and only if $E[T_{AP}(R_A)] > E[T_{PA}(R_P)]$.

\textsuperscript{11}Also, since $E[r] = 0$, the model does not directly capture factor risk premia, but a positive premium would implicitly be captured in a negative $D$; this expected return advantage is not a welfare advantage. Even when $D < 0$, if As overvalue the risky asset and Ps are rational, being an A rather than a P decreases an individual’s true expected utility (owing to excessive risk-taking, and an insufficient reward for bearing risk). So the return penalty to active trading $D$ underestimates the welfare loss from active trading. Greater transaction costs of active trading (not modeled here) would also be reflected in $D$. There is evidence that U.S. and Taiwan investors underperform in their individual stock trades (Barber and Odean (2000b), Barber et al. (2009)), but that active individual fund investors in Sweden outperform passive investors (Dahlquist, Martinez, and Söderlind (2012)).
To see how the population evolves unconditionally, by (13) and direct calculation,

\[ T_{AP}(R_A) - T_{PA}(R_P) = a\beta (R_A^3 - R_P^3) + B(R_A^2 - R_P^2) + C(R_A - R_P) \]

\[ = a\beta [\beta_A^3 - \beta_P^3] + 3r^2(\beta_A^2\epsilon_A - \beta_P^2\epsilon_P) + 3r(\beta_A\epsilon_A^2 - \beta_P\epsilon_P^2) + \epsilon_A^3 - \epsilon_P^3 \]

\[ \quad + B[(\beta_A - \beta_P)^2] + 2r(\beta_A\epsilon_A - \beta_P\epsilon_P) + \epsilon_A^2 - \epsilon_P^2 \]

\[ \quad + C[(\beta_A - \beta_P)r + \epsilon_A - \epsilon_P] \]

\[ \quad + D\{-(r\beta_A + \epsilon_A)[3a\beta(r\beta_A + \epsilon_A) + 2B] - C\} + D^2[3a\beta(r\beta_A + \epsilon_A) + B] \]

\[ - aD^3\beta. \quad (15) \]

Taking the expectation over \( r, \epsilon_A \) and \( \epsilon_P \) and multiplying by \( 2n/\chi \), the expected change in frequency satisfies

\[ \left( \frac{2n}{\chi} \right) E[\Delta f] = E[T_{AP}(R_A) - T_{PA}(R_P)] \]

\[ = a\beta [\beta_A^3 - \beta_P^3\gamma_1\sigma_r^3 + \gamma_1 A\sigma_A^3 - \gamma_1 P\sigma_P^3] + B[(\beta_A^2 - \beta_P^2)\sigma_r^2 + (\sigma_A^2 - \sigma_P^2)] \]

\[ + Da\beta [-3\sigma_A^2 - D^2 - 3\sigma_r^2\beta_A^2] + D^2B - DC, \quad (16) \]

recalling that \( \sigma \) denotes standard deviation and \( \gamma_1 \) denotes skewness.

### 2.7.1 Comparative Statics

To gain insight into the determinants of the reproductive success of \( A \) versus \( P \) strategies, we describe comparative statics effects on the growth in the active population fraction.

**Proposition 2** If \( D \approx 0 \), then under the parameter constraints of the model, the expected increase in the fraction of \( A \):

1. Decreases with the return penalty to active trading \( D \);

2. Increases with factor skewness, \( \gamma_{1r} \), if \( \beta_A > \beta_P \);

3. Increases with active idiosyncratic skewness, \( \gamma_{1A} \); and decreases with passive idiosyncratic skewness, \( \gamma_{1P} \), so long as \( \sigma_P \neq 0 \);

4. Increases with active idiosyncratic volatility, \( \sigma_A \); and decreases with passive idiosyncratic volatility, \( \sigma_P \), so long as \( \sigma_P \neq 0 \) and \( \gamma_{1P} \) is sufficiently close to zero;

5. Increases with the factor loading of the active strategy, \( \beta_A \), and decreases with the factor loading of the passive strategy, \( \beta_P \);

6. Increases with the variance of the common factor, \( \sigma_r^2 \).
7. Increases with SET, $\beta$;

8. Increases with the sensitivity of receptiveness to returns, $b$;

9. Increases with attention of receivers to extremes, $a$;

10. Increases with the conversability, $\gamma$, of trading strategies.

In addition, the expected rate of increase in the fraction of $A$ increases with the susceptibility of receivers, $c$, if $D < 0$, and decreases with susceptibility if $D > 0$.

If we view the active strategy as the choice of investments with certain risk characteristics such as high volatility or skewness motivated by overoptimism about certain stocks, then the predictions in Proposition 2 about the frequency of $A$ become predictions that the trading strategies adopted by $A$ become overpriced. Specifically, the equilibrium model of Section 3 assumes that the actives have overoptimistic beliefs about certain risky assets, whereas passive investors have rational expectations, and shows that the evolution toward active investing depresses the expected returns of the risky assets. In discussing here the partial equilibrium comparative statics predictions about the frequency of $A$, we will make use of the intuitive extension (motivated by the equilibrium model) to draw predictions about expected returns.

To show Part 1, we differentiate (16) with respect to $D$, the return penalty to active trading, to obtain

$$
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial D} = -3a\beta(\beta^2_A\sigma_r^2 + \sigma^2_A) + D(-3a\beta D + 2B) - C
$$

$$
< 0
$$

if $D < 0$ or $D$ is positive but not too large.\(^\text{12}\) So the success of $A$ decreases with $D$, i.e., a greater average return penalty to active trading makes $A$ less contagious.

For Part 2, differentiating with respect to factor skewness $\gamma_{1r}$ gives

$$
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \gamma_{1r}} = a\beta^3 \sigma_r^3 (\beta^3_A - \beta^3_P) > 0,
$$

\(^\text{12}\)The ambiguity for large $D$ results from a spurious effect: for sufficiently large negative $R$, the slope of the quadratic receiving function turns negative. This can make a larger return penalty to active trading, $D$, more successful in transforming $Ps$ to $As$ by making sufficiently big losses more likely to convince the receivers to switch type.
since $\beta_A > \beta_P$. Thus, the advantage of $A$ over $P$ is increasing with factor skewness. Intuitively, extreme high returns are especially likely to be sent, to be noticed, and to convert the receiver when noticed. Since $A$ has a greater factor loading than $P$, factor skewness is magnified in $A$ relative to $P$, making $A$ more contagious.

For Part 3, differentiating with respect to active idiosyncratic skewness $\gamma_{1A}$ gives

$$\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \gamma_{1A}} = a\beta\sigma^3_A > 0.$$  \hspace{1cm} (19)

Thus, the advantage of $A$ over $P$ is increasing with the idiosyncratic skewness of $A$. The intuition is similar to that of Part 2. Differentiating with respect to passive idiosyncratic skewness $\gamma_{1P}$ gives

$$\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \gamma_{1P}} = -a\beta\sigma^3_P,$$  \hspace{1cm} (20)

which is negative so long as $\sigma_P \neq 0$. So by a similar intuition, greater idiosyncratic skewness of $P$ opposes the spread in the active strategy.

Part 3 implies that conversation especially encourages demand for securities with high skewness. Mitton and Vorkink (2007) document that underdiversified investors tend to choose stocks with high skewness—especially idiosyncratic skewness. Goetzmann and Kumar (2008) find that underdiversified investors tend to prefer stocks that are volatile and positively skewed.

Examples of skewed securities include options, and ‘lottery stocks’, such as loss firms (Teoh and Zhang (2011)) or real option firms that have a small chance of a jackpot outcome. Consistent with investors favoring positively skewed stocks, ex ante return skewness is a negative predictor of future stock returns (Conrad, Dittmar, and Ghysels (2009), Eraker and Ready (2013)). There is also evidence from initial public offerings (Green and Hwang (2012)) and general samples (Bali, Cakici, and Whitelaw (2009)) that lottery stocks are overpriced, and that being distressed (a characteristic that leads to a lottery payoff distribution) on average predicts negative abnormal returns (Campbell, Hilscher, and Szilagyi (2008)). Boyer and Vorkink (2011) find that the ex ante skewness of equity options is a negative cross-sectional predictor of option abnormal returns.

There are other other possible explanations for the overvaluation of skewness. In Brunnermeier and Parker (2005), agents who optimize over beliefs prefer skewed payoff distributions. In Barberis and Huang (2008), prospect theory preferences with probability weighting creates a preference over portfolio skewness. This induces a demand for ‘lottery’
(high idiosyncratic skewness) stocks by virtue of their contribution to portfolio skewness. Our approach differs in that there is no inherent preference for skewness. Instead, biases in the transmission process cause the purchase of lottery stocks to be contagious.

This difference results in distinct empirical implications about trading in lottery stocks. In our setting, greater social interaction increases contagion, thereby increasing the holdings of lottery stocks. For example, individuals with greater social connection (as proxied, for example, by population density, participation in investment clubs, or regular church-going) will favor such investments more.

Consistent with a possible effect of social contagion, individuals who live in urban areas buy lottery tickets more frequently than individuals who live in rural areas (Kallick et al. (1979)). Furthermore, there is evidence suggesting that the preference for high skewness stocks is greater among urban investors, after controlling for demographic, geographic, and personal investing characteristics (Kumar 2009).\(^{13}\)

For Part 4, differentiating with respect to active idiosyncratic volatility \(\sigma_A\) gives

\[
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \sigma_A} = 3a\beta\gamma_1A\sigma_A^2 + 2(B - 3aD\beta)\sigma_A
\]

if \(D \approx 0\) or \(D < 0\). Thus, if \(D\) is sufficiently small, the growth of \(A\) increases with active idiosyncratic volatility \(\sigma_A\). Greater return variance increases the effect of SET on the part of the sender. Although not required for the result, high salience to receivers of extreme returns (reflected in \(a\)) reinforces this effect, since it is the extreme high returns that are disproportionately communicated.\(^{14}\)

The finding of Goetzmann and Kumar (2008) discussed above, that underdiversified investors prefer stocks that are more volatile, is consistent with Part 4. A further empirical implication of Part 4 is that in periods in which individual stocks have high idiosyncratic volatility, \textit{ceteris paribus} there will be greater holding of and volume of trade in individual stocks. Intuitively, during such periods \(A\) have more large gains to report selectively.

\(^{13}\)Kumar (2009) empirically defines lottery stocks as stocks with high skewness, high volatility, and low price. His findings therefore do not distinguish the effects of skewness versus volatility.

\(^{14}\)By the same token, differentiating with respect to passive idiosyncratic volatility \(\sigma_P\) gives

\[
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \sigma_P} = -3a\beta\gamma_1P\sigma_P^2 - 2B\sigma_P
\]

if \(\sigma_P > 0\) and \(\gamma_1P \geq 0\) (or if \(\gamma_1P\) is not too negative). Passive idiosyncratic volatility encourages the spread of \(P\) at the expense of \(A\) owing to self-enhancing transmission bias, as reflected in the \(B\) term.

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This implication is in sharp contrast with the prediction of portfolio theory, which suggests that in periods of high idiosyncratic volatility, the gains to holding a diversified portfolio rather than trading individual stocks is especially large. Tests of this prediction, especially if a shift in idiosyncratic volatility can be attributed to fundamentals, would therefore distinguish competing theories.

The idiosyncratic volatility puzzle is the finding that stocks with high idiosyncratic risk earn low subsequent returns (e.g., Ang et al. (2006, 2009)). There is also evidence that this apparent overpricing is stronger for firms held more heavily by retail investors (Jiang, Xu, and Yao (2009), Han and Kumar (2013)), for whom we would expect conversational biases to be strong. Thus, the theory offers a possible explanation based upon social interactions for the idiosyncratic volatility puzzle: the high returns generated by volatile stocks are heavily discussed, which increases the demand for such stocks, driving up their prices.

A plausible individual-level explanation for these findings is that realization utility or prospect theory with probability weighting creates a preference for volatile portfolios and stocks (Barberis and Huang (2008), Boyer, Mitton, and Vorkink (2010)). However, consistent with social contagion playing at least some role, in tests using extensive controls, the preference for high volatility is greater among urban investors (Kumar (2009); see also footnote 13).

For Part 5, differentiating with respect to the factor loading of the active strategy, \( \beta_A \), gives

\[
\left( \frac{2n}{\chi} \right) \frac{\partial E[\Delta f]}{\partial \beta_A} = 3a \beta_A^2 \gamma_1 \sigma_r^2 + 2 \beta_A \sigma_r^2 B - 6a \beta_A \sigma_r^2 D > 0 \tag{23}
\]

if \( D < 0 \) or \( D \approx 0 \). So a greater factor loading for \( A \) increases the spread of \( A \), since the wider distribution of return outcomes encourages the sending of high, influential returns.\(^{15}\)

Baker, Bradley, and Wurgler (2011) report that in the U.S., high beta stocks have substantially underperformed low beta stocks over the past 41 years; Frazzini and Pedersen (2013) also find that high beta stocks underperform and low beta stocks overperform. This

\(^{15}\)By the same token, differentiating with respect to the factor loading of the passive strategy, \( \beta_P \), gives

\[
\left( \frac{2n}{\chi} \right) \frac{\partial E[\Delta f]}{\partial \beta_P} = -3a \beta_P^2 \gamma_1 \sigma_r^2 - 2 \beta_P \sigma_r^2 B < 0. \tag{24}
\]

So a greater factor loading for \( P \) opposes the spread of \( A \), since the wider distribution of return outcomes encourages the spread of \( P \).
evidence is consistent with investors excessively favoring investment in stocks with high loadings.

For Part 6, differentiating with respect to the variance of the common factor, $\sigma_r^2$ gives

$$
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \sigma_r^2} = B(\beta_A^2 - \beta_P^2) > 0.
$$

(25)

So greater volatility of the common factor favors the spread of $A$. Greater factor volatility outcomes encourages the spread of the strategy with the greater loading, $A$, by creating greater scope for SET to operate.

This implies that, ceteris paribus, there will be greater stock market participation in time periods and countries with more volatile stock markets. This contrasts with the conventional theory, in which greater risk, ceteris paribus reduces the benefit to participation. Our theory therefore suggests that bubble periods attract greater investor participation in speculative markets in part because of, not despite, high market volatility.

Overall, the findings on factor loadings and the different components of volatility suggest that volatility will be overvalued in the economy. As such, the model does not offer any help in explaining the equity premium puzzle—the high returns on the U.S. equity market.

For Part 7, we differentiate with respect to $\beta$, the strength of SET. (This reflects how tight the link is between the sender’s self-esteem and performance). Recalling by (7) that $B$ is an increasing function of $\beta$, gives

$$
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial \beta} = a(\gamma_1 A \sigma_A^3 - \gamma_1 P \sigma_P^3) + a \sigma_r^2 (\beta_A^3 - \beta_P^3) \gamma_1 r + \big[ (\beta_A^2 - \beta_P^2) \sigma_r^2 + \sigma_A^2 - \sigma_P^2 \big] \\
+ D(-3a \beta A^2 - 3a \beta A^2 \sigma_r^2 - C) + D^2 B - aD^2 \beta \\
> 0
$$

(26)

if $D \approx 0$. So greater SET increases the evolution toward $A$, because SET causes greater reporting of the high returns that make $A$ enticing for receivers. $A$ generates high returns through higher factor loading, idiosyncratic volatility, or more positive idiosyncratic skewness.

For Part 8, differentiating with respect to the sensitivity of receivers to returns, $b$, gives

$$
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial b} = \beta[(\beta_A^2 - \beta_P^2) \sigma_r^2 + \sigma_A^2 - \sigma_P^2] + D(D\beta - \gamma) \\
> 0
$$

(27)
if \( D \approx 0 \). Greater sensitivity of receivers to returns helps \( A \) spread by magnifying the effect of SET (reflected in \( \beta \)), which helps \( A \) because of the higher volatility of \( A \) returns. The analysis therefore implies, for example, that active trading will be more popular when extrapolative beliefs are stronger (past returns are perceived to be more informative about the future).

For Part 9, recall that the quadratic term of the receiving function \( a \) reflects greater attention on the part of the receiver to extreme profit outcomes communicated by the sender. Differentiating with respect to \( a \) gives

\[
\left( \frac{2n}{\chi} \right) \frac{\partial E[\Delta f]}{\partial a} = \beta \sigma_r^3 \gamma_{1r} (\beta_A^3 - \beta_P^3) + \beta [\gamma_{1A} \sigma_A^3 - \gamma_{1P} \sigma_P^3] + \gamma [(\beta_A^2 - \beta_P^2) \sigma_r^2 + \sigma_A^2 - \sigma_P^2] \\
- 3D \beta (\beta_A^2 \sigma_r^2 + \sigma_A^2) + D^2 \gamma - D^3 \beta > 0 \tag{28}
\]

if \( D \approx 0 \). So greater attention by receivers to extreme outcomes, \( a \), promotes the spread of \( A \) over \( P \) because \( A \) generates more of the extreme returns which, when \( a \) is high, are especially noticed and more likely to convince the receivers to switch strategy. This effect is reinforced by SET, which causes greater reporting of extreme high returns.

For Part 10, differentiating with respect to conversability \( \gamma \) gives

\[
\left( \frac{2n}{\chi} \right) \frac{\partial E[\Delta f]}{\partial \gamma} = a [(\beta_A^2 - \beta_P^2) \sigma_r^2 + \sigma_A^2 - \sigma_P^2] - bD + aD^2 \\
> 0 \tag{29}
\]

if \( D \approx 0 \). Greater conversability \( \gamma \) helps the active strategy spread because of the greater attention paid by receivers to extreme returns (\( a > 0 \)). Extreme returns are more often generated by the \( A \) strategy. So an unconditional increase in the propensity to report returns tends to have a greater influence when the sender is \( A \).

The model therefore predicts that under the conditions of Part 10, active trading will tend to increase with unconditional increases in the tendency for people to talk about their investment performance. For example, the rise of communication technologies, media, and such social phenomena as ubiquitous computing, stock market chat rooms, investment clubs, and blogging should favor active trading. This suggests that the rise of these phenomena may have contributed to the internet bubble.

Also, trading outcomes are a trigger for conversation about trading, so over time as markets become more liquid and trading becomes more frequent, we expect conversation about outcomes to become more frequent. The trend toward greater availability of real-
time reporting and discussion of financial markets on television and through the internet should induce more rapid evolution toward more active investing.

If greater general sociability is associated with greater comfort in discussing performance information, then in any given conversation it increases the unconditional probability that the sender will discuss returns; i.e., it increases $\gamma$. So the model predicts that this will increase evolution toward active trading. In a study of 49 countries, Eleswarapu (2004) reports that stock market turnover is higher in countries with greater population density after controlling for legal and political institutions and income.

Also consistent with this implication is recent evidence that participation in online communities is associated with riskier financial decisions (Zhu et al. (2012)). Using field studies, the authors found greater risk-taking in bidding decisions and lending decisions by participants in “discussion forums” (Prosper.com) and in “discussion boards” and “chat rooms” (eBay.de). Risk-taking was increasing with how active the participants were in the community.

There is also survey evidence that differences in sociability within society affects stock market participation (Hong, Kubik, and Stein (2004)). The degree of social interactiveness is measured by self-reports of interacting with neighbors and of attending church. Socials were more likely to invest in the stock market after controlling for wealth, race, education and risk tolerance. Furthermore, survey evidence from ten European countries indicates that household involvement in social activities increases stock market participation (Geor- garakos and Pasini (2011)).

As discussed in the introduction, another reasonable way to interpret the active versus passive distinction is that active strategies are more conversable (less conventional, more affect-triggering, or more arousing). This distinction could be incorporated formally by replacing $\gamma$ in the sending function with $\gamma_A$ and $\gamma_P$, where $\gamma_A > \gamma_P$. However, the model generates a survival advantage for $A$ even without a conversability advantage. Since it is clear that $\gamma_A > \gamma_P$ favors the spread of $A$, we simply assert this conclusion while maintaining the simplicity of a single $\gamma$ for the remaining analysis. Intuitively, $\gamma_A > \gamma_P$, ceteris paribus, causes adopters of $A$ to evangelize to $P$s more often than the other way of around, which favors evolution of the population toward $A$.

Lastly, for the unnumbered final statement of the Proposition, differentiating with respect to the susceptibility of receivers $c$ gives

$$\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f]}{\partial c} = -D\beta$$

$$> 0$$

(30)
if $D < 0$; the inequality is reversed if $D > 0$. The reason is that greater susceptibility increases the likelihood that the receiver is transformed given that the sender sends. Owing to SET (as reflected in the $\beta$ term above) the probability that $A$ sends is increased relative to the probability that $P$ sends when the returns of $A$ are increased owing to $D < 0$. (This condition will hold if there is a risk premium for the active strategy, even if the premium is not fully commensurate with the risk.) An implication of this is that when there are stronger pressures toward conformity (hence, more susceptible receivers), there is a stronger tendency for the culture to evolve toward the active strategy.

The model further predicts that there will be overvaluation of stocks with ‘glamour’ characteristics that make them attractive topics of conversation, such as growth, recent IPO, sports, entertainment, media, and innovative consumer products (on underperformance of IPO and small growth firms, see Loughran and Ritter (1995) and Fama and French (1993)). In contrast, there will be neglect and underpricing of unglamourous firms that are less attractive topics of conversation, such as business-to-business vendors or suppliers of infrastructure. Conversational transmission biases can therefore help explain several well known empirical puzzles about investor trading and asset pricing.

Related predictions about the effects of investor attention have been made before (Merton (1987)). A distinctive feature here is that the prediction is based on social interaction. These effects should therefore be stronger in times and places with higher social interaction. This point provides additional empirical predictions about the effects on return anomalies of population density, urban versus rural localities, pre- and post-internet periods, national differences in self-reported degrees of social interaction, popularity of investment clubs and chat rooms, and so forth.

### 2.7.2 The Evolutionary Success of Active Investing

The intuition underlying the comparative statics provides insight into the basic issue of whether evolution favors $A$ or $P$. The next proposition follows immediately by (16) and the parameter constraints of the model ($\beta_A > \beta_P \geq 0$, $\sigma_A^2 > \sigma_P^2$, $\gamma_{1A} > 0$, $\gamma_{1P} \approx 0$, and $\gamma_{1r} \geq 0$).

**Proposition 3** If the return penalty to active trading $D$ is sufficiently close to zero, then under the parameter constraints of the model, on average the fraction of active investors increases over time.

This comes from a combination of effects. Based on the previous comparative statics, owing to SET, the spread of $A$ over $P$ is favored by its higher factor loading $\beta_i$, idiosyncratic
skewness $\gamma_i$, and idiosyncratic volatility $\sigma_i$. A strategy that is more volatile (either because of greater loading on a factor or because of idiosyncratic risk) magnifies the effect of SET in persuading receivers to the strategy.$^{16}$ Owing to greater attention to extremes ($a > 0$), skewness (which generates strongly noticed high returns) further reinforces the success of A, but SET promotes the spread of A even if $a = 0$.

An additional direct extrapolation effect which does not rely on SET further promotes the success of A. This further effect only operates if $a > 0$ (salience of extreme news). If $a = 0$, in the absence of SET ($\beta = 0$), and if the expected returns of the two strategies are the same, the transformation of P investors to A resulting from overextrapolation by receivers of high returns is exactly offset by transformations in the other direction when returns are low. Even though receivers believe that past performance is a strong indicator of future strategy value, investors attend equally to the good and bad returns produced by the strategies. So the expected change in the fraction of active investors from a meeting is zero.$^{17}$

However, if $a > 0$, the receiving function is convex, so that high returns have a stronger effect on the upside than low returns have on the downside. Owing to the higher variance of A, it generates extreme returns more often, which intensifies this favorable effect.

To see these effects algebraically, eliminate SET in the model by setting $\beta = 0$. Then the expected change in frequency of A is, up to a multiplicative constant,

$$E[T_{AP}(R_A) - T_{PA}(R_P)] = a\gamma[\beta^2_A - \beta^2_P]\sigma_r^2 + (\sigma^2_A - \sigma^2_P)] - Db\gamma.$$

Setting aside the mean return term $Db\gamma$, we see that even without SET, there tends to be growth in the frequency of A if there is attention to extremes ($a > 0$; the direct attention-to-extremes effect). However, there is no inherent tendency for high skewness strategies to spread. This can also be seen from the comparatives statics of equations (18)-(20), in which the effects of skewness are eliminated when $\beta = 0$.

In summary, SET promotes A owing to its higher variance and (if $a > 0$), its higher skewness; the direct extrapolation effect promotes A only when $a > 0$, and only via a variance effect, not a skewness effect.

$^{16}$Since the population size $n$ is finite, the population fraction evolves stochastically. However, if the population size is large, over many generations $G$ (i.e., letting both the number of generations $G$ and $n/G$ grow large) almost surely the fraction of type A will grow.

$^{17}$More generally, when expectations are extrapolative, whichever strategy has higher mean return will, ceteris paribus, tend to spread owing to the persuasiveness of higher returns. However, this does not generate the key results that active strategies in the sense of high variance, skewness, and personal engagement tend to spread, all else equal (e.g., for given expected returns).
In general, models in which there is contagious adoption of innovations lead to S-shaped adoption curves (Griliches (1957), Young (2009)). Although we do not formally derive it, there is a similar implication here when a new form of active trading becomes available. In contrast, other forms of adoption (such as independent trial and error experimentation) in general do not yield S-shaped adoption curves (Henrich (2001)).

Individual investors are probably relatively strongly influenced by social interactions rather than independent analysis and investigation. This suggests that the predictions of Propositions 2 and 3 that social interaction favors active investing will apply more strongly to individual investors than to professionals.

2.7.3 An Alternative Specification for the Receiving Function

We have assumed throughout that a receiver is influenced only by the sender’s return, not own return. This assumption may seem unrealistic, but should matter little for qualitative conclusions, as each type has an equal chance of becoming the sender.

Here we examine an alternative specification where the receiving function depends on the difference between the returns of the sender and the receiver. Specifically, we now assume that when a sender of type $A$ meets with a receiver of type $P$ and communicates return $R_A$, $P$ is converted to type $A$ with probability $r_{AP}(R_A) = a(R_A - R_P)^2 + b(R_A - R_P) + c$. Under this specification, the expected change in frequency is, up to a multiplicative constant,

$$\left(\frac{2n}{\chi}\right) E[\Delta f] = E[T_{AP}(R_A) - T_{PA}(R_P)]$$

$$= a\beta[(\beta_A - \beta_P)(\beta_A^2 + \beta_P^2)\gamma_{1A}\sigma_r^3 + \gamma_{1A}\sigma_A^3 - \gamma_{1P}\sigma_P^3] + B'[\beta_A^2 - \beta_P^2]\gamma_{1P}\sigma_r^2 + (\sigma_A^2 - \sigma_P^2)]$$

$$+ Da\beta[-3\sigma_A^2 - 3\sigma_P^2 - D^2 - 3\sigma_r^2(\beta_A - \beta_P)^2] + D^2 B' - DC',$$  \hspace{1cm} (32)

where

$$B' = b\beta$$

$$C' = 2b\gamma + c\beta.$$  \hspace{1cm} (33)

We have verified that Propositions 1, 2 (except the last item about the effect of conversability) and 3 remain valid when the receiving function depends on the return difference between the sender and the receiver.
2.8 Local Bias and Familiarity Bias

Proximate and familiar events and issues are attractive topics of conversation. This fascination is also reflected in local reporting in the news media. In experimental studies of deliberation, people talk more about information that is already shared than about information that is unique to an individual (e.g., Stasser and Titus (1985), Stasser, Taylor, and Hanna (1989)). People also prefer to find common ground in conversations by discussing jointly familiar topics (Fast, Heath, and Wu (2009)). So in meetings with fellow locals, we expect locally familiar firms to be perceived as legitimate and attractive conversation topics.

A slight extension of the model to include local and non-local investors generates local investment biases. In such a setting, the high conversability of local stocks to local investors, combined with the tendency of local investors to talk to each other, promotes local stock holding and trading.

Consider a setting in which there are two assets, an individual stock and the market portfolio, and in which some investors (‘locals’) are located near the firm’s headquarters and some (‘outsiders’) are not. Locals find the individual stock more conversable than do outsiders. Let $A$ be investing in the individual stock, and $P$ be holding the market portfolio.

Assume further that locals and outsiders never talk to each other, so that we can apply the basic model separately to locals and outsiders. (We do not consider how equilibrium prices are determined by the local and outside supplies and demands for the stock.) Then the higher conversability of the local stock implies a stronger tendency for the local than for the outsider population to evolve toward $A$. For some parameter values, the analysis of earlier sections implies that local investors invest in the local stock and outsiders do not.

**Result 1** In the model of this subsection, there exists a non-empty open set of parameter values such that the local population evolves toward $A$, and the outsider population will evolve toward $P$.

To see this, consider the values $\beta_A = 1.2, \beta_P = 1, \sigma_r = 0.2, \sigma_A = 0.4, \sigma_P = 0$, and $\gamma_{1r} = \gamma_{1A} = \gamma_{1P} = 0$. These parameter values do not seem empirically implausible. Let the parameters for the sending and receiving functions be $a = 5, b = 1, c = 0.5$, and $\beta = 5$.

Let the two populations differ only in the conversability parameter $\gamma$; let $\gamma = 0.6$ for local investors and $\gamma = 0.2$ for outsiders. Then $E[T_{AP}(R_A) - T_{PA}(R_P)] = 0.1692$, $E[T_{AP}(R_A) - T_{PA}(R_P)] = -0.0310$. Thus, by (16), on average the fraction of investors within the local population who hold the individual stock increases, whereas for outsiders the population shifts towards holding the market portfolio.
So avoidance by outsiders of a stock need not be driven by any fear, aversion, or unawareness on their part about it. A high conversability of local stocks to locals, together with a propensity for people to be persuaded by others in conversation, is enough to drive home bias. In contrast with existing theories of home bias, our theory implies that the shape of the social network and other social characteristics of market participants are key determinants of home bias.

There is evidence of familiarity and local biases in investing, as with home bias in favor of domestic over foreign stocks (Tesar and Werner (1995)). Huberman (2001) provides evidence that investors tend to choose locally familiar stocks. The analysis here implies that such effects will be stronger when social interaction is more geographically localized. This suggests that the rise of new forms of internet communication such as social networking websites should reduce local bias.

Finally, this analysis can be reinterpreted as reflecting clustering in a conversational social network in which linkage is based on conceptual rather than geographical affinity. We can view locals as those who are more familiar with an investment, and outsiders as those who are less familiar with it. If we further assume that those who are familiar with it are especially prone to talking about it to each other as compared with outsiders, a similar implication follows that those who are more familiar with the investment invest more in it.

For example, employees often voluntarily hold company stock, sacrificing diversification. It seems quite plausible that a firm’s employees talk to each other—even to geographically distant fellow employees—more than they talk to random outside members of the population. So conversational bias can explain the company stock puzzle.

3 Equilibrium Trading and Returns

So far our, in our notion of activity versus passivity, A can refer to either some static action such as holding a given risky asset, or to a general dynamic pattern of investing, such as day trading, margin investing, stock picking, market timing, sector rotation, dollar cost averaging, technical analysis, and so forth. To derive equilibrium implications for trading and prices, we now specialize the interpretation of A to refer to placing a higher valuation than $P$ upon risky assets.

In the next subsection, we consider the case of a single risky asset. This provides the groundwork for Subsection 3.2, in which there are many risky assets, each investor invests in only one of them per period, and individuals are randomly selected in pairs for conversation. In this setting, by the law of large numbers across investors and assets, the
evolution of the population is, in the limit, deterministic.

3.1 Active and Passive Returns

We assume that there is a riskfree asset and a risky asset, each of which generates a terminal value one period later and liquidates. The riskfree asset is in zero net supply, with return denoted $R_F$. The risky asset $S$ has a terminal value $V$ which is optimistically perceived by $A$ to have expected value $V_A$, and by the $P$s to be $V_P$, where $V_A > V_P$. Both types agree about the variance of risky asset return. A new realized terminal value is redrawn independently each period, and investors myopically optimize each period based upon their current beliefs. We distinguish agent expectations, denoted by $E_i[]$ or ‘bar’ variables with subscripts, from true expectations, denoted by unscripted $E[]$.

An investor receives one unit of the numeraire at the start of each period, which he invests fully in the two assets; he consumes the payoffs of these assets at the end of each period. Letting $w_A$ and $w_P$ be the portfolio weights placed by each type on the risky asset, the returns achieved by an $A$ or $P$ are

$$R_A = (1 - w_A)R_F + w_A R_S$$
$$R_P = (1 - w_P)R_F + w_P R_S.$$  (34)

The investors’ mean-variance decision problem is

$$\max_{w_i} E_i[R_i] - \left( \frac{\nu}{2} \right) var(R_i), \quad i = A, P,$$  (35)

where $\nu$ is the coefficient of absolute risk aversion, and the $i$ subscript on the expectation reflects the different beliefs of $A$ and $P$ about the expected value of the risky asset. Both perceive the return variance to be $\sigma_S^2$.

Let $\bar{R}_S$ denote the expectation by type $i$ of the return on the risky asset. Substituting for the $R_A$ and $R_P$ from (34) gives the optimization problems

$$\max_{w_i} (1 - w_i)R_F + w_i \bar{R}_S - \left( \frac{\nu}{2} \right) w_i^2 \sigma_S^2, \quad i = A, P.$$  (36)

Differentiating with respect to $w_i$ and solving gives

$$w_i = \frac{\bar{R}_S - R_F}{\nu \sigma_S^2}, \quad i = A, P.$$  (37)

Substituting for the $w$’s in (34) using (37) gives

$$R_A - R_F = \lambda(R_P - R_F),$$  (38)
where

\[ \lambda \equiv \frac{\bar{R}_{SA} - R_F}{\bar{R}_{SP} - R_F}. \]  \hspace{1cm} (39)

It follows by (39) that

\[ R_A = \lambda R_P + (1 - \lambda)R_F \]
\[ = \lambda R_P + (1 - \lambda)E[R_P] - D \]  \hspace{1cm} (40)

where

\[ D \overset{\text{def}}{=} (1 - \lambda)w_P(E[R_S] - R_F). \]  \hspace{1cm} (41)

Taking the expectation of both sides of (40), we see that \( D \) is the return penalty to active trading, in analogy with equation (13) of the basic model. In (39), since \( A \)s are more optimistic than \( P \)s about the risky asset, either \( \lambda > 1 \) (if its denominator is positive) or \( \lambda < 0 \) (if its denominator is negative). If the risky asset is not too overpriced, it will earn a positive risk premium over the riskfree rate and will be perceived to do so by the \( P \)s, implying \( \lambda > 1 \). It follows that \( D < 0 \), a negative return penalty to active trading. If the risky asset is so overpriced that its expected return is below the riskfree rate, and the \( P \)s rationally perceive this to be the case, then there is a positive return penalty to active trading, \( D > 0 \).

3.2 Market Equilibrium

Each period each individual is newly endowed with one unit of the numeraire to invest, and that individuals consume all their investment payoffs at the end of each period, at which point the old assets vanish and new investment opportunities appear. Assume the per capita supply of risky asset is normalized to be one unit of the numeraire. Then the per capita market clearing condition requires that

\[ fw_A + (1 - f)w_P = 1. \]  \hspace{1cm} (42)

The expected return as perceived by type \( i \) is \( \bar{R}_{Si} = (\bar{V}_i - p)/p \), \( i = A, P \), so substituting for the \( w_i \)'s from (37), and solving for the price of the risky asset \( p \) gives

\[ p = \frac{f\bar{V}_A + (1 - f)\bar{V}_P}{1 + \nu\sigma_s^2 + R_F}. \]  \hspace{1cm} (43)
By (43) and the definition of return,

\[ \bar{R}_{SA} = \frac{V_A}{p} - 1 = \frac{(1 - f)(V_A - V_P) + V_A(\nu\sigma^2_S + R_F)}{fV_A + (1 - f)V_P} \]

(44)

and

\[ > R_F \]  

(45)

since \( V_A > fV_A + (1 - f)V_P \).

Similar steps yield

\[ \bar{R}_{SP} = \frac{f(V_P - V_A) + V_P(\nu\sigma^2_S + R_F)}{fV_A + (1 - f)V_P} \]

(46)

Since the first term in the numerator is negative and the second is positive, depending upon parameter values \( \bar{R}_{SP} \) can be greater or less than \( R_F \). Since the passives are less optimistic, they view the risky asset as overpriced, so they underweight it relative to the holdings of the actives, \( w_P < w_A \). However, since the risky asset is in positive net supply, there is aggregate risk from holding it. So the Ps may still regard it as commanding a positive expected return premium as long as the As are not too optimistic (which would drive the price too high). Specifically, by (37) and (46), \( w_P > 0 \) are both possible.

### 3.3 Evolutionary Dynamics and the Stable Fraction of Active Investors

To derive the dynamics of the fraction of actives, we now extend the equilibrium setting to allow for many ex ante identical stocks with independent normal returns. The analysis can be viewed as providing an equilibrium foundation for the partial equilibrium model of earlier sections, for the special case of the return assumptions in (13) where there is no factor risk (\( \beta_A = \beta_P = 0 \) and zero skewness).

Models in evolutionary game theory often hypothesize an infinitely large population of interacting agents, usually represented as a continuum. These models often assign revision protocols to agents describing how their behaviors change in response to what they observe, and derive deterministic evolutionary dynamics for the system, in the form of differential or difference equations for population shares (Sandholm (2010)). Changes in shares are viewed as averages over large numbers of individual strategy switches.

In this approach, randomness at the individual level is caused by the matching and the switching processes. In our model, there is an additional source of randomness: the payoffs.
of the strategies. In this section we assume that the risks of the stocks held by different individuals are diversifiable, so that the system evolves deterministically.

Each individual holds only one stock, each of which is held by many investors. $A$ and $P$ refer to greater or lesser optimism about the given stock that the investor considers. The independence across stocks implies that the payoffs to the $A$ and $P$ strategies are independent across those investors who trade in different stocks.

Pairs of individuals are randomly selected for social interactions from the entire population (not just holders of a given stock). With many stocks and many investors per stock, random fluctuations in returns and the randomness of the pairings are diversified away, so by the law of large numbers the fraction converting to $A$ evolves deterministically. The interior stable fraction adopting $A$ reflects a balance between two forces. On the one hand, owing to SET, the $A$ strategy tends to spread. On the other hand, when overoptimism is prevalent, risky securities become overpriced, and therefore tends to generate lower returns; such returns do not attract emulation.

Specifically, we assume that there are $N = nm$ investors, $m$ risky assets (stocks) with identical and independently distributed payoffs, and that each stock has $n$ investors, where we will take the limit as $m$ and $n$ becomes large. During each period, each investor holds only one stock, along with the riskfree asset. The assumption that diversification is incomplete is in the spirit of theories of limited investor information processing in which investors hold only subsets of available assets (Merton (1987)); we take this to an extreme for simplicity. Based on his beliefs, in equilibrium even a $P$ takes a non-null position in his risky stock, as in the previous subsection.

In each period $t$, a fraction $f_t$ of the investors in each stock are active, meaning that they hold overly optimistic belief $\overline{V}_A$ about the mean stock payoff. The $P$ investors in each stock hold the correct belief $\overline{V}$. Our assumption that the $A$s are mistaken tilts the model against the spread of $A$, but we will see that $A$ still survives.

Each period investors are randomly assigned to stocks such that the fraction of $A$ investors in each stock is equal. With many investors, we can ignore discreteness issues in such assignments.

Under these assumptions, we can analyze equilibrium trading in each stock independently based on that stock’s investor base, so we omit notation to identify the specific stock. Risky assets start anew each period, so there is no repeated learning about the prospects of a given asset. In addition, investors do not draw inferences from price when forming demand for risky assets.

Let the degree of optimism of a type $A$ investor for his chosen stock be denoted by
$\kappa = \nabla_A / \nabla - 1 > 0$, where $\kappa$ is constant over time. For simplicity of notation and without loss of generality, we set the riskfree rate $R_F = 0$, and set the parameter $a$ for the nonlinear term in the receiving function to zero (i.e., we eliminate the special salience of extreme news).\footnote{The parameter $a$ was only needed for the model’s skewness predictions, but the return distributions in this section are Gaussian. Allowing for $a > 0$ makes expressions more cumbersome without additional insight. We have verified that Proposition 4 still holds when $a > 0$.}

Since stocks payoffs are independent and the investor bases of different stocks do not overlap, at each time $t$ the analysis of market equilibrium is, stock by stock, identical to that in Sections 3.1 and 3.2. The optimal holdings $w_A, w_P$, and the true equilibrium expected return $E(R_S)$ all decrease with the fraction $f$ of A investors in the population, because active (overoptimistic) investors drive up the equilibrium price, $p$, of each stock:

$$p_t = \left( \frac{1 + f_t \kappa}{1 + \nu \sigma^2_S} \right) \nabla \tag{47}$$

$$E_t(R_S) = \frac{\nu \sigma^2_S - f_t \kappa}{1 + f_t \kappa} \tag{48}$$

$$w_{At} = \frac{(1 + \kappa)\nu \sigma^2_S + (1 - f_t)\kappa}{(1 + f_t \kappa)\nu \sigma^2_S} \tag{49}$$

$$w_{Pt} = \frac{\nu \sigma^2_S - f_t \kappa}{(1 + f_t \kappa)\nu \sigma^2_S}, \tag{50}$$

where the quantities above are the same for all stocks.

Each period, a fraction $\theta$ of the investors are randomly selected to meet simultaneously in pairs without regard to type or to which stocks they are holding. When $N$ is large, the probability that a mixed pair is drawn (one A, one $P$) is almost exactly $2f(1 - f)$, and almost surely they hold different stocks. In the next period, anyone who is converted to A becomes optimistic about whatever stock he is assigned to, and anyone who is converted to $P$ acquires objective beliefs about whatever stock he is assigned to.

So during a given period $[t, t + 1]$, there are $\theta N$ meetings of pairs of individuals. Let the meetings be indexed by $\omega$, and let $\zeta$ denote the function that specifies which stock is held by each individual in a given mixed meeting, as a function of the meeting and each investor’s type. In a given meeting $\omega$ between an $A$ holding stock $\zeta_{A\omega}$ and a $P$ holding stock $\zeta_{P\omega}$, the returns experienced by these individuals are

$$R_{A\omega} = w_{At} R_{S\zeta_{A\omega}}, \quad R_{P\omega} = w_{Pt} R_{S\zeta_{P\omega}}. \tag{51}$$

The sending and receiving functions are as in Section 2. So after $\theta N$ simultaneous meetings of pairs during a given period $[t, t + 1]$, by (10) the change in the population frequency of
A is (up to a stochastic error that shrinks with $N$), and suppressing $t$ subscripts on return variables,

$$f_{t+1} - f_t = \left( \frac{X}{2N} \right) \theta_N \sum_{\omega=1}^{\theta_N} T_{AP}(R_{A\omega}) - T_{PA}(R_{P\omega})$$

$$= \left( \frac{X}{2N} \right) \theta_N \sum_{\omega=1}^{\theta_N} B \left( R_{A\omega}^2 - R_{P\omega}^2 \right) + C(R_{A\omega} - R_{P\omega}),$$

where $B = b\beta$ and $C = b\gamma + c\beta$. Substituting equations (49)-(51) into the above gives

$$f_{t+1} - f_t = \left( \frac{X}{2N} \right) \frac{1}{N} \sum_{\omega=1}^{\theta_N} B(w_{At}^2 R_{S\xi\omega}^2 - w_{Pt}^2 R_{S\xi\omega}^2) + C(w_{At} R_{S\xi\omega} - w_{Pt} R_{S\xi\omega}).$$

Letting $n$ and $m$ (and hence the population size $N = nm$) approach infinity, we obtain a deterministic dynamic for the fraction of actives. Since stock returns are identical and independently distributed, we can apply the law of large numbers to the summation on the RHS of the last equation above, replacing each term being averaged with its expectation. Then the dynamic for the population frequency of $A$ becomes

$$\frac{f_{t+1} - f_t}{f_t(1 - f_t)} = \theta(w_{At} - w_{Pt})\{B(w_{At} + w_{Pt})[E_t(R_S)^2 + \sigma_S^2] + CE_t(R_S)\}, \quad (52)$$

where $E_t(R_S)$ is given in equation (50), and where by (49) and (50), $w_{At} - w_{Pt}$ and $w_{At} + w_{Pt}$ can be expressed as functions of $f_t$,

$$w_{At} - w_{Pt} = \frac{\kappa(1 + \nu \sigma_S^2)}{(1 + f_t \kappa) \nu \sigma_S^2}, \quad (53)$$

$$w_{At} + w_{Pt} = \frac{(2 + \kappa) \nu \sigma_S^2 + (1 - 2f_t) \kappa}{(1 + f_t \kappa) \nu \sigma_S^2}. \quad (54)$$

By (52), the net conversion rate from $P$ to $A$, $(f_{t+1} - f_t)/f_t$, is a decreasing function of $f_t$. Specifically, it is the product of $1 - f_t$ and the right hand side of (52), which is a decreasing function of $f_t$, since $w_{At} - w_{Pt}$, $w_{At} + w_{Pt}$ and $E_t(R_S)$ all decrease with $f_t$.

When $f$ is small, the stocks have positive expected return premia, which means that strategy $A$ earns high expected returns relative to $P$, i.e., $D < 0$. In this circumstance, other investors tend to convert to $A$ (becoming optimistic) as they hear about the high returns experienced using $A$. However, as the fraction of $A$s becomes sufficiently large, the expected return premium on the risky asset declines or even turns negative, which limits the spread of $A$.

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19 If $E[R_S] - R_F > 0$, then $w_p > 0$ and $\lambda = w_A/w_P > 1$, and $D \equiv (1 - \lambda)w_p(E[R_S] - R_F) < 0$.
So long as the optimistic belief is not too extreme \( \kappa < \nu \sigma^2_S \), \( E(R_S) \) is positive, and so is the right hand side of (52), regardless of the value of \( f \) \( (0 \leq f \leq 1) \). Thus, owing to SET, the fraction of \( A \) increases indefinitely, and type \( A \) dominates the population.

However, if the \( A \)s have sufficiently optimistic beliefs \( \kappa > \nu \sigma^2_S \), as \( f \) grows the risky assets become highly overpriced, driving \( E(R_S) \) negative. This results in an expected return penalty to \( A \). As the actual and reported returns on \( A \) diminish, so does the net conversion rate from \( P \) to \( A \), which becomes negative when the fraction \( f \) of active investors becomes too large. So when the \( A \) belief is sufficiently optimistic, there exists a stable fraction \( f^* \in (0, 1) \) of type \( A \) such that if \( f = f^* \), the net conversion rate from \( P \) to \( A \) is zero; if \( f < f^* \), \( \Delta f > 0 \); and if \( f > f^* \), \( \Delta f < 0 \).

By equation (52) and the fact that \( w_{At} - w_{Pt} \) given in (53) is always positive, the stable fraction \( f^* \) satisfies \( H(f^*) = 0 \), where

\[
H(f) \equiv B(w_{At} + w_{Pt})[E_t(R_S)^2 + \sigma^2_S] + CE_t(R_S).
\] (55)

Substituting equations (48)-(50) into the above, \( H(f^*) = 0 \) can be equivalently written as \( G(f^*) = 0 \), where

\[
G(f) \equiv B[(2+\kappa)\nu \sigma^2_S + (1-2f)\kappa][(\nu \sigma^2_S - f \kappa)^2 + \sigma^2_S(1+f \kappa)^2] + C \nu \sigma^2_S (1+f \kappa)^2 (\nu \sigma^2_S - f \kappa).
\] (56)

\( G \) is a cubic polynomial in \( f \), with \( G(0) > 0 \). When the \( A \)s have sufficiently optimistic beliefs, i.e., when \( \kappa > \nu \sigma^2_S \) is sufficiently large, \( G(1) < 0 \), and the discriminant of \( G \) is negative, so that there exists a unique \( f^* \in (0, 1) \) satisfying \( G(f^*) = 0 \).

The expected return premium on the risky assets at the stable fraction \( f^* \) must be negative, because the net conversion from \( P \) to \( A \) is still positive when \( E(R_S) = 0 \). Intuitively, SET would cause conversion to \( A \) until everyone adopts \( A \), unless \( A \) has an offsetting adverse effect on expected returns that opposes such conversion. We summarize these results as follows.

**Proposition 4** If the type \( A \) investors are sufficiently optimistic, then there exists a stable fraction \( f^* \in (0, 1) \), such that \( \Delta f > 0 \) for \( f < f^* \), and \( \Delta f < 0 \) for \( f > f^* \). Corresponding to \( f^* \), the expected return premium on the risky asset is negative.

Since the stable fraction \( f^* \) of \( A \) satisfies \( H(f^*) = 0 \), we can use use (55) to calculate how \( f^* \) varies with key model parameters. It is straightforward to verify that

\[
\frac{\partial H}{\partial f} < 0, \quad \frac{\partial H}{\partial \kappa} < 0, \quad \frac{\partial H}{\partial c} < 0, \quad \frac{\partial H}{\partial \gamma} < 0.
\] (57)
The first two inequalities in (57) derive from the fact that \( E(R_S) \) decreases with the fraction of actives and with their optimism \( \kappa \). The greater the fraction of actives, or the more optimistic they are about stock payoffs, the more that stocks are overpriced. The last two inequalities in (57) result from the fact that the expected return on the risky asset is negative at the stable fraction of actives.

Applying the implicit function theorem to \( H(f^*) = 0 \) using the inequalities in (57) gives

\[
\frac{\partial f^*}{\partial \kappa} < 0, \quad \frac{\partial f^*}{\partial c} < 0, \quad \frac{\partial f^*}{\partial \gamma} < 0. \tag{58}
\]

We therefore have:

**Proposition 5** If the type A investors are sufficiently optimistic, the stable fraction \( f^* \) of A decreases with optimism \( \kappa \), conversability \( \gamma \), and receiver susceptibility, \( c \).

Intuitively, greater overoptimism causes stocks to become more overpriced (for given frequency of A. The low returns that they experience limit the spread of A.

Greater susceptibility, \( c \), helps transform the receiver only if the sender actually sends. In equilibrium A earns lower expected returns than P; owing to SET, this reduces the sending probability by type A relative to P, which reduces the probability that the receiver is converted.

Owing to the lower equilibrium expected returns of A, greater conversability \( \gamma \) causes greater spread of bad news about A rather than P. This reduces the stable frequency of A.

Proposition 2 Part 10 and the unnumbered final statement of Proposition 2 provided conditions under which the frequency of A increases with \( \gamma \) and \( c \). A required condition is that A earn sufficiently high expected returns compared to P (i.e., \( D < 0 \)). Here A earns lower expected returns than P, \( D > 0 \), which implies opposite comparative statics.

The conclusion that \( D > 0 \) here comes from our simplifying assumption that the susceptibility of receivers, \( c \), is the same regardless of whether the sender was an A or a P, so that the probability that a receiver is converted depends only on the sender’s return. However, a more plausible assumption is that a risk averse receiver who recognizes that A is riskier than P will be less willing to convert, for any given return, if the report came from an A. For example, a report of a 4% annual return might be much more attractive if it is about a riskfree asset than about a risky tech IPO. So we would expect receivers to be less susceptible to messages that come from an A. This would be reflected by having the receiver susceptibility parameter \( c \) in the receiving function be lower if the sender was an A than a P, \( c_{SA} < c_{SP} \).
All the asset pricing equations in Section 4.1 and 4.2 continue to hold when the receiver susceptibility parameter depends on the sender type. Under this assumption, the dynamic for the population frequency of $A$ becomes

$$\frac{f_{t+1} - f_t}{\theta f_t(1 - f_t)} = B(w^2_A - w^2_P)[E_t(R_S)^2 + \sigma^2_S] + b\gamma E_t(R_S) + \beta(c_{SA}w_A - c_{SP}w_P)E_t(R_S) + (c_{SA} - c_{SP})\gamma. \tag{59}$$

The stable frequency $f^*$ now satisfies $H'(f^*) = 0$, where

$$H'(f) \equiv B(w^2_A - w^2_P)[E_t(R_S)^2 + \sigma^2_S] + b\gamma E_t(R_S) + \beta(c_{SA}w_A - c_{SP}w_P)E_t(R_S) + (c_{SA} - c_{SP})\gamma. \tag{59}$$

If the difference in susceptibility is sufficiently large,

$$c_{SP} - c_{SA} > \frac{B\kappa(2 + \kappa)(1 + \nu\sigma^2_S)}{\gamma\nu}, \tag{60}$$

then the RHS of (59) is negative, and hence the net conversion from $P$ to $A$ is negative, when evaluated at the fraction of $A$ corresponding to $E(R_S) = 0$. Since both the net conversion rate from $P$ to $A$ and $E(R_S)$ decrease with $f$, it follows that when the difference in susceptibility is sufficiently large, the expected return premium on the risky assets at the stable fraction $f^*$ is positive. Intuitively, a lower receiver susceptibility for messages from $A$ handicaps $A$ in spreading through the population.

So, when $c_{SA} < c_{SP}$, the effect of SET, which favors spread of $A$, is opposed by the lower susceptibility of receivers to messages from As. If the handicap is large enough, the population dynamic has a stable frequency in which the net conversion from $P$ to $A$ becomes zero, even when the equilibrium expected return of the risky assets is positive. In such an equilibrium $A$ earns a higher risk premium than $P$. The conclusion that in equilibrium there will be a positive risk premium seems plausible for many applications.

Given the positive equilibrium risk premium corresponding to the stable fraction $f^*$ of $A$ in this circumstance, it follows that

$$\frac{\partial H'}{\partial f} < 0, \frac{\partial H'}{\partial \kappa} < 0, \frac{\partial H'}{\partial \gamma} > 0, \frac{\partial H'}{\partial b} > 0, \frac{\partial H'}{\partial c_{SA}} > 0, \frac{\partial H'}{\partial c_{SP}} < 0. \tag{61}$$

Applying the implicit function theorem to $H'(f^*) = 0$ and the inequalities in (61) gives

$$\frac{\partial f^*}{\partial \kappa} < 0, \frac{\partial f^*}{\partial \gamma} > 0, \frac{\partial f^*}{\partial b} > 0, \frac{\partial f^*}{\partial c_{SA}} > 0, \frac{\partial f^*}{\partial c_{SP}} < 0. \tag{62}$$

We therefore have:
Proposition 6 When the receiver susceptibility to messages from A is sufficiently lower than P (satisfying (60)), the expected return premium on the risky asset is positive, at the stable fraction $f^*$ of A. The stable fraction $f^*$ of A decreases with optimism $\kappa$, increases with conversability $\gamma$, sensitivity of receptiveness to return, $b$, and receiver susceptibility to messages from A, $c_{SA}$, and decreases with receiver susceptibility to messages from P, $c_{SP}$.

In summary, the differences in comparative statics predictions in the equilibrium setting as compared with that of Section 2.7.1 derive from the fact that in equilibrium the expected return on a risky asset is below the riskfree rate.\textsuperscript{20} In the analysis with $c_{SA} = c_{SP}$, this is a necessary condition for an interior stable fraction of A, as otherwise, owing to SET, the fraction of A would grow toward one. However, when $c_{SA} < c_{SP}$ by a sufficiently large amount, a positive risk premium is restored, and the comparative statics in the equilibrium setting is consistent with the earlier comparative statics in Proposition 2.

For example, intuitively, greater sensitivity $b$ of receivers to returns tends to promote A by magnifying the effect of SET, which is biased in favor of A since A generates the extreme high returns that SET can operate on. This is the effect highlighted in Part 8 of Proposition 2. A possible opposing effect again is that if A earns lower returns on average, which discourages emulation, then greater sensitivity of receivers magnifies this effect. However, if $c_{SA} < c_{SP}$ by enough to make the risk premium positive, A in equilibrium earns a higher expected return, so that there is no opposing effect. Thus, as stated in Proposition 6, the stable population frequency of A increases with sensitivity $b$ of receivers when $c_{SA} < c_{SP}$ by enough to make the risk premium positive.

3.4 Trading Volume

We now generalize the previous model to provide implications for volume of trade, by allowing differences in optimism about the risky asset among A investors. We develop our conjectures heuristically here. One interpretation of A versus P considered previously is that the As are unduly optimistic about a given risky security, whereas the Ps are not. An alternative interpretation which we focus on here is that A has a more general belief about whether stock picking or market timing are worthwhile activities. Investors who believe that these are worthwhile will conduct analyses that provide them with signals about the value of the risky asset. In contrast, passive investors by assumption share some common

\textsuperscript{20}For Proposition 2 Part 10, the conclusion of a positive effect of conversability on $f$ does not actually require that A earn a higher expected return than P, because $a > 0$ contributes positively (see equation (29)). Since $a = 0$ in the equilibrium analysis, we have excluded that effect here by assumption.
prior belief (some conventional view prevalent in society), do not investigate further, and hence remain in agreement. In consequence, actives form divergent beliefs about the asset whereas passive investors do not.

We therefore allow the As to have heterogeneous expectations about the value of the risky asset,

$$\nabla^{A_k} = \nabla^A + \psi^k, \tag{63}$$

where $k$ refers to an individual type A investor, $\nabla^{A_k}$ is the expectation of investor $k$ of the terminal cash flow of the security, $\psi^k$ is uniformly distributed on an interval $[-u, u]$ which is centered at zero. The parameter $u$ captures the amount of disagreement among the As.

Owing to the diversity of perceptions among the As, they trade with each other. This effect tends to cause volume of trade to increase with the frequency of $A$ in the population. However, owing to the difference in belief between As and Ps, there is also trading between individuals of different types.

The diversity of the As makes the analysis of evolution of the population more complex. To simplify, we let $u$ approach zero, so that the evolution of the population is arbitrarily well approximated by a setting in which the As are identical.

We additionally let the difference in beliefs of the As and Ps become arbitrarily close to zero more rapidly than $u$ does, i.e., $(\nabla_A - \nabla_P)/u \to 0$. This captures in extreme form the idea that the dispersion in beliefs among the actives is wider than the difference between the average beliefs of the As and the Ps. Under this assumption, volume is dominated by trading amongst A’s rather than between types. As a result, as the frequencies of the different types shift, the qualitative prediction that volume of trade increases with the fraction of As remains valid.

Thus, under appropriate conditions, the comparative statics predictions from Proposition 2 about the conditions under which the expected fraction of As grows also provide predictions about what determines increased trading volume. Similarly, the earlier analysis of local bias in investment also implies greater trading of stocks that are geographically local or otherwise familiar to investors.

As discussed in Subsection 2.3, the basic model implies a frothy churning of beliefs as investing ideas are transmitted from person to person. Even if $A$ does not end up dominating the population, stochastic fluctuation in population fractions of $A$ and $P$ is a continuing source of turnover. In consequence, the model implies excessive volume of trade even in the absence of overconfidence, and that such volume is increasing with proxies for social connectedness.
4 Concluding Remarks

Individual investors often invest actively, increasing their risk without commensurate increase in return. Social interaction seems to exacerbate the bias toward active trading. In the model presented here, biases in the social transmission of behaviors favor active over passive trading strategies.

We argue that the spread of investment strategies can be understood in terms of sender and receiver functions for the transmission of information about the strategies and their performance. These functions describe the factors that cause an investor to talk about an investment idea, or to be receptive to such an idea upon hearing about it. In the model, senders’ propensity to communicate their returns to receivers, and receivers’ propensity to be converted, are increasing in sender return. Receivers’ propensity to attend to and be converted by the sender is increasing and convex in sender return.

Owing both to the multiplicative effect of these increasing functions, and the convexity of the receiving function, the rate of conversion of investors to active investing is convex in sender return. Unconditionally, active strategies (high variance, skewness, and personal involvement) dominate the population unless the mean return penalty to active investing is too large. Thus, the model can explain overvaluation of ‘active’ asset characteristics even if investors have no inherent preference over them. The model also can explain several empirical puzzles about investor trading and offers implications about how social interaction affects several asset pricing anomalies and volume of trade.

Conversations are influenced by chance circumstances, subtle cues, and even trifling costs and benefits to the transactors. This suggests that small variations in social environment can have large effects on economic outcomes. For example, the model suggests that a shift in the social acceptability of talking about one’s successes, or of discussing personal investments more generally, can have large effects on risk taking and active investing. It would be interesting to model feedback from the frequency of active investors to the acceptability of discussing one’s investment successes, which could result in multiple equilibria with different amounts of active investing.

Much of the empirical literature on social interaction in investment focuses on whether information or behaviors are transmitted, and perhaps on what affects the strength of social contagion. Our approach suggests that it is also valuable to measure how biases in the transmission process affect the relative success of different kinds of behaviors.

More broadly, the approach offered here illustrates the how cultural evolution can help explain stylized facts about investing and pricing. It would be interesting to extend the
approach to study shifts in popularity of different money management vehicles, such as mutual funds, ETFs, and hedge funds.

Our approach also offers a microfoundation for research on fluctuations in investor sentiment toward different kinds of investment strategies. For example, observers have often argued that social interactions contribute to bubbles. Notably, the millennial high-tech stock market boom coincided with the rise of investment clubs and chat rooms. If the sending and the receiving functions of our model depend on the sender’s return over multiple periods (rather than just current period return), there can be overshooting and correction. So our model, and more generally the social finance approach, offers a possible framework for modeling how the spread of investment ideas cause bubbles and crashes.
Appendices

A Endogenizing the Receiving and Sending Functions

We now consider explicitly the determinants of the sending and receiving functions, and derive the assumed functional forms endogenously.

A.1 The Sending Function

To derive a sending function that reflects the desire to self-enhance, we assume that the utility derived from sending is increasing with own-return. Conversation is an occasion for an individual to try to raise the topic of return performance if it is good, or to avoid the topic if it is bad. Suppressing \( i \) subscripts, let \( \pi(R, x) \) be the utility to the sender of discussing his return \( R \),

\[
\pi(R, x) = R + \frac{x}{\beta'},
\]

where \( \beta' \) is a positive constant, and random variable \( x \) measures whether, in the particular social and conversational context, raising the topic of own-performance is appropriate or even obligatory.

The sender sends if and only if \( \pi > 0 \), so

\[
s(R) = \Pr(x > -\beta'R|R) = 1 - F(-\beta'R),
\]

where \( F \) is the distribution function of \( x \). If \( x \sim U[\tau_1, \tau_2] \), where \( \tau_1 < 0, \tau_2 > 0 \), then

\[
s(R) = \frac{\tau_2 + \beta'R}{\tau_2 - \tau_1} = \frac{\tau_2}{\tau_2 - \tau_1} + \beta R,
\]

where \( \beta \equiv \beta'/(\tau_2 - \tau_1) \), and where we restrict the domain of \( R \) to satisfy \(-\tau_2/\beta' < R < \tau_1/\beta'\) to ensure that the sending probability lies between 0 and 1. This will hold almost surely if \(|\tau_1|, |\tau_2|\) are sufficiently large. Equation (66) is identical to the sending function (4) in Subsection 2.2 with

\[
\gamma \equiv \frac{\tau_2}{\tau_2 - \tau_1}.
\]

A.2 The Receiving Function

A convex increasing shape for the receiving function can derive from the combination of two effects: greater receiver attention to extreme return outcomes (inducing convexity),
and, conditional upon paying attention, greater persuasiveness of higher return. Greater attention to extreme outcomes can be captured by having receiver attention be a positive quadratic function of the sender’s return,

\[ A(R) = c_1 R^2 + c_2, \quad c_1, c_2 > 0. \]

Conditional on the receiver attending, assume that the receiver’s probability of converting to the sender’s type is an increasing linear function of sender return,

\[ B(R) = e_1 R + e_2, \quad e_1, e_2 > 0. \]

In other words, the receiver interprets sender return as providing information about the desirability of the sender’s strategy. This inference may be largely invalid, but is tempting, as reflected in the need for the standard warning to investors that “past performance is no guarantee of future results.” Overweighting of small samples (the law of small numbers) is a consequence of representativeness (Tversky and Kahneman (1974)), the tendency to expect similarity between the characteristics of a sample and the underlying population.

The law of small numbers should attenuate the degree to which a receiver discounts for a sender’s upward selection in reporting returns. A receiver who thinks that even a single return observation is highly informative will adjust less for sender suppression of bad news. For example, in the limiting case in which one return observation is viewed as conclusive about the strategy’s quality, selection bias notwithstanding, a favorable return report will be taken at face value.

As with the law of small numbers, the vividness of personal stories causes insufficient discounting of selection bias. People tend to neglect the abstract information contained in ‘base rates’ (general statistical information about the population) in favor of small samples of vivid cases (Borgida and Nisbett (1977)).

With these assumptions on the \( A \) and \( B \) functions,

\[ r(R) = A(R)B(R) \]

is a cubic function with positive coefficients. This implies positive coefficients on the quadratic Taylor approximation to \( r(R) \), as in equation (5) in Section 2.

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\(^{21}\)In the experiments of Hamill, Wilson, and Nisbett (1980), subjects were asked to rate a population (welfare recipients, or prison guards) for its characteristics after being exposed to a vivid case example involving a single member of the population. Exposure to the vivid example affected the views of subjects about the entire population even when subjects were told that the case was highly atypical of the population.
References


