Lab 5: Solutions

(a) primal: \( \min 2x_1 - 4x_2 \)
\[ \begin{align*}
\text{s.t.} & \quad x_1 + x_a \leq 1 \\
& \quad x_1 \geq 0
\end{align*} \]

\( \Rightarrow \)
\[ \begin{align*}
\text{s.t.} & \quad x_1 + x_a^+ - x_a^- \leq 1 \\
& \quad x_a^+, x_a^- \geq 0
\end{align*} \]

dual: \( \min y \)
\[ \begin{align*}
\text{s.t.} & \quad y \geq -2 \\
& \quad y \geq 4 \\
& \quad y \leq -4 \\
& \quad y \geq 0
\end{align*} \]
\( \Rightarrow \)
\( y = 4 \)
\( y \geq 0 \)
\( \Rightarrow \)
\( y^* = 4 \)

(b)
\( (5 - Ax^*)^T y^* = 0 \)
\( (A^T y^* - c)^T x^* = 0 \)
\( (4 \begin{bmatrix} 4 & -2 \\ -4 & -4 \end{bmatrix})^T x^* = 0 \)
\( \begin{bmatrix} 6 \\ 0 \end{bmatrix}^T \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = 0 \)
\( \Rightarrow \)
\( x_1^* = 0 \)

\( (1 - (x_1^* + x_3^+ - x_3^-)) y^* = 0 \)
\( (1 - (x_2^+ - x_2^-)) y = 0 \)
\( \Rightarrow \)
\( x_2^+ - x_2^- = 1 \)
\( \Rightarrow \)
\( x_2^* = 1 \)
c.) Dual is infeasible when $c_i < -4$

In this case, primal is unbounded (although, in general, primal could be infeasible).
Problem 2

5. Imagine a company creates carbohydrate, fiber, and protein pills to satisfy the army's dietary requirements. The dual seeks to maximize company's profit subject to constraints that pills are no more expensive than food items they are replacing.

ii) \( b^+ y^* = 2.08 \) (by strong duality)

iii) We must have \( y_a^* = 0 \) because optimal objective function value is insensitive to changes in \( b_2 \) around 40 (see plot above).