Kortum (1997 Ema): “Research, Patenting, and Technological Change”

Chad Jones
Stanford GSB
Overview

• Construct a growth model consistent with these facts:
  ◦ Exponential growth in research (scientists)
  ◦ No growth in the number of patents granted to U.S. inventors
    ⇒ large decline in Patents per Researcher
  ◦ Exponential growth in output per worker
Patents in the U.S.

Total in 2013: 302,000
U.S. origin: 147,000
Foreign share: 51%

1915 = 1950 = 1990 \approx 40,000
How does Kortum do this?

- Quality ladder model, a la Aghion and Howitt (1992)
  - Each idea is a proportional improvement in productivity (ten percent rather than ten units). E.g. \( q \equiv 1.10 \)

\[
Y_t = q^{N_t} K_t^\alpha L_t^{1-\alpha}, \quad A_t \equiv q^{N_t}
\]

\[
\log A_t = N_t \log q
\]

\[
\Rightarrow \frac{\dot{A}_t}{A_t} = \dot{N}_t \log q
\]

- Also, make ideas harder to obtain over time \( \Rightarrow \) it takes more and more researchers to discover the next idea
  - So TFP growth tied to growth in number of researchers. (Also, Segerstrom 1998 AER)
Quality Ladders (Aghion-Howitt / Grossman-Helpman)
Other Insights

• Search model
  ◦ Ideas = draws from a probability distribution
  ◦ All you care about is the best idea (Evenson and Kislev, 1976)

• Technical: Extreme Value Theory and Pareto Distributions
  ◦ Key to exponential growth is that the stationary part of the search distribution have a Pareto upper tail
  ◦ The probability of drawing a new idea that is 2% better than the frontier is invariant to the level of the frontier
  ◦ Incomes versus heights
Pr \left[ q > \theta \bar{q} \mid q > \bar{q} \right] = \theta^{-\alpha}

if \( F(q) \) is Pareto(\( \alpha \))
Extreme Value Theory

- Let $N$ be the number of draws from a distribution, and consider the distribution of the largest draw as $N \rightarrow \infty$.
- For a distribution with unbounded support, the max will go to infinity, so we have to “normalize” it somehow.
- Extreme Value Theorem (e.g. Galambos 1987) If a limiting distribution exists, then it takes one of three forms: Fréchet, Weibull, Gumbel.
- Kind of like the Central Limit Theorem (normalized mean is asymptotically normal).
Fundamental Example

- Suppose $x^*$ is your income, equal to the maximum of $N$ iid draws from some distribution $F(\cdot)$.
- What is the distribution of $x^*$?

$$G(z) \equiv \Pr[x^* < z]$$

$$= \Pr[x_1 < z] \cdot \Pr[x_2 < z] \cdot \ldots \cdot \Pr[x_N < z]$$

$$= F(z)^N.$$  

- Suppose $F(\cdot)$ is Pareto: $F(z) = 1 - (z/\gamma)^{-\alpha}$.

$$G(z) = \left(1 - (z/\gamma)^{-\alpha}\right)^N$$

But this goes to zero as $N \to \infty$. So we need to normalize somehow.
\[ G(z) = (1 - (z/\gamma)^{-\alpha})^N \]

- Recall \( e^y \equiv \lim_{N \to \infty} (1 + y/N)^N \). Now choose some normalized value \( y \) such that

\[
\left( \frac{z}{\gamma} \right)^{-\alpha} = \frac{y}{N}
\]

\[ \Rightarrow z = \gamma(y/N)^{-1/\alpha} \]

- Therefore

\[ G(y) = \Pr \left[ x^* < \gamma(y/N)^{-1/\alpha} \right] = (1 - y/N)^N \]

\[ \rightarrow e^{-y}. \]
• Now redefine $z \equiv \gamma y^{-1/\alpha}$. As $N$ gets large

$$
\Pr [x^* < N^{1/\alpha} z] = e^{-(z/\gamma)^{-\alpha}} = e^{-(1-F(z))}
$$

$$
\Rightarrow \Pr [N^{-1/\alpha} x^* < z] = e^{-(z/\gamma)^{-\alpha}}
$$

• And this is the Fréchet distribution!

• Therefore, as $N$ gets large

$$
E[N^{-1/\alpha} x^*] = \gamma \Gamma(1 - 1/\alpha)
$$

$$
\Rightarrow E[x^*] \approx N^{1/\alpha} \gamma \Gamma(1 - 1/\alpha)
$$

• So the maximum value scales as $N^{1/\alpha}$

• Note: If $F(x)$ does not have a Pareto upper tail, then the scaling is less than a power function of $N$. 
Model
The Economic Environment

Preferences

\[ U_0 = \int_0^\infty e^{-\rho t} \exp(\int_0^1 \log C_{jt} \, dj) \, dt \]

Production

\[ C_{jt} = q_{jt} \ell_{jt} \]

Resource constraint

\[ \int_0^1 \ell_{jt} \, dj + R_t = L_t = L_0 e^{nt} \]

Research

Poisson process, next slide
Research and New Ideas

- An idea is a quality \( q \sim F(q; K) \) and a sector \( j \sim \text{Uniform}[0,1] \)

- Discovery is a Poisson process

\[
\begin{align*}
R_t & \quad \text{Researchers} \\
R_t \, dt & \quad \text{Flow of new ideas per unit time} \\
R_t (1 - F(q; K)) \, dt & \quad \text{Flow of ideas that exceed quality level } q
\end{align*}
\]

- The length of time until an innovation occurs is exponentially distributed with parameter \( R_t \)

- \( K \) is cumulative stock of research ("knowledge")

\[
\dot{K}_t = R_t
\]
Key Assumption 2.1

\[ Pr(Q \leq q; K) \equiv F(q; K) = 1 - S(K)(1 - F(q)) \]

\[ H(q) \equiv 1 - F(q) = Pr(Q > q) \]

\[ \tilde{H}(q; K) \equiv 1 - F(q; K) = Pr(Q > q; K) = S(K)H(q) \]

- \( S(K) \): Spillover function. Ex: \( S(K) = K^\gamma \)

- \( F(q) \): Stationary search distribution

- If \( \gamma = 0 \), then \( F(q; K) = F(q) \)

- As \( K \) grows, more of the mass is concentrated at higher values of \( q \).
Proposition 2.1

- The distribution $G_1$ of the state of the art productivity for producing in sector $j$ is, for a fixed $K$,

$$G_1(z; K) = \exp\{- (1 - F(z))\Sigma(K)\}$$

where $\Sigma(K) \equiv \int_0^K S(x)dx$ (cumulative spillovers).

- Remarks
  - $G_1(z; K)$ is an Extreme Value Distribution
  - Also the distribution of max productivity across sectors.
  - Research enters through $K$.
  - With Poisson process, things aggregate nicely.
Proof

- $G_1(z)$ is probability frontier is less than $z$
- What is probability that no discovery occurs?

$$Pr(\text{No discovery}) = e^{-R(s)ds}$$

$$Pr(\text{No discovery} \geq z) = e^{-R(s)(1-F(z;K(s)))ds}$$

$$\Rightarrow$$

$$G_1(z; K(s + ds)) = G_1(z; K(s))e^{-R(s)(1-F(z;K(s)))ds}$$

- Integrate this differential equation to get the result.
 Allocation of Resources

• An allocation in this economy is \( \{R_t, \{\ell_{jt}\}\} \).

• To see many of the useful results, we can focus on a Rule of Thumb allocation:

\[
R_t = \bar{s}L_t
\]

\[
\ell_{jt} = \bar{\ell} = (1 - \bar{s})L_t
\]

• Optimal to allocate labor equally across sectors given symmetry.
Analyzing the Economy

Constant patents with growing research?
Constant Patents??

• What fraction of new ideas are improvements (patentable)?

\[ p(K) = \int_{q_0}^{\infty} (1 - F(z; K)) \, dG_1(z; K) \]

prob idea exceeds \( z \)

• Substituting for \( G_1(\cdot) \) and making a change of variables \( x \equiv S(K)(1 - F(z)) \) when integrating gives

\[ p(K) = \frac{S(K)}{\Sigma(K)} \cdot (1 - e^{-\Sigma(K)/S(K)}) \]

• The fraction of new ideas that will be improvements depends on the spillover function
  ○ Independent of the stationary search distn \( F(q) \)!
Remarks on $p(K) = \frac{S(K)}{\Sigma(K)} \cdot (1 - e^{-\Sigma(K)/S(K)})$

- Independence of $F(q)$ is well-known in theory of recordbreaking (example: track and field)
  - Depends on the rate at which the stationary distribution shifts out (better shoes, track, nutrition)
  - Partial intuition: the distn of records itself depends on $F(\cdot)$, but what fraction get broken depends on how quickly we march down the tail

- Example: $S(K) = K^\gamma \Rightarrow \Sigma(K) = K^{1+\gamma}/(1 + \gamma)$
  $\Rightarrow S/\Sigma = (1 + \gamma)/K$

  $$p(K) = \frac{1 + \gamma}{K} \cdot (1 - e^{-K/(1+\gamma)})$$

  $\Rightarrow$ Looks like $1/K$ for $K$ large and $\gamma = 0$...
Glick (1978): Math of Record-Breaking

• Begins with a very simple example...

• Consider a sequence of daily weather observations — temperatures

• The first is obviously a record high

• The 2nd has a 50% chance of being a record (viewed before any data are recorded)

• Exchangeability: The probability that day \( n \) is a record is \( 1/n \)

• Independent of the distribution of temperatures.
Patenting

- $R(t)$ ideas, $p(K)$ improve, so total patenting is

\[ I_t = R_t \cdot p(K_t) = R_t \frac{S(K_t)}{\Sigma(K_t)} \left(1 - e^{-\Sigma(K_t)/S(K_t)}\right) \]

So the rise in $R_t$ can be offset by a decline in $p(K_t)$.

- Proposition 3.1 says that for $I_t$ to be constant while $R$ grows at rate $n$, $S(K)$ must be a power function.

\[ I_t = R_t \frac{1 + \gamma}{K_t} \left(1 - e^{-K_t/(1+\gamma)}\right) \]

\[ \dot{K}_t = R_t \Rightarrow \frac{\dot{K}_t}{K_t} = \frac{R_t}{K_t} \rightarrow n \]

\[ I^* = n(1 + \gamma) \]
Analyzing the Economy

Exponential income growth with constant patents?
Productivity

- Given symmetry, a productivity index is

\[ A(K_t) \equiv \int_{q_0}^{\infty} zdG_1(z; K_t) \]

Proportional to output per worker.

- This does depend on the shape of \( F(q) \). Examples:

1. Pareto (incomes): \( H(q) = q^{-1/\lambda} \)
   \[ \Rightarrow A(K) = c_1 K^{\lambda(1+\gamma)} \]

2. Exponential (heights): \( H(q) = e^{-q/\lambda} \)
   \[ \Rightarrow A(K) = c_0 + c_1 \log K \]

3. Uniform (bounded): \( H(q) = 1 - q/\lambda \)
   \[ \Rightarrow A(K) = c_0 - \frac{c_1}{K^{1+\gamma}} \]
Growth Implications

- **Pareto:**
  \[
  \frac{\dot{A}_t}{A_t} = \lambda (1 + \gamma) \frac{\dot{K}_t}{K_t} \rightarrow \lambda (1 + \gamma)n
  \]
  Sustained exponential growth! \((G_1(\cdot)\) is Fréchet).

- **Exponential:**
  \[
  \dot{A}_t = c_1 \frac{\dot{K}_t}{K_t} \rightarrow c_1 n \Rightarrow \frac{\dot{A}_t}{A_t} \rightarrow 0
  \]
  Arithmetic growth, but not exponential growth.

- **Uniform:**
  \[
  K \rightarrow \infty \Rightarrow A \rightarrow c_0
  \]
  Stagnation — no long run growth.
Growth (continued)

- Whether or not model can sustain growth depends on the shape of the upper tail of \( f(q) \).

**Proposition 3.2:** As the stock of research \( K \) approaches infinity, the limiting form of the distribution of the technological frontier \( G_1 \) is either Fréchet, Gumbel, or Weibull (subject to normalizing sequences). In all three cases productivity satisfies 
\[
\lim_{K \to \infty} \left\{ \frac{(A_{k',K}/A_K) - 1}{(A_{k'',K}/A_K) - 1} \right\} < \infty \text{ for any } 0 < k'' < k' < \infty.
\]
If and only if the limiting form of \( G_1 \) is Fréchet does productivity satisfy
\[
\lim_{K \to \infty} \left( \frac{A_{kK}}{A_K} \right) = k^b
\]
for some \( b > 0 \) and for all \( k > 0 \). Stationary search distributions \( F \) leading to the Fréchet have unbounded upper support and satisfy 
\[
\lim_{x \to \infty} \left\{ \frac{(1 - F(xa))/(1 - F(x))}{(1 - F(xa))/(1 - F(x))} \right\} = a^{-b}, \text{ for all } a > 0 \text{ and for some } b > 0.
\]
If and only if the upper tail of \( f(q) \) is a power function, then exponential growth can be sustained.
\[
(A(kK)/A(K) = k^b).
\]
Remarks

- In a very different setup, Kortum gets the same result we got from the Romer model with $\phi < 1$: per capita income growth is tied to the rate of population growth.
  - $I_t = R_t \cdot p(K_t)$: growth in number of researchers is exactly offset by increased difficulty of finding a useful new idea.
  - If $F(q)$ is Pareto, then $H(q) = q^{-\alpha}$. So $\Pr(\text{Idea is a 5% improvement} \mid \text{Idea is an improvement})$ is constant.
    $\Rightarrow$ Ideas are proportional improvements (a la quality ladder models)
    $\Rightarrow$ a constant flow of patents is consistent with exponential growth.

- What about $\phi = 1$ case? Kortum emphasizes that there is no $F(\cdot)$ such that the limiting distribution yields $A = e^{\lambda K}$. 

Kortum (1997) – p. 28
Remarks (continued)

• Potential problem: total patents in U.S. is growing in recent decades?

• Kortum analyzes equilibrium and optimal allocations
  ◦ Knowledge spillovers mean the equilibrium may feature too little investment in research.
  ◦ Counterbalancing that is a business stealing effect: some of the new innovator’s profits come at the expense of existing entrepreneurs.
Applications

- Pareto and Fréchet distributions show up in many places now in economics

  - Zipf’s Law: Size = 1/Rank for cities, etc. (Gabaix 1999). \( \text{Pr}[\text{Size} > s] \sim 1/s \), which is Pareto with parameter = 1. (Pareto \( \Rightarrow \) exp growth and exp growth \( \Rightarrow \) Pareto!)

  - Eaton and Kortum (2002 Ema) on trade

  - Erzo Luttmer (2010) on the size distribution of firms

  - Several recent Lucas papers (with Alvarez, Buera, and Moll); Perla and Tonetti (2014)


  - Jones and Kim (2018 JPE) on top income inequality

Harhoff, Scherer, Vopel “Exploring the tail...”
Zipf’s Law for U.S. Cities (Gabaix 1999)
Zipf’s Law for U.S. Firms (Luttmer 2010)

For 1992, 2000, and 2006
Pareto Distribution for Top Incomes

INCOME RATIO: $\frac{\text{MEAN}(Y \mid Y > Z)}{Z}$

Kortum (1997) – p. 34